

Hybrid Method with correlator bounding

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RBC-UKQCD Collaborations

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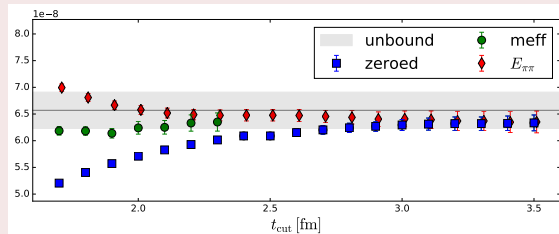


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Bounding method:[Lehner '15; Borsanyi et al '16, Blum et al '18]

Signal of the vector-vector correlation function deteriorates for large values of t . Replace noisy tail of the correlator by low and high bounds.

$$C_{\text{bound}}(t) = C(t_{\text{cut}})e^{-(t-t_{\text{cut}})E_b} \quad \text{for } t > t_{\text{cut}}$$

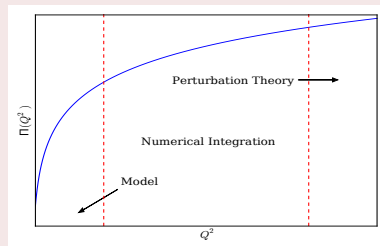


- ⇒ Vary t_{cut} , Smooth transition?
- ⇒ Different boundings (choices of E_b)
- ⇒ Improve bounds by GEVP?

Hybrid method:[Golterman, Maltman, Peris '14]

Largest contribution to a_μ from $Q^2 \sim m_\mu^2/4$.
Relative precision of $\Pi(Q^2)$ worst for small Q^2 .

⇒ **HYBRID METHOD**



Vary cuts in Q^2 , low Q^2 parameterisations (Padé vs Conformal Polynomials, different orders), matchings ((correlated?) fit vs discrete moments).

⇒ Done for strange contribution in [RBC/UKQCD '16]

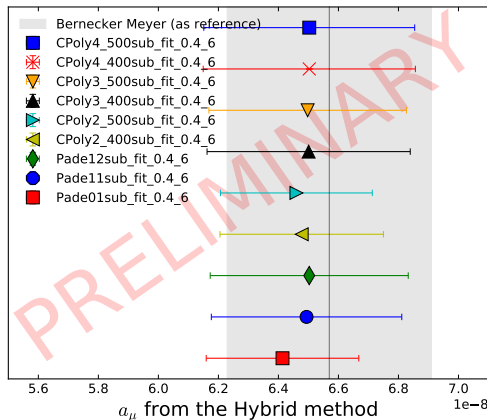
Consistency check via the Hybrid Method

Recent RBC/UKQCD paper [Blum et al '18] uses
Time-Momentum representation [Bernecker, Meyer '11]
⇒ Hybrid Method as additional consistency check.

⇒ Improvement in low Q^2 region?
⇒ Correlated fits improve error

Can we simultaneously profit from **bounding
method** and **hybrid method**?

Many knobs to turn from which we can gain insight
on systematics.

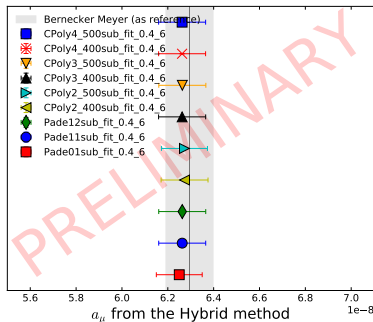


Hybrid method for unbound correlator (2+1 Iwasaki
DWF with $m_\pi = 139$ MeV, $a^{-1} = 1.7295(38)$ GeV).

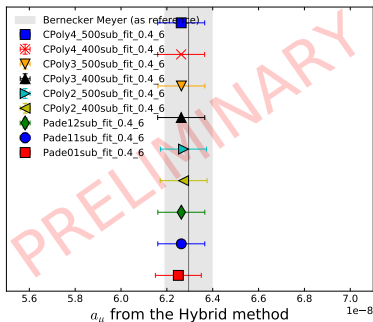
Preliminary hybrid method results

Replace raw correlation function $C(t)$ by

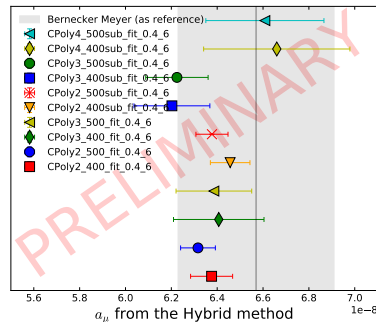
$$C_{\text{bound}}(t) = C(t_{\text{cut}})e^{-(t-t_{\text{cut}})E_b} \quad \text{for } t > t_{\text{cut}}$$



$C(t) = 0$ for $t > 3.0$ fm; low



$E_b = E_{\pi\pi}$ for $t > 3.0$ fm; high



Correlated fits (unbound).

All results for 2+1 Iwasaki DWF ($48^3 \times 96$) with $m_\pi = 139$ MeV, $a^{-1} = 1.7295(38)$ GeV

- Vary bounding parameters:
 - t_{cut}
 - bounding prescription
 - combine with GEVP study (\rightarrow A. Meyer)
- Vary hybrid parameters
 - $Q_{\text{low}}, Q_{\text{high}},$
 - Choices or low Q^2 parameterisations (Padé vs Conformal Polynomials)
 - The way their parameters are determined: moments matching vs fits, correlated vs uncorrelated
- Continuum Limit
- Combine with other contributions
- Full error budget

From lattice correlators to HVP

$$a_{\mu}^{\text{HVP,LO}} = 4\alpha^2 \int_0^{\infty} dQ^2 f(Q^2, m_{\mu}) \Pi(\hat{Q}^2)$$

$$\hat{\Pi}(Q^2) = \Pi(Q^2) - \Pi(0)$$

$$Q^2 \Pi(Q^2) = \sum_{t=-T/2}^{T/2-1} e^{-iQt} C(t)$$

$$\hat{\Pi}(Q^2) = \sum_{t=-T/2}^{T/2-1} \left(\frac{\cos Qt - \textcolor{red}{1}}{Q^2} + \frac{\textcolor{blue}{1}}{2} t^2 \right) C(t)$$

$$C(t) = \frac{1}{3} \sum_{i=1}^3 Z_V^2 \sum_f \sum_{\mathbf{x}} Q^f \left\langle V_i^f(\mathbf{x}, t) V_i^f(\mathbf{0}, 0) \right\rangle$$

zero-momentum and zero-mode subtraction.

Low Q^2 parameterisations

$$R_{mn}(\hat{Q}^2) = \Pi_0 + \hat{Q}^2 \left(\sum_{k=0}^{m-1} \frac{a_k}{b_k + \hat{Q}^2} + \delta_{mn} c \right)$$

$$R_{mn}^{\text{sub}}(\hat{Q}^2) = \hat{Q}^2 \left(\sum_{k=0}^{m-1} \frac{a_k}{b_k + \hat{Q}^2} + \delta_{mn} c \right)$$

where $n = m, m+1$

$$P_n^E(\hat{Q}^2) = \Pi_0 + \sum_{k=1}^n p_k w^k$$

where

$$w = \frac{1 - \sqrt{1-z}}{1 + \sqrt{1+z}} \quad z = \frac{\hat{Q}^2}{E^2}$$