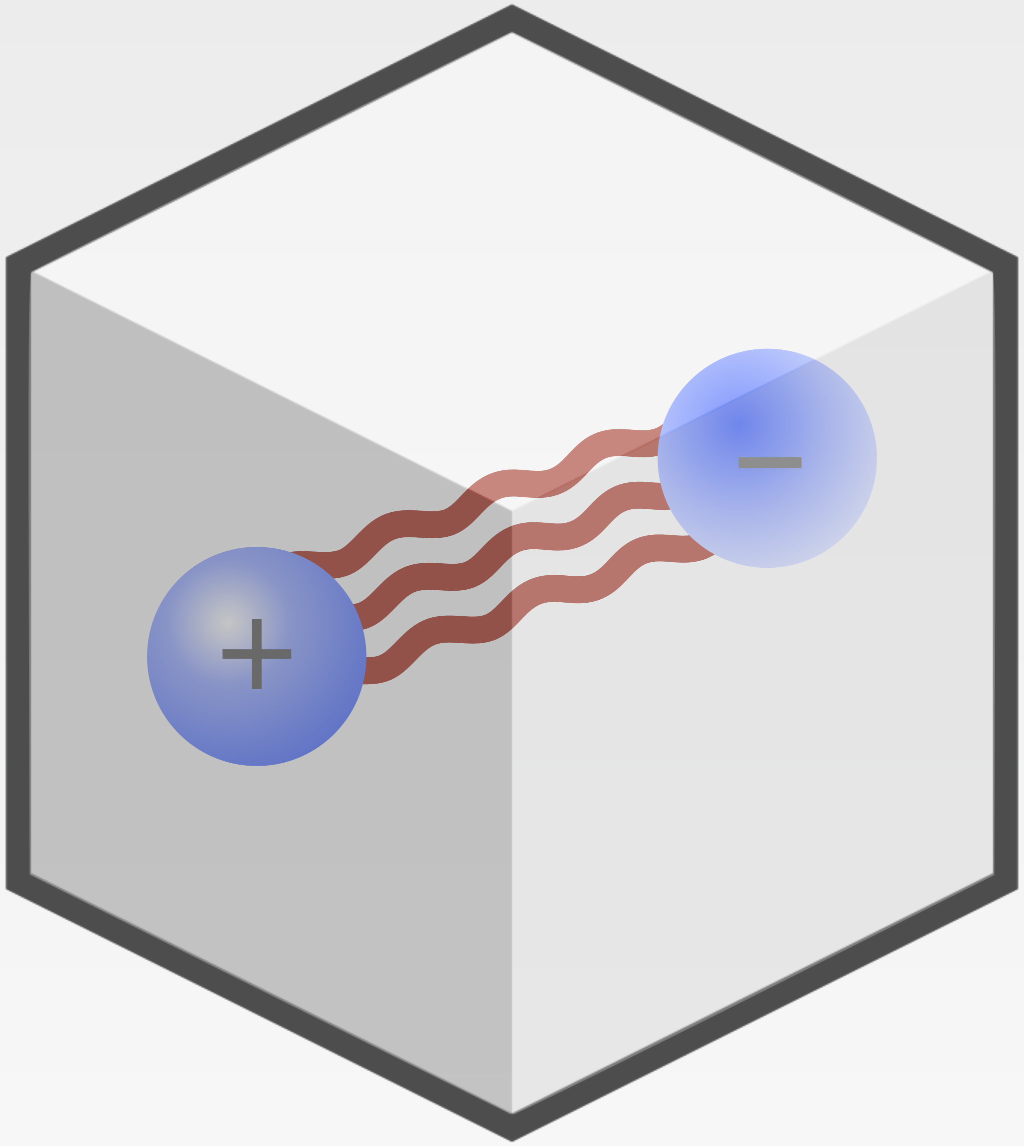

Analytical HVP finite volume corrections from two pion states

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THE UNIVERSITY *of* EDINBURGH

- General problem
- QCD HVP finite-volume effects
- QED HVP finite-volumes effects
- Outlook

General problem

Motivation

- Lattice calculations done in a finite-volume space-time. Results **contaminated by finite-size effects**.
- Volume: **IR cutoff**. Finite-size effects related to IR properties of correlation functions.
- What analytical knowledge can we build for the HVP?

Finite-volume effects in a nutshell

- Finite volume: momentum quantisation $\mathbf{k} = \frac{2\pi}{L}\mathbf{n}$

$$\mathcal{A} = \int \mathrm{d}^3\mathbf{k}\, f(\mathbf{k}) \longrightarrow \mathcal{A}_{\mathrm{FV}} = \frac{1}{L^3}\sum_{\mathbf{k}} f(\mathbf{k})$$

- Poisson summation formula:

$$\Delta\mathcal{A} = \left(\tfrac{1}{L^3}\textstyle\sum_{\mathbf{k}} - \int \tfrac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3}\right) f(\mathbf{k})$$
$$= \textstyle\sum_{\mathbf{n}\neq\mathbf{0}} \hat{f}(L\mathbf{n})$$

- In a nutshell: FV effects very much related to the decay at large distance of $\hat{f}(\mathbf{x})$.

Finite-volume effects in a nutshell

- $\hat{f}(\mathbf{x})$ decay rate related to smoothness of $f(\mathbf{k})$.

- Easy (essentially part integration):

$f(\mathbf{k})$ smooth $\Rightarrow$ $\hat{f}(\mathbf{x})$ faster than any power.
$f(\mathbf{k})$ has non-differential points $\Rightarrow$ $\hat{f}(\mathbf{x})$ power-like.

- A bit harder (Paley-Wiener theorems):

$f(\mathbf{k})$ analytic in a band $\Rightarrow$ $\hat{f}(\mathbf{x})$ decays exponentially.

- Physics:
  Energy gap: exponential FV effects.
  Momentum singularities: power-like FV effects.

Rough expectations for the HVP

- Long-distance part of the HVP dominated by 2-pion states propagating. *[Aubin et al., PRD 93(5), 2016]*

- Pure QCD Euclidean HVP: 2-pion integrand **analytic up to the first pion pole**.

Expectations: $\exp(-M_\pi L)$ $(\sim 2\%$ at $M_\pi L = 4)$

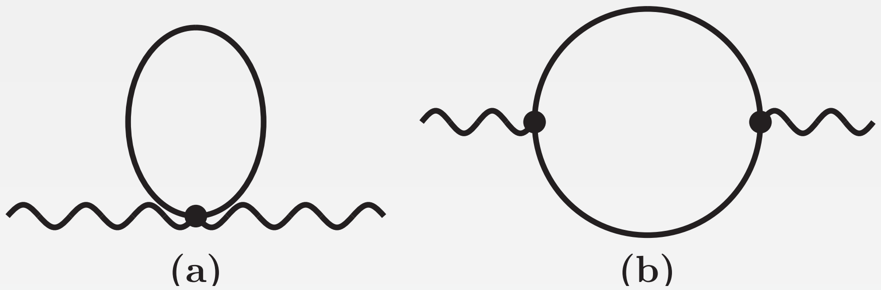
- QED corrections: **analytic expect for the photon pole**.

Expectations: $\dfrac{e^2}{L^3|\mathbf{k}|} \sim \dfrac{e^2}{L^2}$ $(\sim 0.6\%$ at $M_\pi L = 4)$

QCD HVP finite-volume effects

LO effects

- LO: scalar QED vacuum polarisation



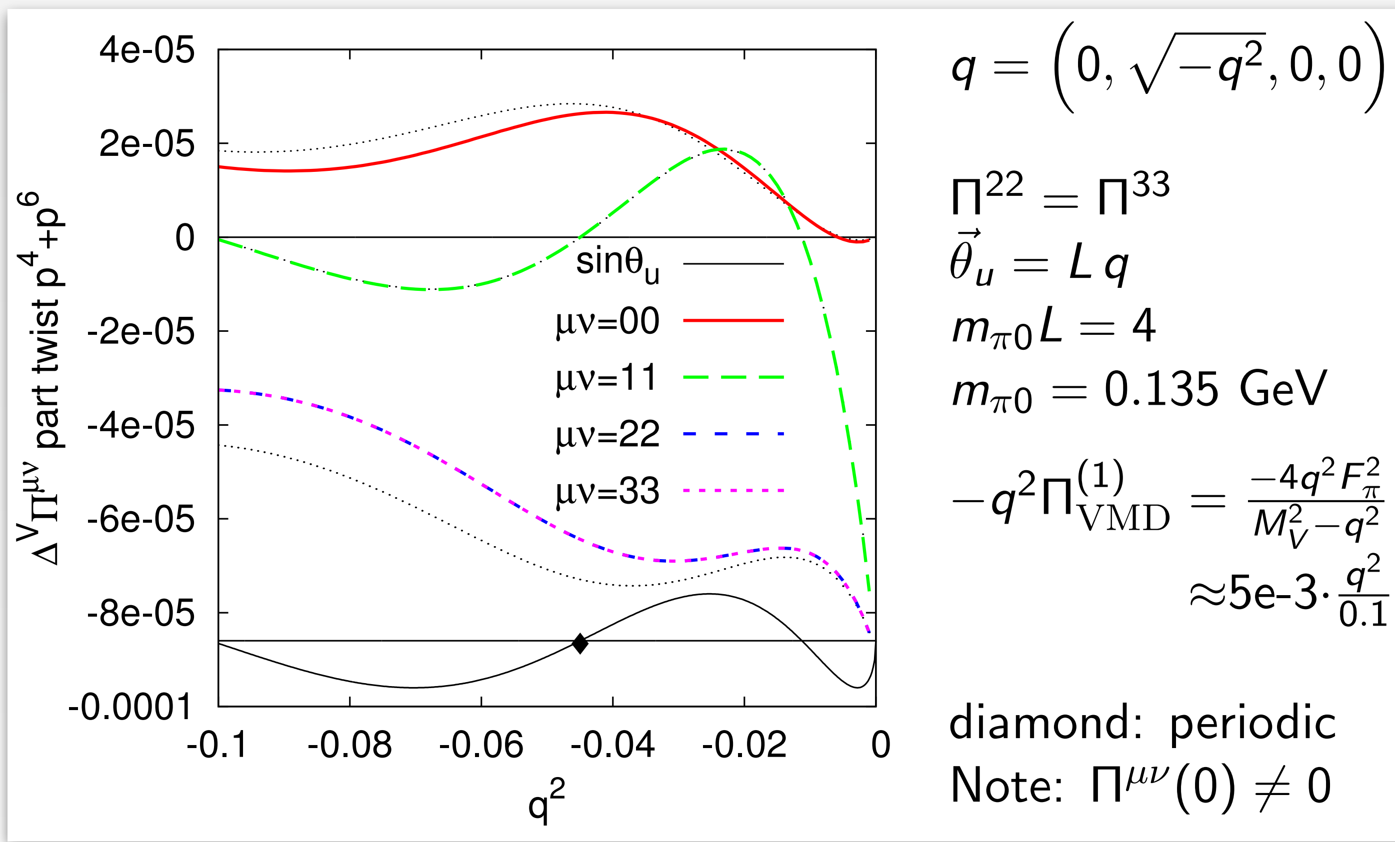
- Finite-volume: **constant (harmonic) mode** allowed $Z \sim \exp(-M_\pi L)$.

- **Important to subtract** before extracting $\Pi(q^2)$.
  [Aubin et al., PRD 93(5), 2016]

- After subtraction, LO FV effect of $\sim 2-8\%$ at $M_\pi L = 4$
  Good agreement with naive estimate.
  [Aubin et al., PRD 93(5), 2016]

NLO effects

- 2-loop chiPT finite-volume effects computed. Including with twisted boundary conditions.
  [J. Bijnens & J. Relefors, JHEP 12, 2017] [J. Bijnens, HVP KEK 2018]



- About $\sim 0.5\%$ correction to g-2, $\sim 20\%$ of LO.
  [RBC-UKQCD, PRL, 2018]

Consequences for g-2

- $\sim 2-8\%$ is **very big** compared to O(0.1%) targets.
- It is crucial to go beyond effective approaches, or to quantify precisely their efficiency (NNLO size ok?).
- Important puzzle: **is the resonance contribution small enough** that one can forget about it?
- **More details with Davide** after this talk.

QED HVP finite-volume effects

To appear:
[J. Bijnens, P. Boyle, **J. Harrison**, **N. Hermansson Truedsson**, **T. Janowski**, A. Jüttner, A.P.]
[P. Boyle, **Z. Davoudi**, **J. Harrison**, A. Jüttner, A.P., M. Savage]

Periodic finite-volume QED

- Finite-volume EM with **$\mathbf{QED_L}$ prescription**

$$\mathcal{A} = \int \mathrm{d}^3\mathbf{k}\, \frac{f(\mathbf{k})}{|\mathbf{k}|} \longrightarrow \mathcal{A}_{\mathrm{FV}} = \frac{1}{L^3} \sum_{\mathbf{k}\neq\mathbf{0}} \frac{f(\mathbf{k})}{|\mathbf{k}|}$$

- Poisson summation formula

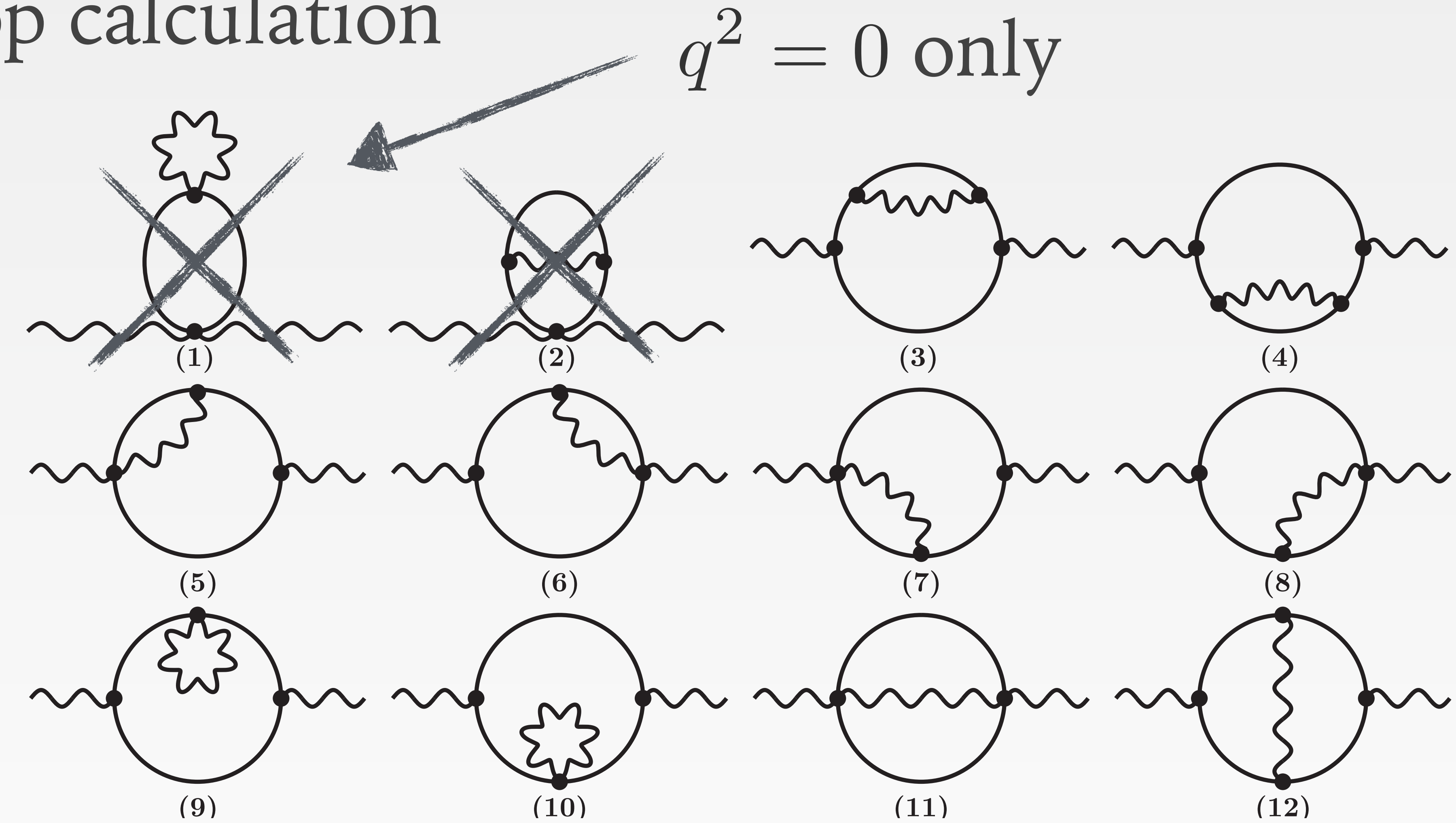
$$\begin{aligned}\Delta\mathcal{A} &= \left(\tfrac{1}{L^3}\textstyle\sum_{\mathbf{k}\neq\mathbf{0}} - \int \tfrac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3}\right) g(\mathbf{k}) \\ &= \left(\textstyle\sum_{\mathbf{n}\neq\mathbf{0}} - \int \mathrm{d}^3\mathbf{n}\right) \hat{g}(L\mathbf{n}) \quad \left(g(\mathbf{k}) = \tfrac{f(\mathbf{k})}{|\mathbf{k}|}\right)\end{aligned}$$

not very useful… give duality relations though.

- Power FV effects can be obtained by low-$\mathbf{k}$ expansion.

Strategy for HVP QED FV effects

- 2-loop calculation



- Very important: Euclidean $q^2 \geq 0$, the only singularity is the photon pole.

Strategy for HVP QED FV effects

$$\Delta\mathcal{D} = \left(\frac{1}{L^3}\sum_{\mathbf{k}\neq\mathbf{0}} - \int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3}\right)\int \frac{\mathrm{d}^3\boldsymbol{\ell}}{(2\pi)^3}\int \frac{\mathrm{d}k_0}{2\pi}\int \frac{\mathrm{d}\ell_0}{2\pi}\, d(k,\ell,q)$$

$$= \frac{a(\boldsymbol{\ell},q)}{|\mathbf{k}|} + b(\boldsymbol{\ell},q) + \cdots$$

VP average diagonal component

VP zero-mode subtracted

Momentum $q = (\sqrt{z}m, \mathbf{0})$

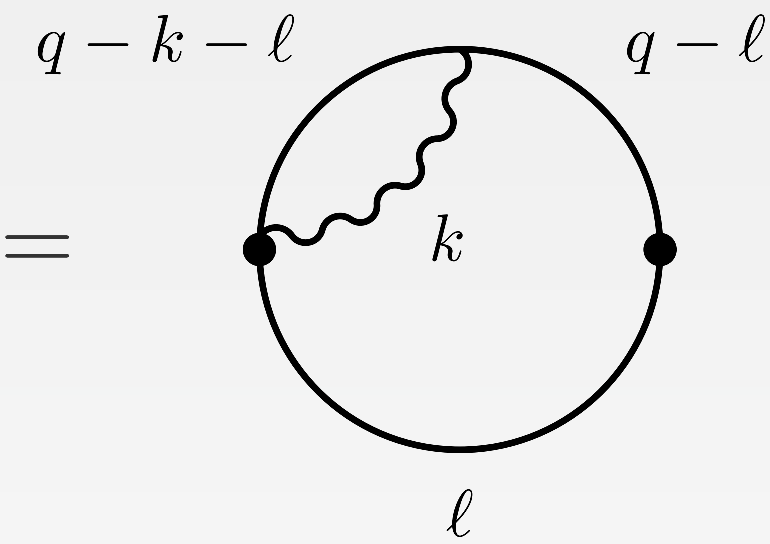
Typical lattice setup

$$= \frac{A(q)}{|\mathbf{k}|} + B(q) + \cdots$$

$$= c_1\frac{A(q)}{2\pi L^2} - \frac{B(q)}{L^3} + \cdots$$

$c_1 = -2.83729748\ldots$

Diagram example

$C =$ 

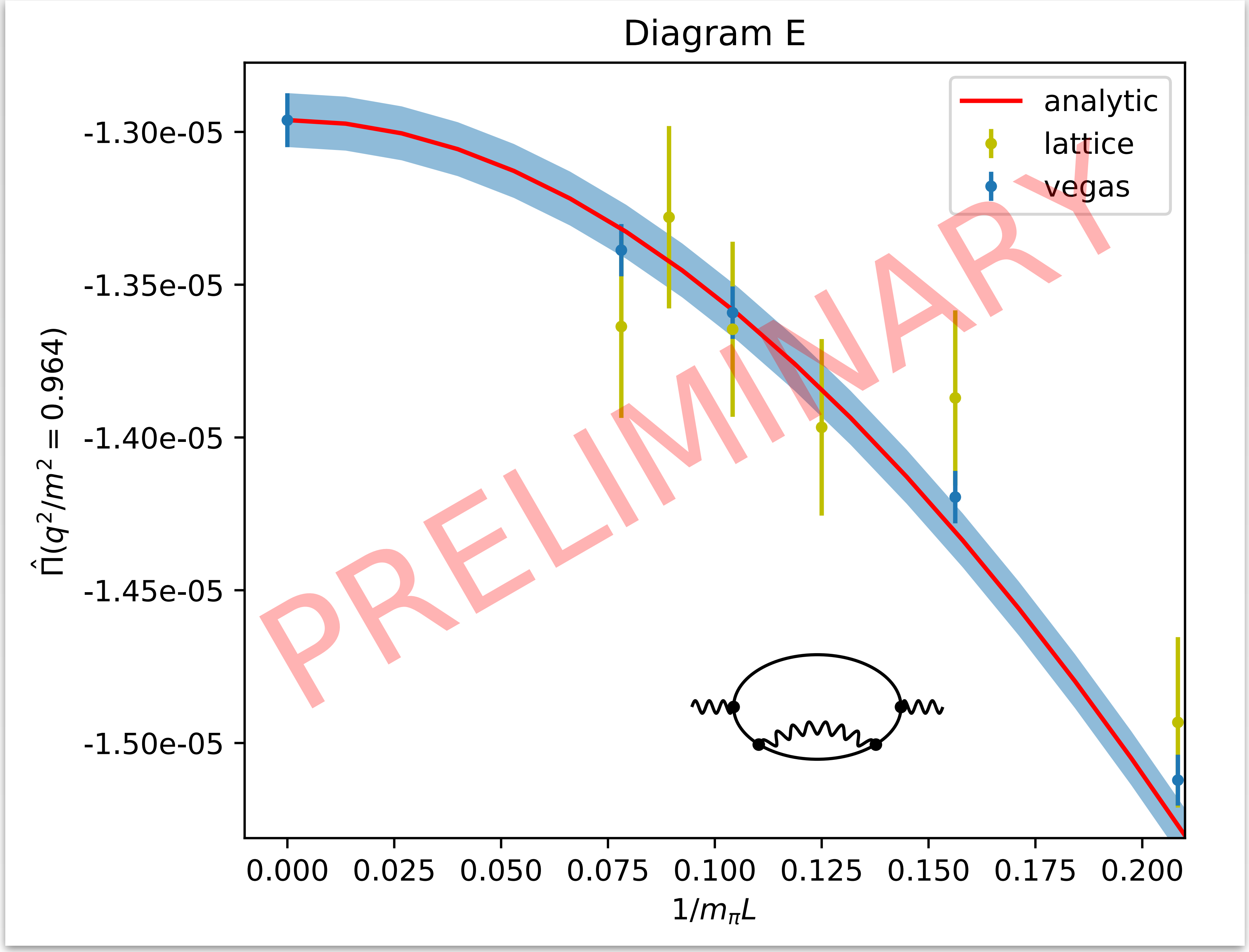
$$\Delta\Pi_C(z) = \frac{c_1\left[\sqrt{z(z+4)} + (z+4)\log(2) - (z+4)\log\left(\sqrt{z}+\sqrt{z+4}\right)\right]}{32\pi^3 L^2 m^2\sqrt{z^3(z+4)}} - \frac{z\left(3z - 8\sqrt{z+4}+12\right) - 8\sqrt{z+4} - \frac{96}{\sqrt{z+4}} + 64}{384\pi L^3 m^3 z^3} + \cdots$$

Total result and LO cancellation

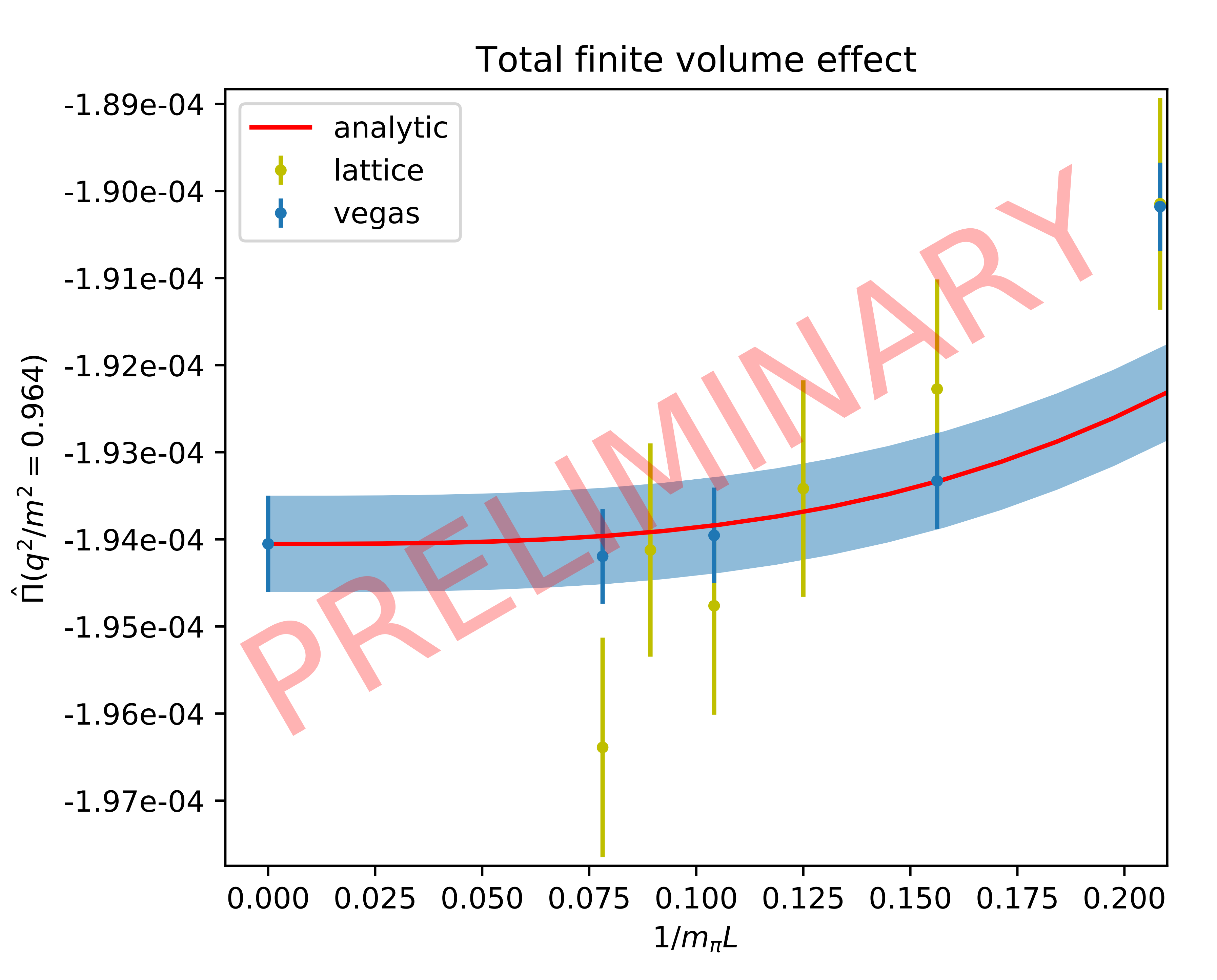
$$\Delta\Pi(z) = \frac{c_1}{\pi m^2 L^2}\left(\frac{16}{3}\Omega_{-1,3} - \frac{1}{3}\Omega_{1,2} - \frac{32}{3}\Omega_{1,3} - \frac{2}{3}\Omega_{3,2} + \frac{16}{3}\Omega_{3,3} - \frac{1}{8}\Omega_{5,1} + \Omega_{5,2}\right) - \frac{1}{m^3L^3}\left(-\frac{128}{3}\Omega_{-2,4} + \frac{256}{3}\Omega_{0,4} - \frac{5}{3}\Omega_{2,2} + \frac{8}{3}\Omega_{2,3} - \frac{128}{3}\Omega_{2,4} - \frac{3}{8}\Omega_{4,1} + \frac{7}{6}\Omega_{4,2} - \frac{8}{3}\Omega_{4,3}\right) + \mathcal{O}\left(\frac{1}{L^4}, e^{-mL}\right)$$

- $\Omega_{\alpha,\beta}$: family of hypergeometric functions of z .
- Using relations between the $\Omega_{\alpha,\beta}$, **the LO cancels**.
- Quite probably **related to the neutrality of the system** (no cancellation with charged current).

Scalar QED numerical check



Scalar QED numerical check



Consequences for g-2

- LO cancellation probably universal
  (we are working on it)
- NLO certainly not universal
  (pion structure intreating with photon zero-mode)
- **QED FV effects very small. Even if wrong by an order of magnitude.**
- Example on RBC-UKQCD result (non-official)

$$a_\mu^{\mathrm{QED}} = -1.0(6.0)_{\mathrm{S}}(0.5)_{\mathrm{C}}(\cancel{1.8})_{\mathrm{V}}(1.7)_{\mathrm{E}} \times 10^{-10}$$

$$(0.02)_{\mathrm{V}}$$

Outlook

Summary

- Finite-volume effects are a critical systematics in lattice calculation of g-2 from the HVP.

- 2-pion contribution well understood at NLO in chiPT and $O(\alpha)$ in QED.

- LO is in the range $\sim 2-8\%$ for g-2.

- NLO chiPT is $\sim 0.5\%$.

- NLO QED looks negligible (sub-0.1%).

Perspectives

- Direct lattice studies of volume scaling important.
- Test of NLO 2 pion effects: can it be used directly to correct data? Is NNLO beyond HVP target precision?
- Beyond 2 pions, resonances? More pions?
- For peace of mind: check QED scaling too.

Thank you!

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