Lattice QCD Study of Exclusive Channels in the Muon HVP

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Fit Procedure

Computation on one physical M_π ensemble, $24^3 \times 64$, $a^{-1} = 1.015\ \mathrm{GeV}$

Local vector current operator:

- Local $\mathcal{O}_0 = \sum_x \bar{\psi}(x)\gamma_\mu \psi(x)$

Three 2π operators with $\mathcal{O}_{1,2,3}$ given by $\vec{p}_\pi \in \frac{2\pi}{L} \times \{(1,0,0),(1,1,0),(1,1,1)\}$

$$\mathcal{O}_n = \left| \sum_{xyz} \bar{\psi}(x) f(x-z) e^{-i\vec{p}_\pi \cdot \vec{z}} \gamma_5 f(z-y) \psi(y) \right|^2$$

Correlators arranged in a 4×4 symmetric matrix:

$\otimes$	$\mathcal{O}_0$	$\mathcal{O}_1$	$\mathcal{O}_2$	$\mathcal{O}_3$
$\mathcal{O}_0$	$C^{(2)}_\rho$	$C^{(3)}_{\rho\to\pi\pi}$	$C^{(3)}_{\rho\to\pi\pi}$	$C^{(3)}_{\rho\to\pi\pi}$
$\mathcal{O}_1$		$C^{(4)}_{\pi\pi\to\pi\pi}$	$C^{(4)}_{\pi\pi\to\pi\pi}$	$C^{(4)}_{\pi\pi\to\pi\pi}$
$\mathcal{O}_2$			$C^{(4)}_{\pi\pi\to\pi\pi}$	$C^{(4)}_{\pi\pi\to\pi\pi}$
$\mathcal{O}_3$				$C^{(4)}_{\pi\pi\to\pi\pi}$

Generalized EigenValue Problem (GEVP) to estimate overlaps & energies

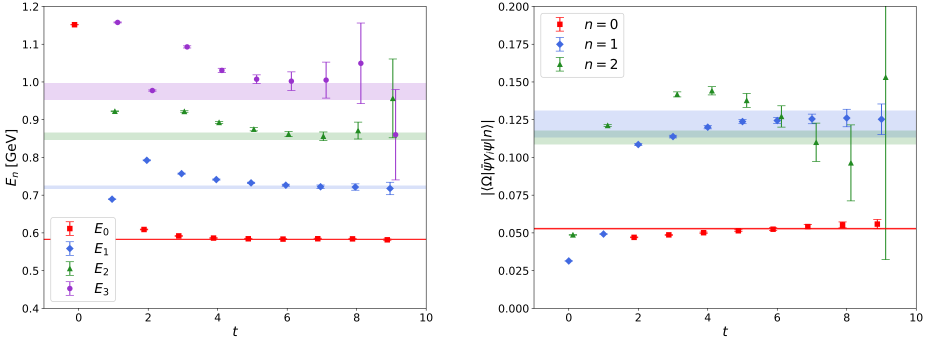
$$C(t)\,V = C(t+\delta t)\,V\,\Lambda(\delta t)\,; \quad \Lambda_{nn}(\delta t) \sim e^{+E_n \delta t}\,,\ V_{im} \propto \langle \Omega | \mathcal{O}_i | m \rangle$$

Full analysis to include correlated exponential fit to entire matrix basis:

$$C^{\mathrm{latt.}}_{ij}(t) = \sum_n^N \langle \Omega | \mathcal{O}_i | n \rangle \langle n | \mathcal{O}_j | \Omega \rangle e^{-E_n t}$$

In practice, only finite N necessary to model correlation function

Spectrum & Overlap



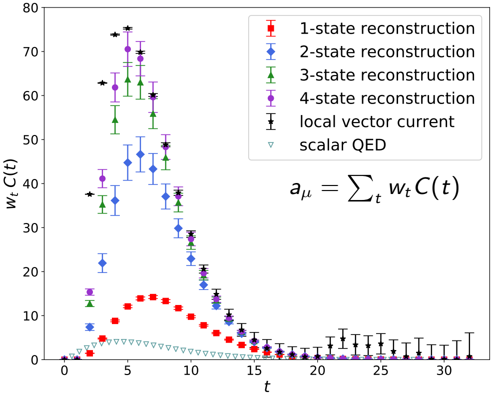
Data from GEVP; bands from correlated 5-state fit

Excited state contamination in GEVP gives curvature, not in clean asymptotic limit

Good agreement between GEVP and correlated fit, precise lowest state

Excited state systematics from comparing 5-state to 6-state fit; < statistics on HVP

Correlation Function Reconstruction

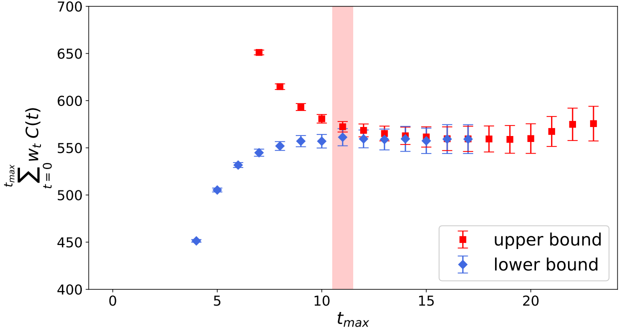


Correlated fit to reconstruct long-distance behavior of
local vector current correlation function $C(t)$

More states $\implies$ better reconstruction

Scalar QED underestimates lowest state ($\sim \pi\pi$)

Improved Bounding Method



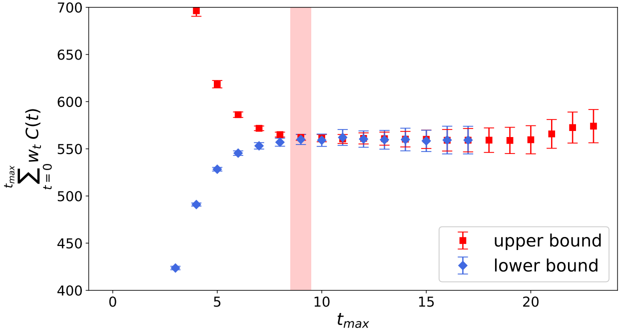
No bounding method: $a_\mu^{HVP} = 577(31)$

Bounding method $t_{\text{max}} = 2.1$ fm, no reconstruction: $a_\mu^{HVP} = 566.8(9.0)$

Very large lattice spacing: $a^{-1} = 1.015$ GeV, finite volume effects

Could expect $10 - 20\%$ discretization errors

Improved Bounding Method



No bounding method:	$a_\mu^{HVP} = 577(31)$
Bounding method $t_{\max} = 2.1\ \mathrm{fm}$, no reconstruction:	$a_\mu^{HVP} = 566.8(9.0)$
Bounding method $t_{\max} = 1.7\ \mathrm{fm}$, 1 state reconstruction:	$a_\mu^{HVP} = 561.0(4.3)$

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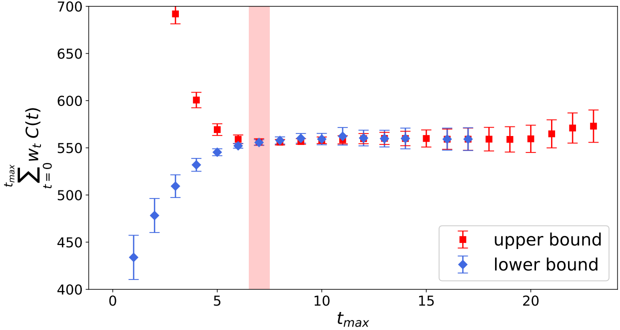

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Bounding method $t_{\max} = 1.7$ fm, 1 state reconstruction: $a_\mu^{HVP} = 561.0(4.3)$
Bounding method $t_{\max} = 1.4$ fm, 2 state reconstruction: $a_\mu^{HVP} = 555.9(2.9)$

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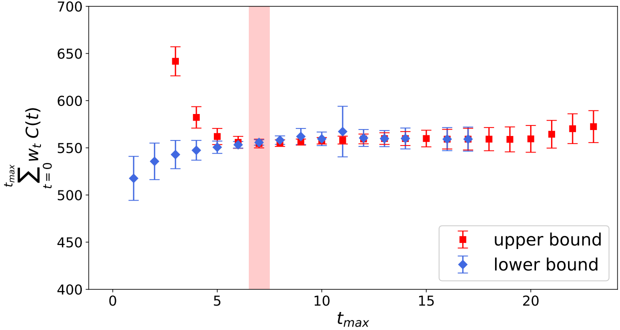
Improved Bounding Method



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Bounding method $t_{\text{max}} = 1.4$ fm, 2 state reconstruction:	$a_\mu^{HVP} = 555.9(2.9)$
Bounding method $t_{\text{max}} = 1.4$ fm, 3 state reconstruction:	$a_\mu^{HVP} = 555.1(3.3)$

Very large lattice spacing: $a^{-1} = 1.015$ GeV, finite volume effects
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BACKUP

Improved Bounding Method

Use known results in spectrum to make a precise estimate of upper & lower bound on a_μ^{HVP}

$$\widetilde{C}(t; t_{\max}, E) = \begin{cases} C(t) & t < t_{\max} \\ C(t_{\max}) e^{-E(t - t_{\max})} & t \geq t_{\max} \end{cases}$$

Upper bound: $E = E_0$, ground state of spectrum

Lower bound: $E = \log[\frac{C(t_{\max})}{C(t_{\max}+1)}]$

Good control over lower states in spectrum with exclusive reconstruction:

Replace $C(t) \to C(t) - \sum_n^N |c_n|^2 e^{-E_n t}$

$\implies$ Long distance convergence now $\propto e^{-E_{N+1} t}$

$\implies$ Smaller overall contribution from neglected states

Add back contribution from reconstruction after bounding correlator