HVP lattice finite-volume corrections

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Second Plenary Workshop of the Muon g-2 Theory Initiative

Helmholtz Institut Mainz

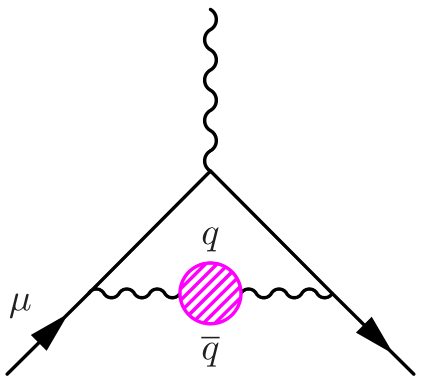
18th - 22nd June 2018

OUTLINE

- Motivations
- Current *status* from Collaborations

Motivations

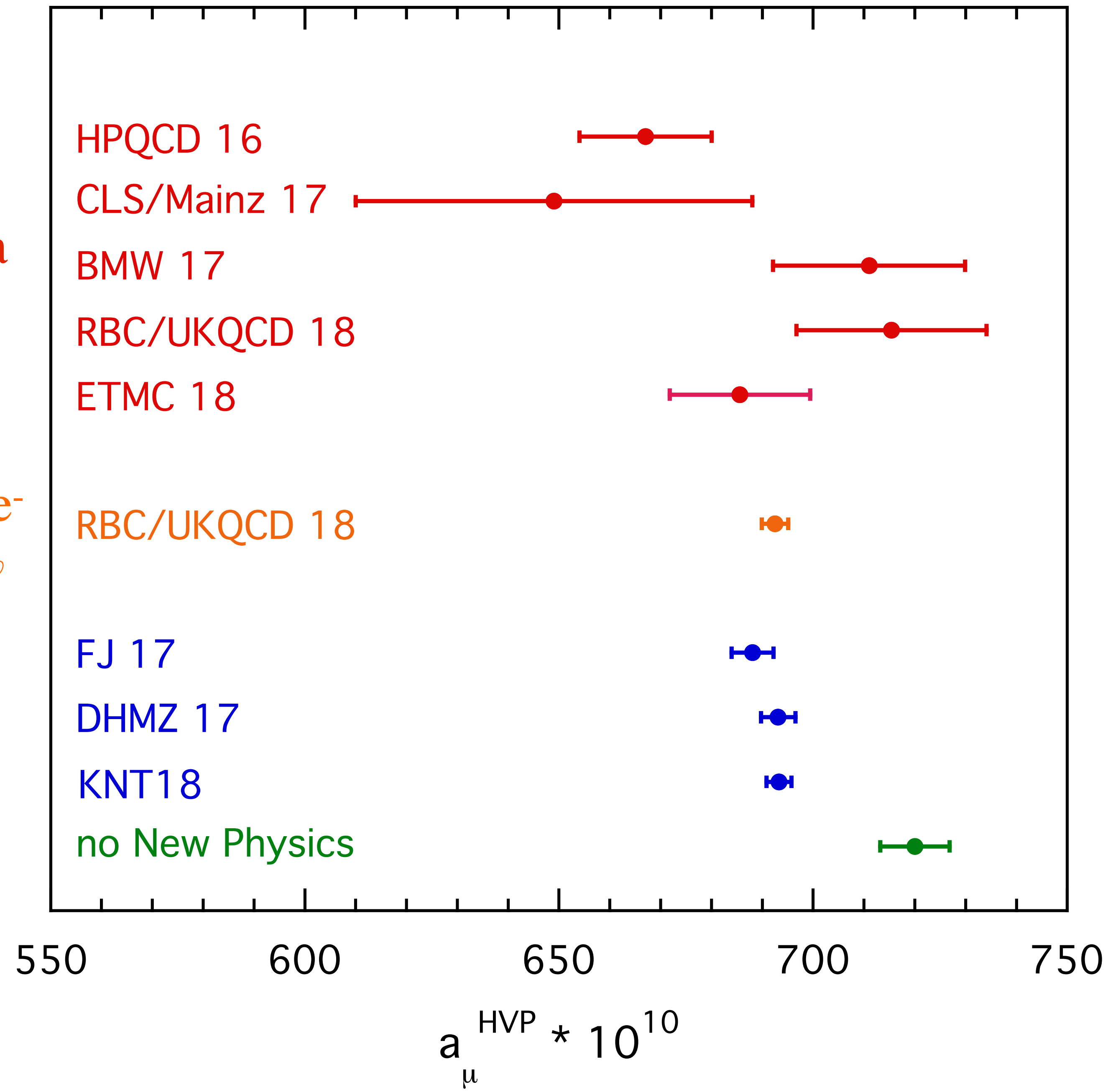
HVP of the muon



lattice data
100%

lattice + e^+e^-
~ 30% + 70%

e^+e^- data
100%



$\gtrsim$ 2%

$\gtrsim$ 0.4%

FVEs cannot be neglected

LO-HVP FVEs

- Chiral Perturbation Theory

  Blum et al. 2018; Borsanyi et al. 2017; Chakraborty et al. 2017

  Bijnens and Relefors 2017; Aubin et al. 2016

- Gounaris-Sakurai parameterisation + Lüscher formalism

  Della Morte et al. 2017

  ETMC, talk by S. Simula

  RBC/UKQCD
  talk by C. Lehner
  (new updates)

- Time-momentum representation

  Izubuchi et al. 2018

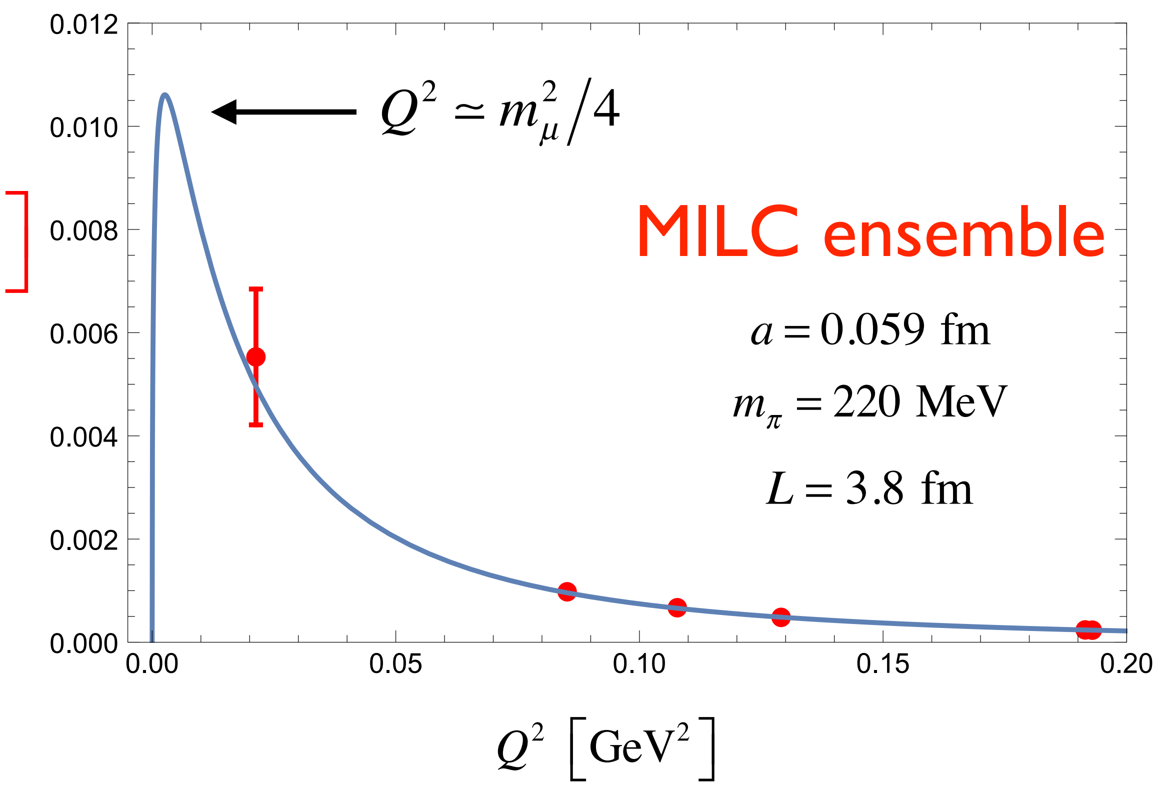
Current *status* from Collaborations

χPT Groups

Aubin et al. 2016

$$a_\mu^{\mathrm{LO,HVP}}\left[Q^2_{\max}\right] = 4\alpha^2_{em}\int_0^{Q^2_{\max}} dQ^2\, f(Q^2)\left[\Pi(Q^2)-\Pi(0)\right]$$

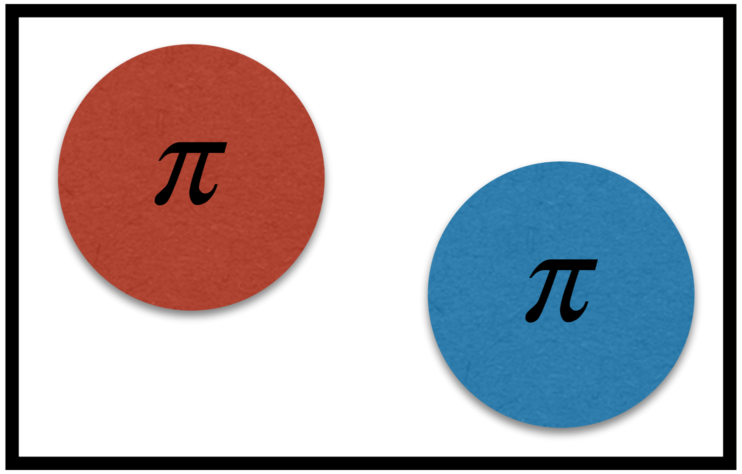
$$\Pi_{\mu\nu}(Q) = \left(Q^2\delta_{\mu\nu} - Q_\mu Q_\nu\right)\Pi(Q^2)$$



NLO χPT; PBCs

connected contribution

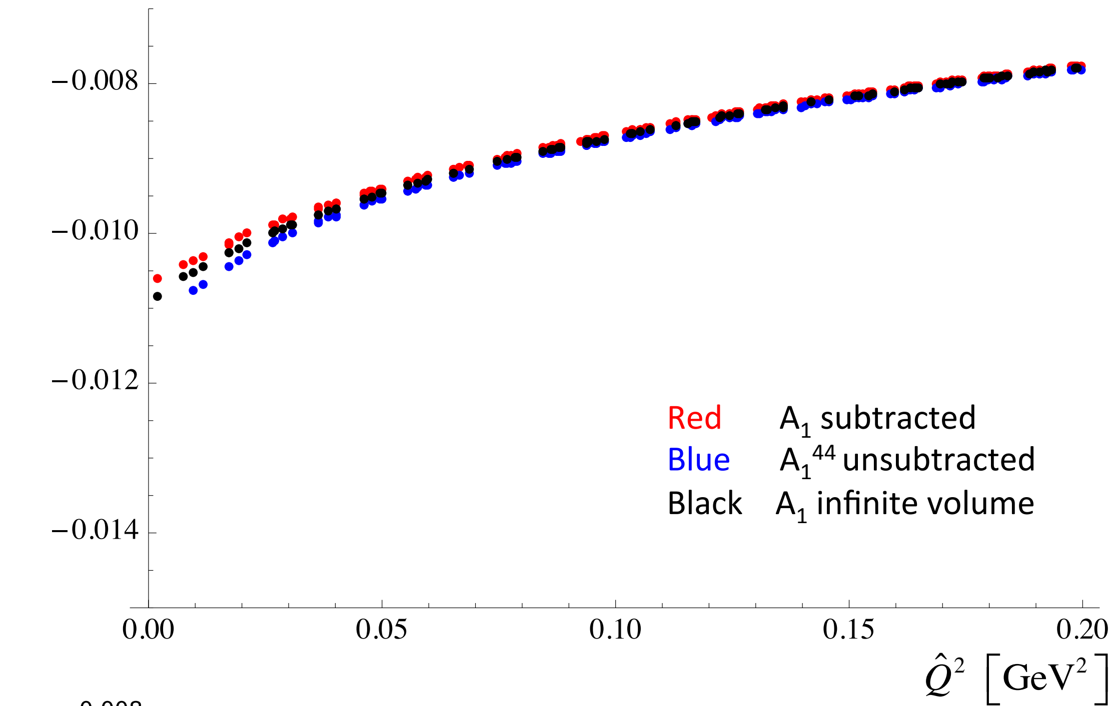
$$\Pi^{\mathrm{ChPT}}_{\mu\nu}(Q) = \frac{10}{9}\, 4\pi\alpha_{em}\Bigg[4\,\frac{1}{L^3T}\sum_p \frac{\sin(p+Q/2)_\mu \sin(p+Q/2)_\nu}{\left(2\sum_\kappa(1-\cos p_\kappa)+m_\pi^2\right)\left(2\sum_\kappa(1-\cos(p+Q)_\kappa)+m_\pi^2\right)} - 2\,\delta_{\mu\nu}\,\frac{1}{L^3T}\sum_p\left(\frac{\cos p_\mu}{\left(2\sum_\kappa(1-\cos p_\kappa)+m_\pi^2\right)}\right)\Bigg]$$

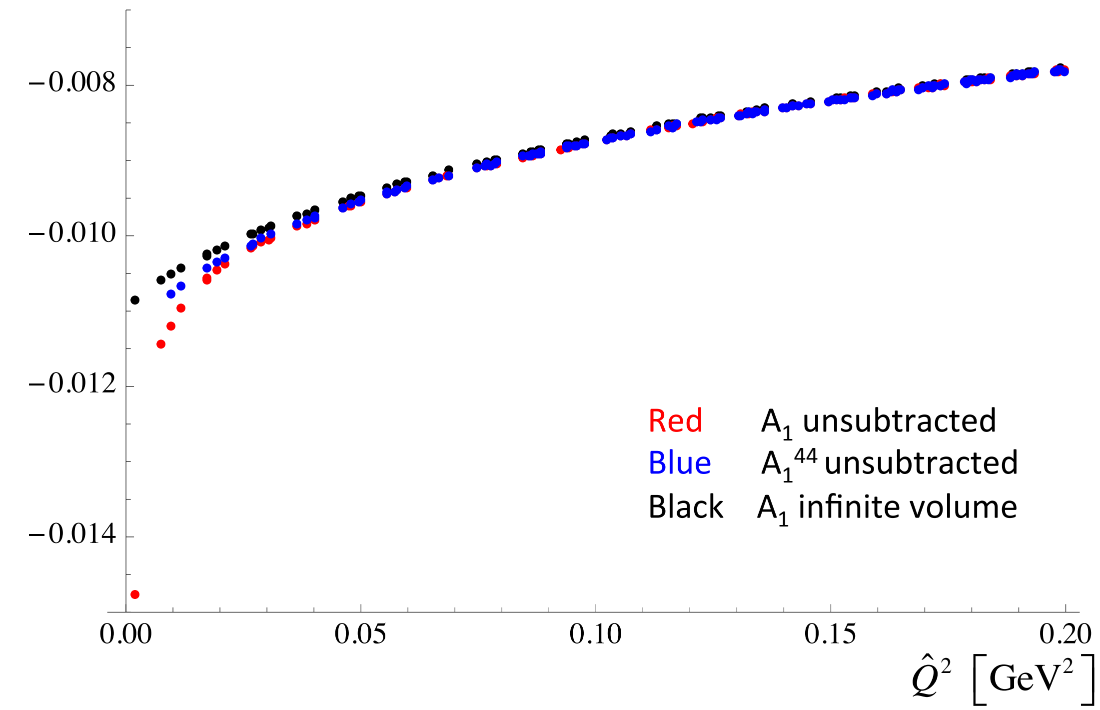


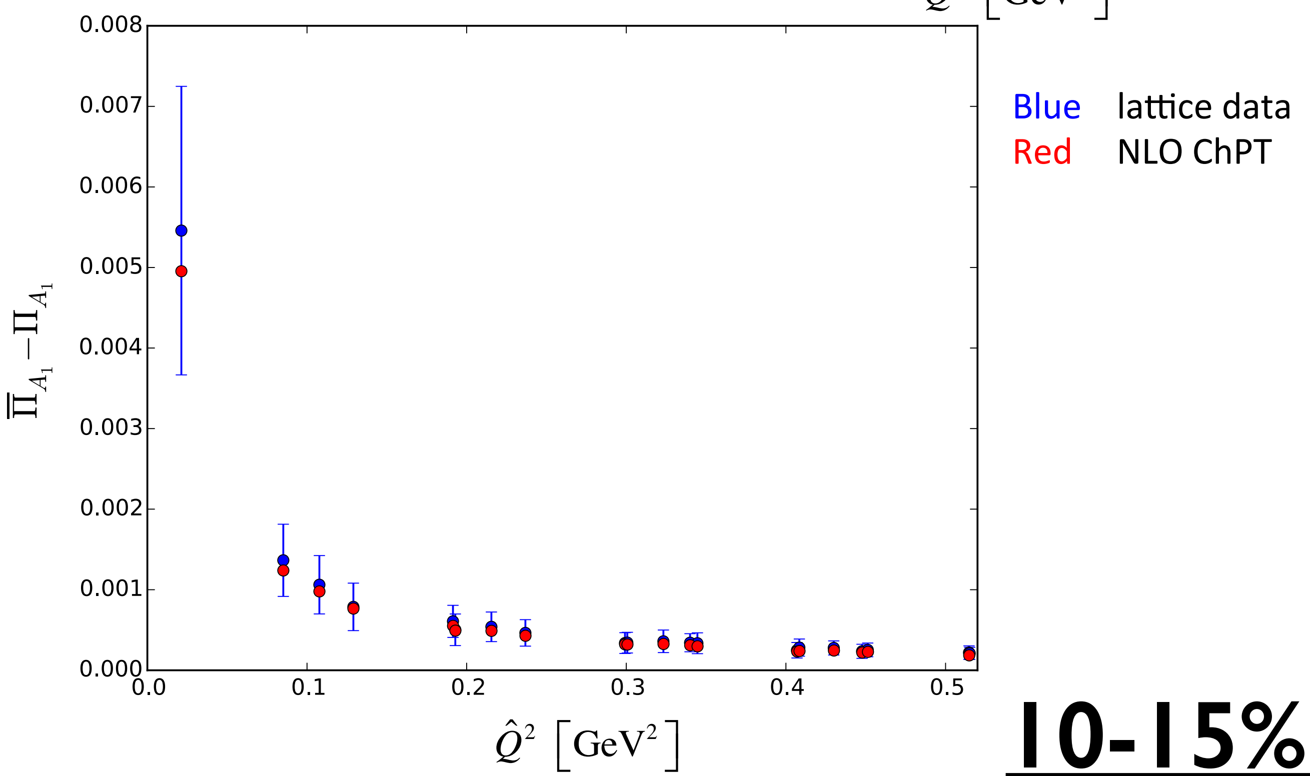
Staggered χPT → weighted average taste-split pion spectrum

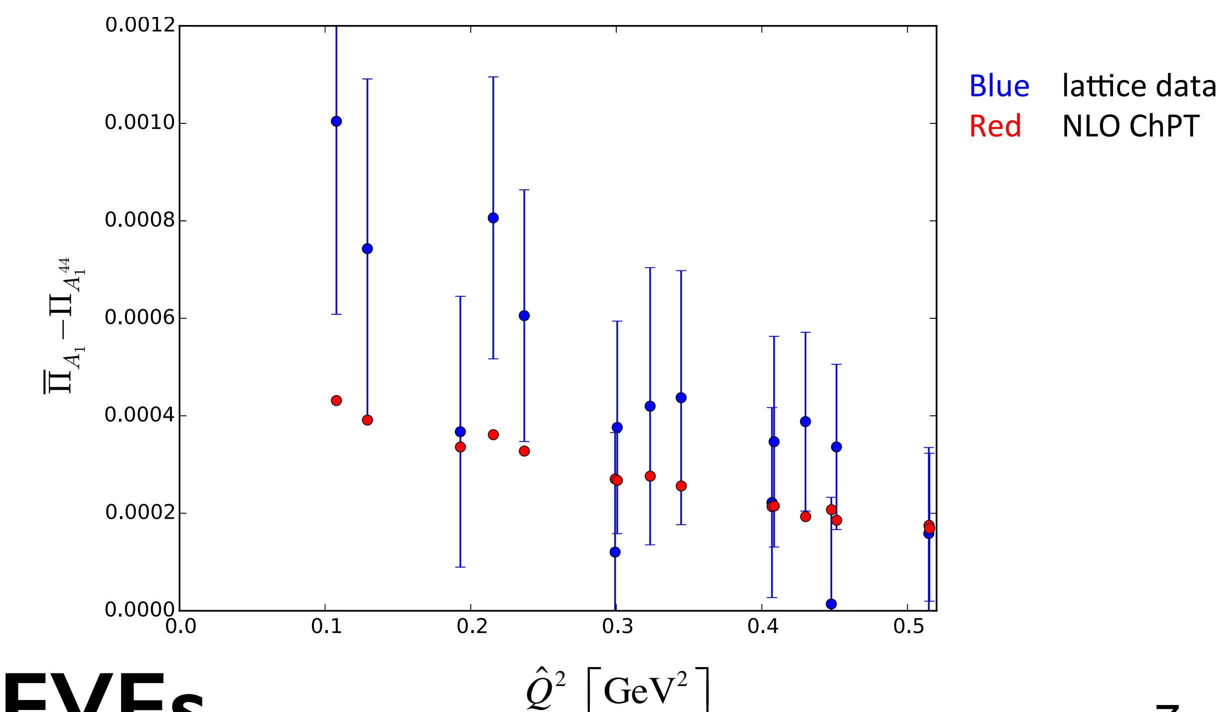
χPT Groups

Aubin et al. 2016 $\quad A_1 : \sum_i \Pi_{ii}$; $A_1^{44} : \Pi_{44}$ $\quad m_\pi L = 4.2$





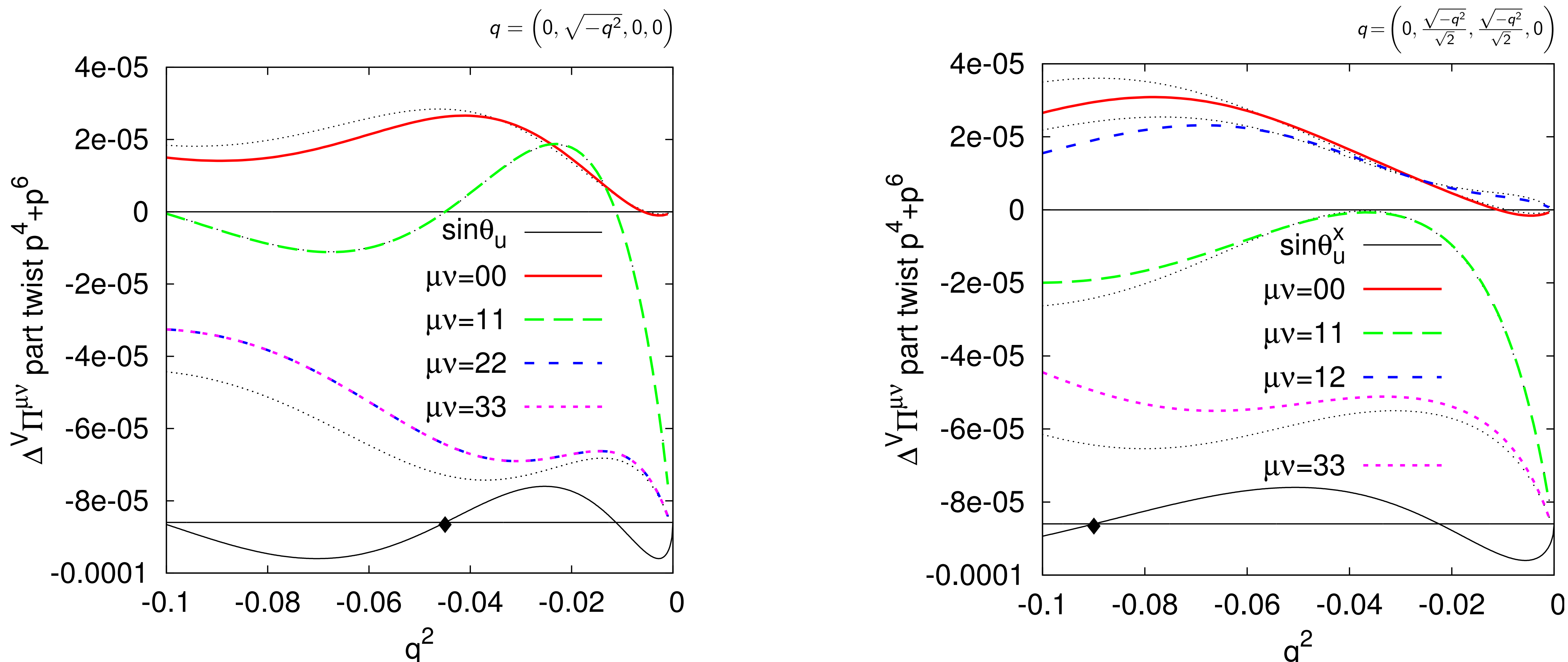




10-15% FVEs

χPT Groups

Bijnens and Relefors 2017 NNLO χPT PQχPT + twisted BCs



$m_\pi L = 4$

FV corrections: different twist angles at same q^2

<u>Small corrections with respect to NLO</u>

<u>FVEs sizeable</u> (few %) for present lattices

RBC/UKQCD Collaboration

Blum et al. 2018

Two ensembles

$$L = 5.4 \div 5.5 \text{ fm}$$
$$T = 10.7 \div 11 \text{ fm}$$

$$a = 0.084 \div 0.114 \text{ fm}$$

Physical mass point $m_\pi L = 3.8 \div 3.9$

FVEs corrected with NLO χPT

Systematic uncertainty from the largest ratio of p^6 to p^4

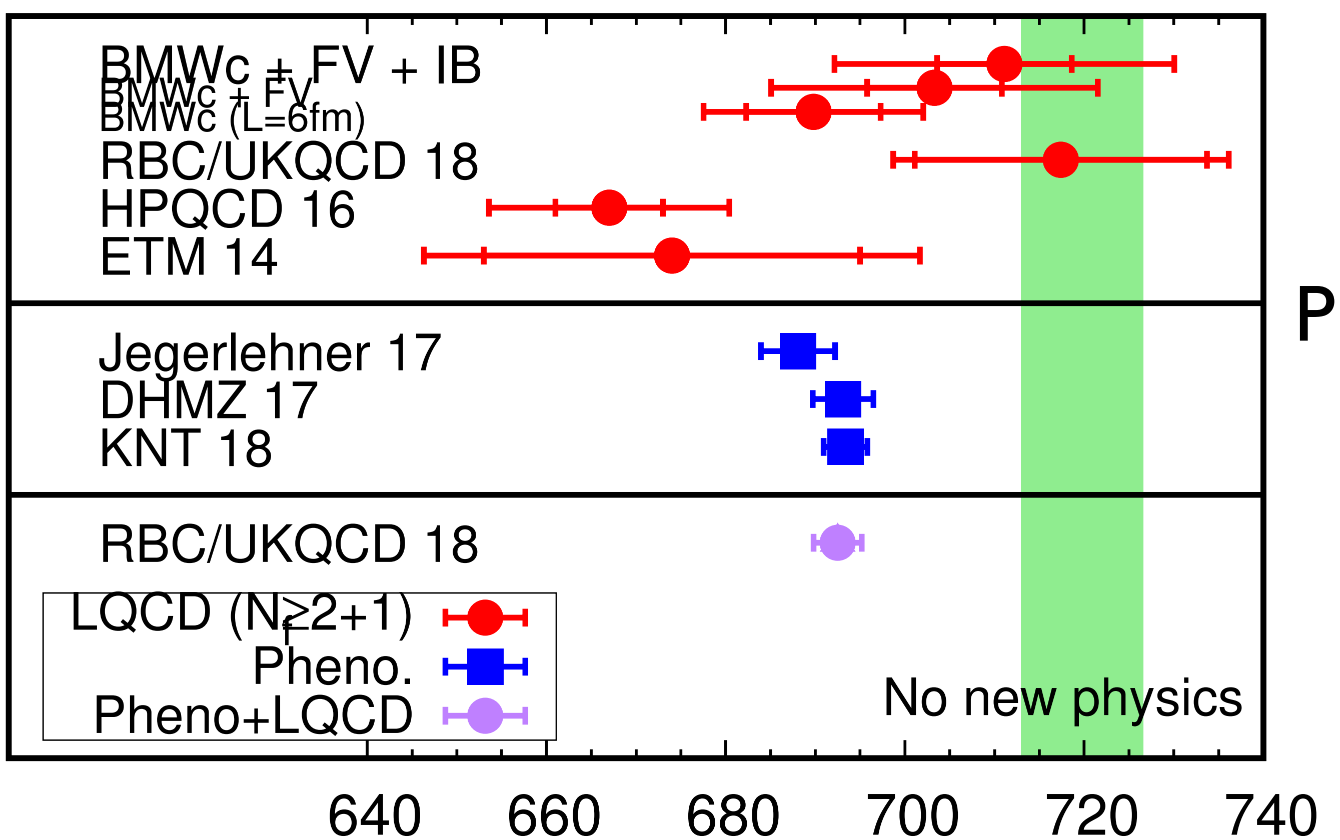
$$\Delta^{FVEs} a_\mu \big|_{conn} (ud) = 15.9(3.7) \cdot 10^{-10}$$

GSL approach: $\Delta^{FVEs} a_\mu \big|_{conn} (ud) = 20(3) \cdot 10^{-10}$

Talk by C. Lehner *(updates)*

BMW Collaboration

$a_\mu^{LO\text{-}HVP} \,.\, 10^{10}$ Borsanyi et al. 2017



$L = 6.1 \div 6.6$ fm

$T = 8.6 \div 11.3$ fm

Physical mass point

$m_\pi L = 4.2 \div 4.5$

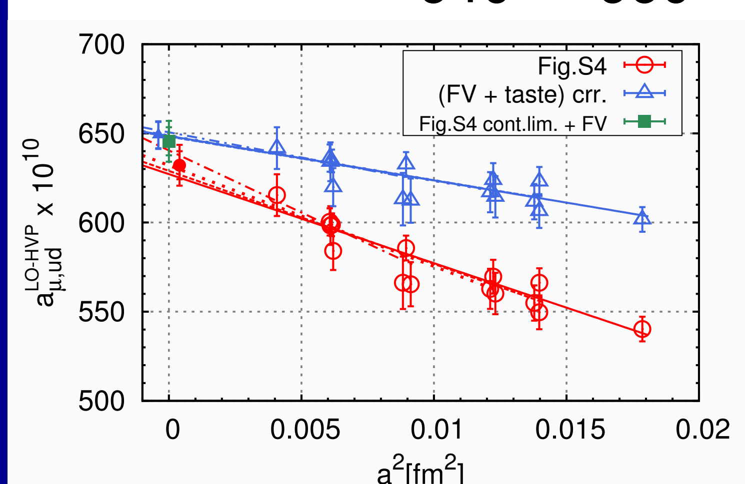
β	a [fm]	$(T \times L/a^2)$
3.7000	0.134	64 × 48
3.7500	0.118	96 × 56
3.7753	0.111	84 × 56
3.8400	0.095	96 × 64
3.9200	0.078	128 × 80
4.0126	0.064	144 × 96

<u>FVEs corrected with</u>

<u>NLO SU(2) SχPT</u>

(*I*=1 channel only)

fixed $m_\pi L \simeq 4.1$



$$\Delta^{FVEs} a_\mu^{I=1}(ud) = 15.0(15.0) \cdot 10^{-10}$$

extrapolated to the continuum limit

(six lattice spacings ranging from 0.064 to 0.134 fm)

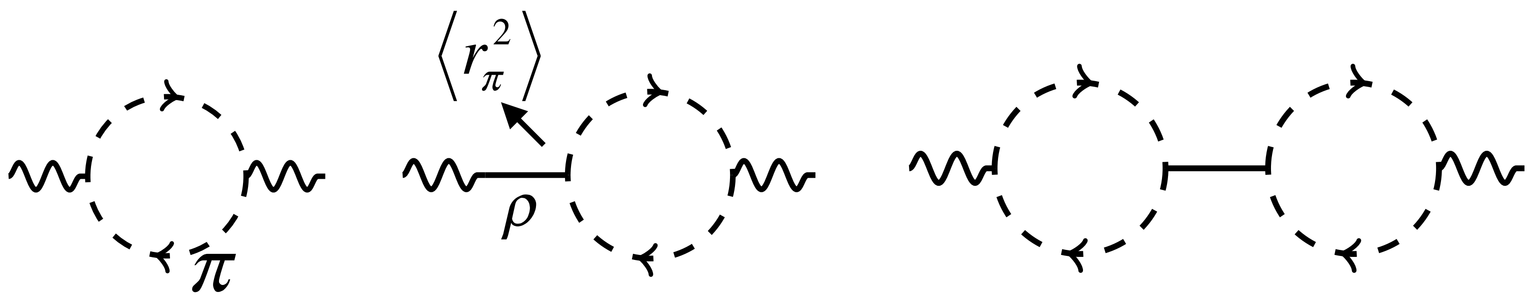
HPQCD Collaboration

Chakraborty et al. 2017

Combined FV and discretisation effects (pion tastes)

NLO SχPT + $\gamma - \rho^0 - \pi^+\pi^-$

mixing to all orders
in leading interactions



5 times larger

$$\hat{\Pi}(-q_E^2, f_\rho, m_\rho, m_\pi) = -\hat{\Sigma}(-q_E^2, m_\pi, m_\pi) + \frac{f_\rho^2}{2m_\rho^2} \frac{q_E^2 \left(1 + g_\rho g_{\rho\pi\pi} \hat{\Sigma}(-q_E^2, m_\pi, m_\pi)\right)^2}{q_E^2 \left(1 + g_{\rho\pi\pi}^2 \hat{\Sigma}(-q_E^2, m_\pi, m_\pi)\right) + m_\rho^2}$$

small FVEs+discr. for
s quark contribution

Largest correction for <u>lightest pion</u> masses: <u>7%</u>

$L = 2.4 \div 5.8$ fm

$T = 7.2 \div 8.6$ fm

$m_\pi = 134 \div 311$ MeV

$m_\pi L = 3.2 \div 5.4$

3 lattice volumes @:

$m_\pi \simeq 220$ MeV
$a = 0.12$ fm

Preliminary

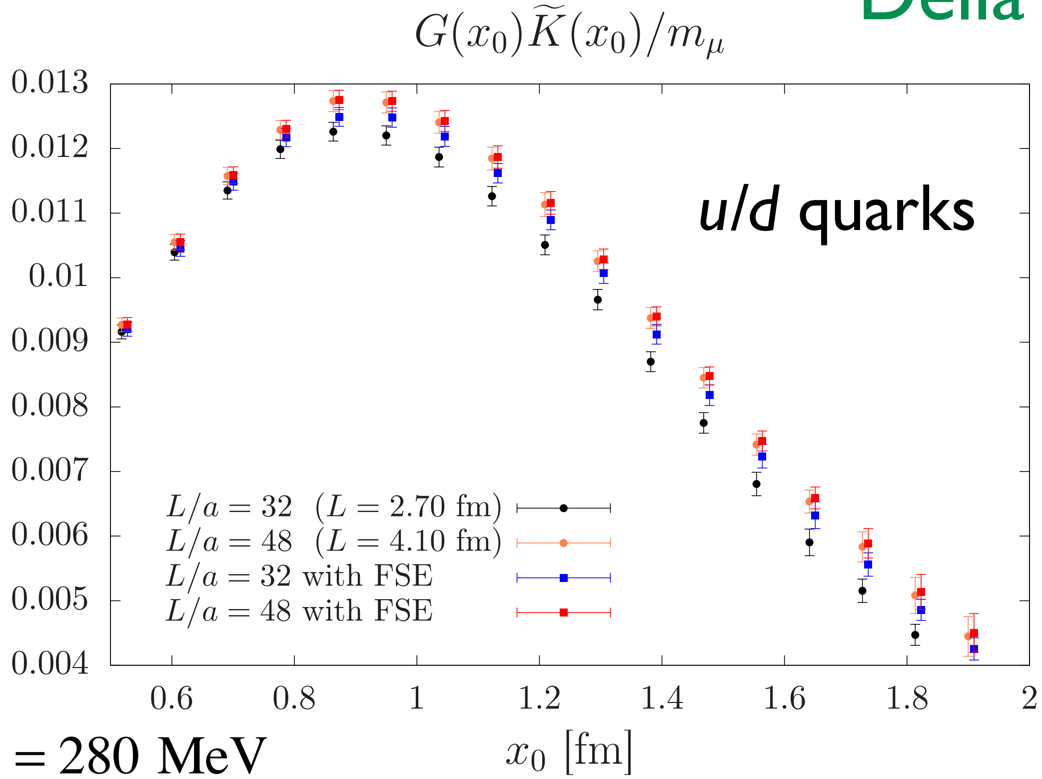
$$a_\mu^{\mathrm{LO,HVP}}\Big|_{conn}(ud) \simeq 610(9) \cdot 10^{-10}$$

Talk by R. S. Van de Water

Uncertainty: ±0.7%

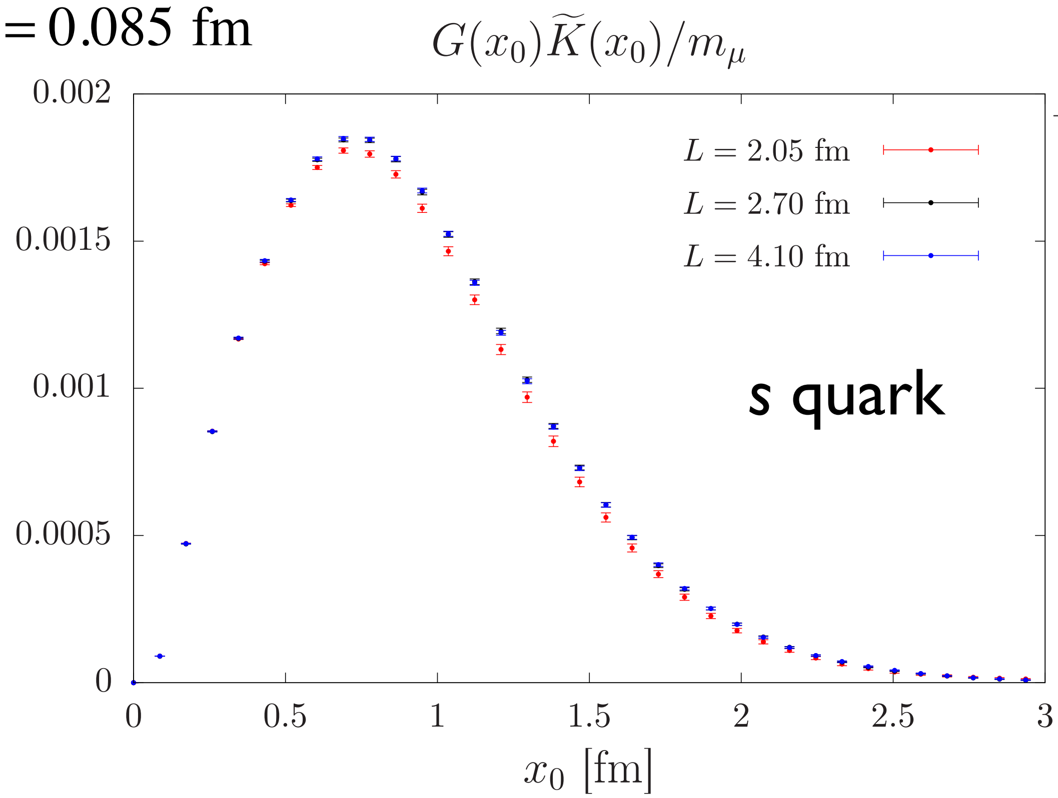
Mainz Group

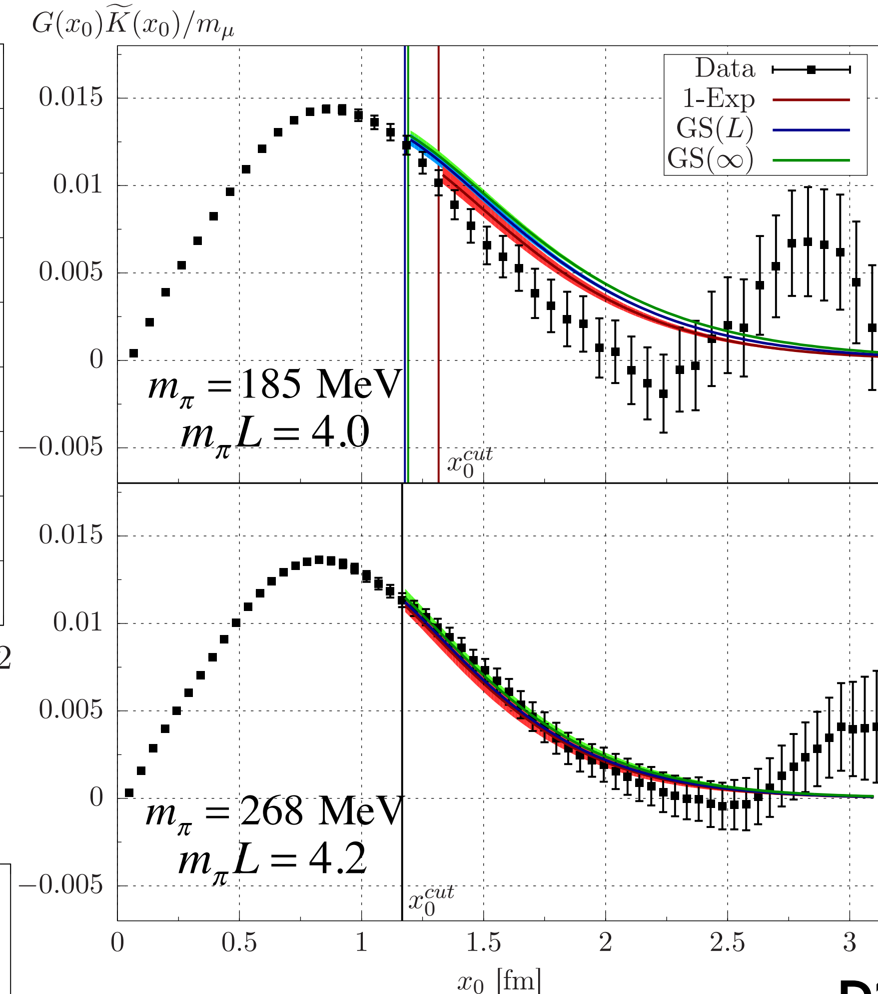
Della Morte et al. 2017



$m_\pi = 280$ MeV

$a = 0.085$ fm





$L = 2.1 \div 4.2$ fm

$T = 4.2 \div 8.4$ fm

$m_\pi = 185 \div 495$ MeV

$m_\pi L = 4.0 \div 6.0$

FVEs corrected with <u>Gounaris-Sakurai parameterisation</u> + <u>Lüscher formalism</u>

parameters: $m_\rho^{\text{exp fit}}$ Γ_ρ^{GS}

<u>FVEs</u>: <u>**5%**</u> shift in a_μ for $m_\pi L \approx 4$ and near-*phys.* point

$\pi\pi$ interactions important for $t > 1$ fm

$$\Delta^{FVEs} a_\mu \simeq 20.4(4.1) \cdot 10^{-10}$$

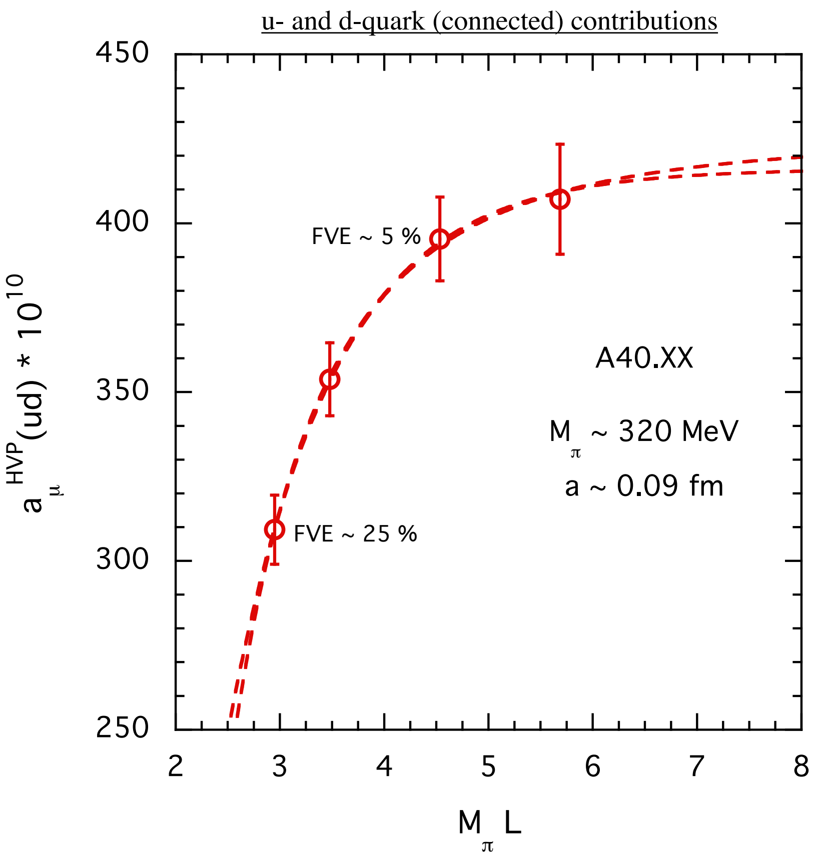
Preliminary

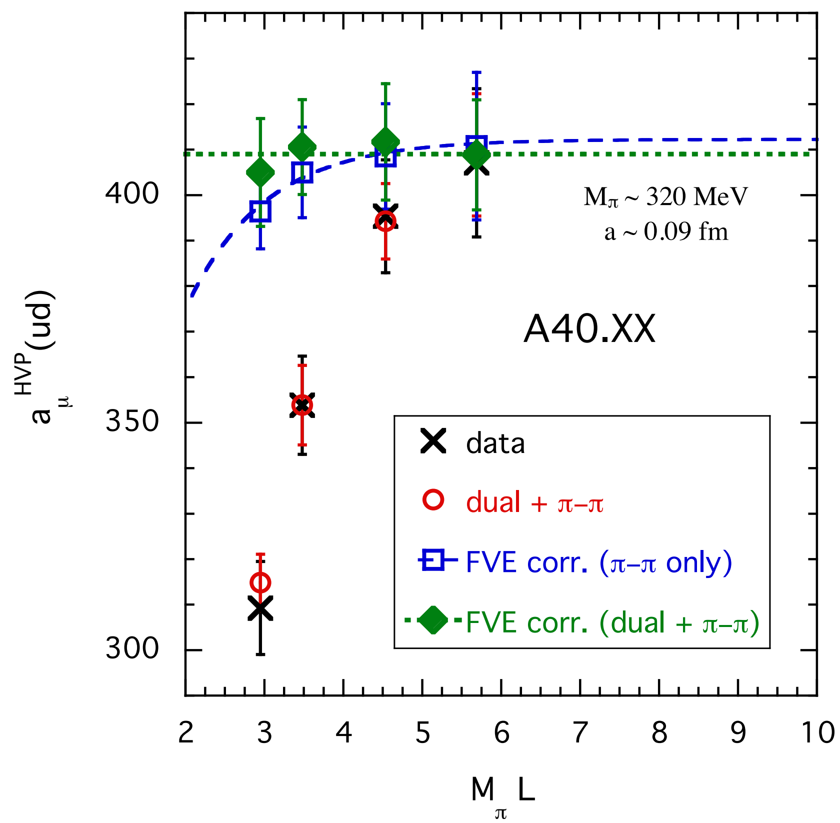
$m_\pi = 140$ MeV $m_\pi L = 4.0$

Talk by H. Meyer

ETM Collaboration

Talk by S. Simula





$L = 1.8 \div 3.5$ fm

$T = 3.5 \div 7.1$ fm

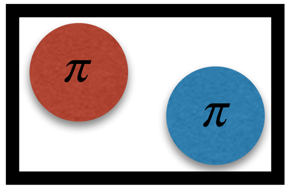
$m_\pi = 223 \div 495$ MeV

$m_\pi L = 3.0 \div 5.8$

FVEs corrected with

<u>2π Lüscher formalism</u>

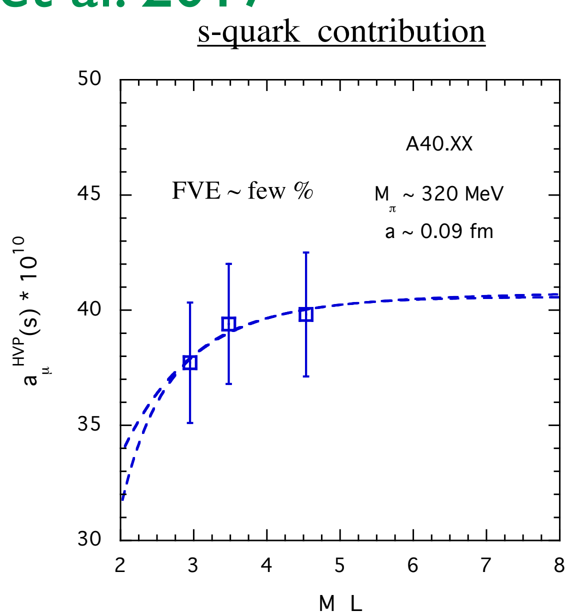
<u>and GS</u> F_π

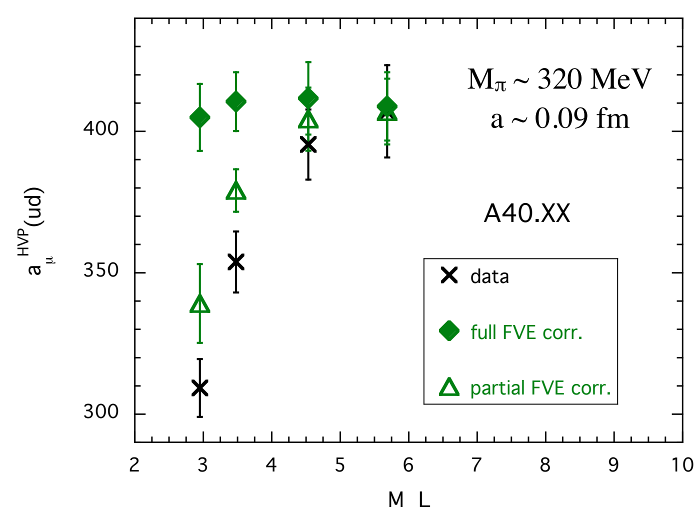


+

<u>dual pQCD contribution</u>

DG et al. 2017





parameters: $m_\rho \quad g_{\rho\pi\pi} \quad R_{dual} \quad E_{dual}$

pure <u>FVEs</u>:

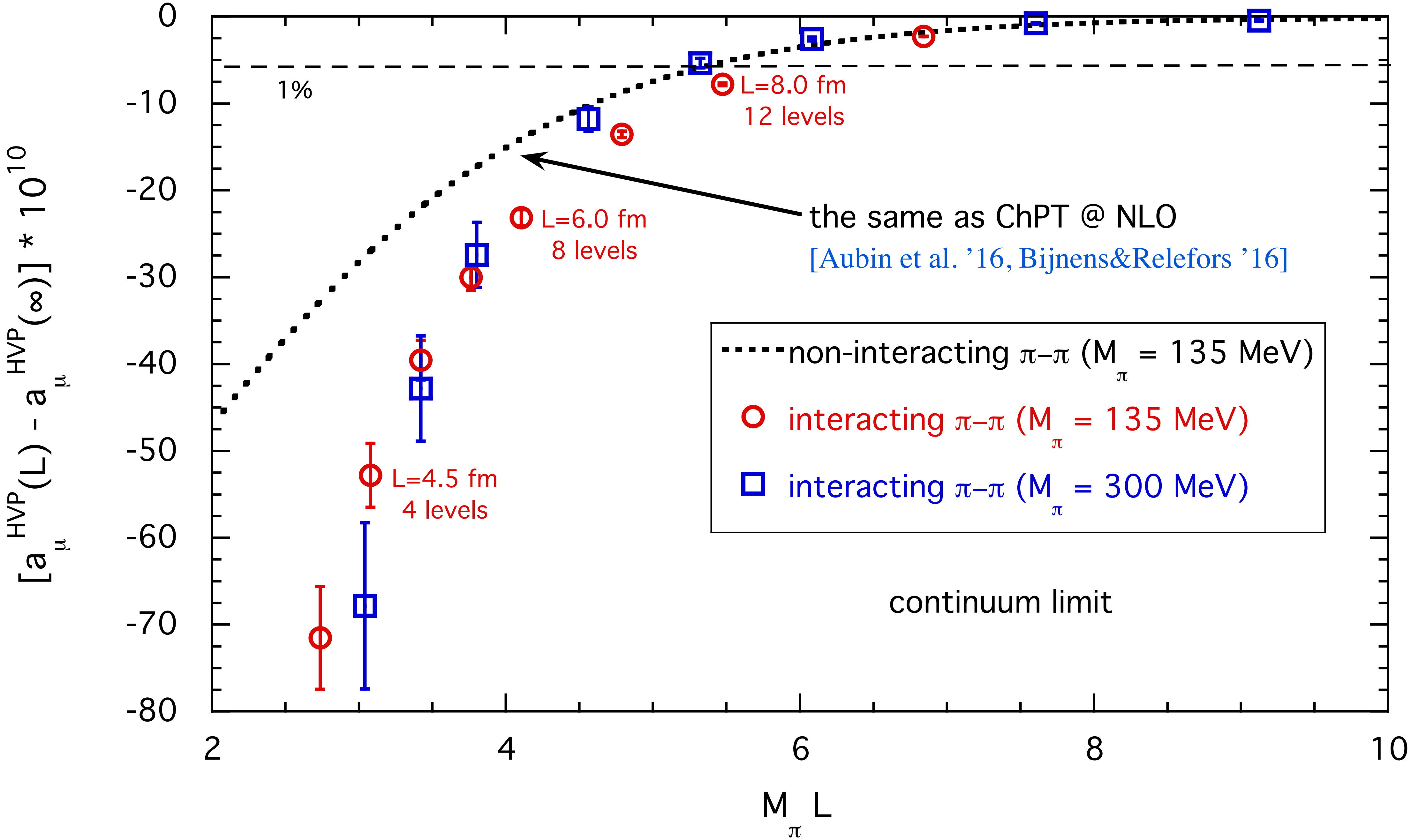
~**<u>5%</u>** correction to a_μ

@ $m_\pi = 135$ MeV $\quad m_\pi L \approx 4$

$a^2 \to 0$

$$a_\mu^{\text{LO,HVP}}\Big|_{conn}(ud) = 622.8(12.8) \cdot 10^{-10}$$

FVE correction @ $a^2 \to 0$



[Francis et al. '13]

non-interacting π-π: $$V_{\pi\pi}^{(L)}(t) - V_{\pi\pi}^{(\infty)}(t) = \frac{M_\pi^4}{3\pi^2} t \sum_{\vec{n}\neq 0} \left\{ \frac{K_2\left[M_\pi\sqrt{L^2\vec{n}^2+4t^2}\right]}{M_\pi^2\left(L^2\vec{n}^2+4t^2\right)} - \frac{1}{M_\pi L|\vec{n}|}\int_1^\infty dy\, K_0\left[M_\pi y\sqrt{L^2\vec{n}^2+4t^2}\right]\sinh\left[M_\pi L|\vec{n}|(y-1)\right] \right\}$$

interacting π-π: dual + π-π representation [note that $\Delta a_\mu^{HVP}(L)$ depends approximately on $M_\pi L$ only]

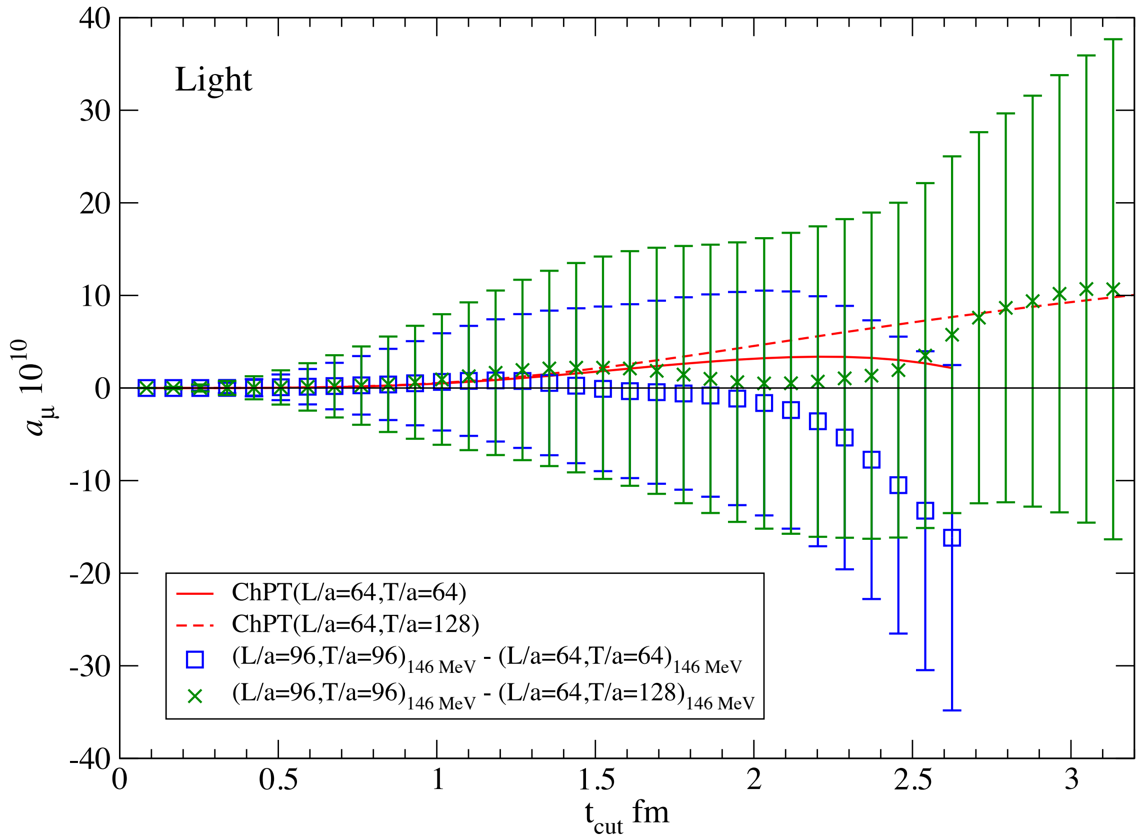
PACS Collaboration

Izubuchi et al. 2018

<u>FVEs estimated using TMR</u>

comparison between two volumes

$a_\mu^{\mathrm{LO,HVP}}$ on L=5.4 fm is $(10 \pm 26) \cdot 10^{-10}$
from L=8.1 fm @ $m_\pi = 146\ \mathrm{MeV}$



Two ensembles

$L = T = 5.4 \div 8.1\ \mathrm{fm}$

$a = 0.085\ \mathrm{fm}$

near-*phys*. mass point

$m_\pi L = 3.8 \div 5.8$

Backward state propagation
(2T=10.8 fm)

↓

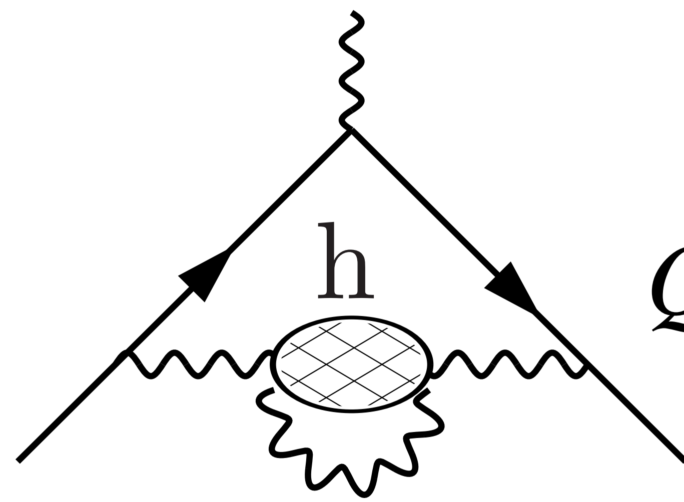
positive contribution to a_μ
4% @ t_{cut}=2.6 fm

IB contribution: FVEs

DG et al. 2017

s/c contribution only

expected to start at *O*(1/L^3)
(IR safe, neutral meson states,
vanishing charge radius)



Blum et al. 2018

photon propagator

QED_L prescription for zero mode subtraction

$$G_{\mathrm{L}}(x) = \frac{1}{V}\sum_k{}' \frac{1}{\hat{k}^2} e^{ikx}$$

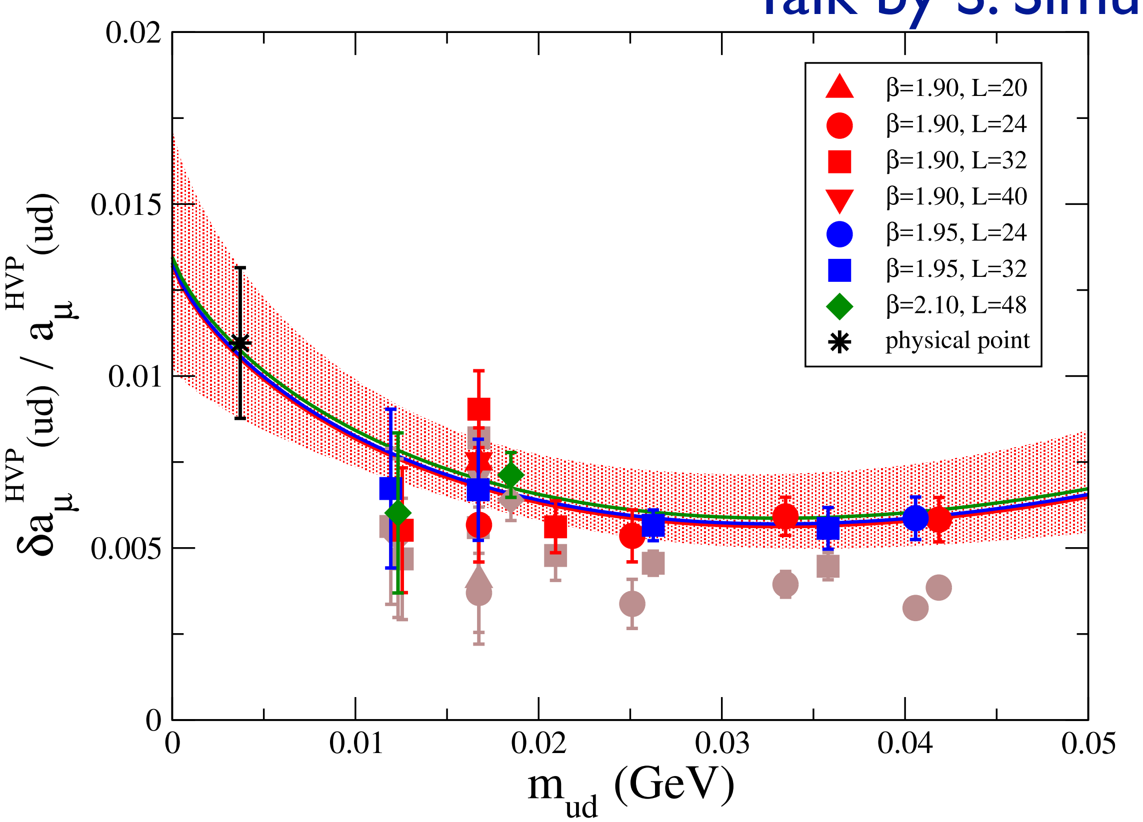
QED_∞

$$G_\infty(x) = \int_{-\pi}^{\pi} \frac{d^4k}{(2\pi)^4} \frac{1}{\hat{k}^2} e^{ikx}$$

Analytical calculation

Talk by A. Portelli

Talk by S. Simula

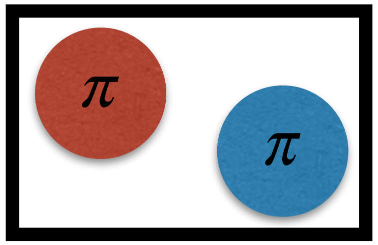


Discussion points

Sizeable FVEs for present lattices in $a_\mu^{\mathrm{LO,HVP}}(ud)$

New systematic lattice study
with several volumes has been performed

Discrepancy between χPT predictions
and lattice determinations with interacting pions



Most Collaborations adopt model estimates so far
(new recent efforts for first-principles determinations)

Small QED FVEs

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