HVP contribution to a_μ

Lattice vs lattice and phenomenology

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HVP from LQCD: introduction

On Euclidean $T \times L^3$ lattice can compute

$$C_{\mu\nu}(x) = \langle J_\mu(x) J_\nu(0) \rangle, \qquad C(t) = \frac{a^3}{3} \sum_{i=1}^{3} \sum_{\vec{x}} C_{ii}(x)$$

w/ $J_\mu = \frac{2}{3}\bar{u}\gamma_\mu u - \frac{1}{3}\bar{d}\gamma_\mu d - \frac{1}{3}\bar{s}\gamma_\mu s + \frac{2}{3}\bar{c}\gamma_\mu c + \cdots$

Naive Fourier transform in box, at discrete finite-volume momenta

$$\Pi_{\mu\nu}(Q) = a^4 \sum_{x_\mu} e^{iQ\cdot x} C_{\mu\nu}(x) = \left(Q_\mu Q_\nu - \delta_{\mu\nu} Q^2\right) \Pi(Q^2)$$

Then (Lautrup et al '69, Blum '02)

$$a_\ell^{\text{LO-HVP}} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty \frac{dQ^2}{m_\ell^2} w(Q^2/m_\ell^2) \hat{\Pi}(Q^2)$$

w/ $\hat{\Pi}(Q^2) \equiv \left[\Pi(Q^2) - \Pi(0)\right]$

Need interpolation of $\hat{\Pi}(Q^2)$ to evaluate integral

HVP from LQCD: Q^2 interpolation

1. Divide into regions and inter/extrapolate $\Pi(Q^2)$ from discrete lattice Q^2 with, e.g. Padés (Golterman et al '14)

$$\Pi(Q^2) = \Pi(0) + \frac{\sum_{n=1}^{N} a_n Q^{2n}}{1 + \sum_{n=1}^{D} b_n Q^{2n}}$$

2. Padés from time moments (HPQCD 14)

$$\Pi_n = \frac{1}{n!}\frac{\partial^n}{\partial Q^{2n}}\Pi(Q^2)|_{Q^2=0} = a\sum_t (-1)^n \frac{t^{2n+2}}{(2n+2)!} C(t)$$

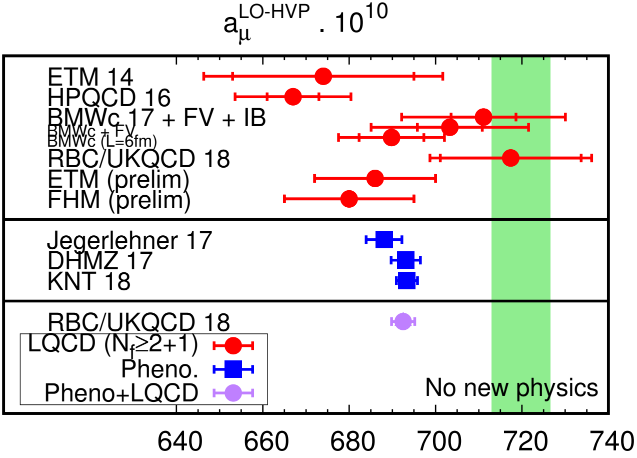
3. Coordinate space calculation, infinite volume kernel (Bernecker et al 11)

$$a_\ell^{\text{LO-HVP}}(Q^2 \leq Q^2_{\text{max}}) = \left(\frac{\alpha}{\pi}\right)^2 \left(\frac{a}{m_\ell^2}\right) \sum_{t=0}^{T/2} W(tm_\ell, Q^2_{\text{max}}/m_\ell^2)\ \mathrm{Re}\,C(t)$$

where

$$W(\tau, x_{\mathrm{max}}) = \int_0^{x_{\mathrm{max}}} dx\, w(x) \left(\tau^2 - \frac{4}{x}\sin^2\frac{\tau\sqrt{x}}{2}\right)$$

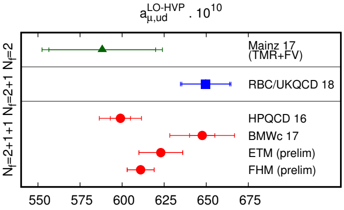
The obvious: $a_\mu^{\text{LO-HVP}}$

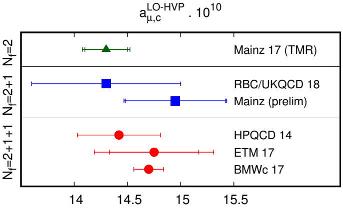


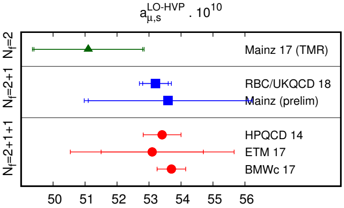
- Lattice errors $\sim$ 2% vs phenomenology errors $\sim$ 0.4%
- Some lattice results suggest new physics others not but all compatible with phenomenology

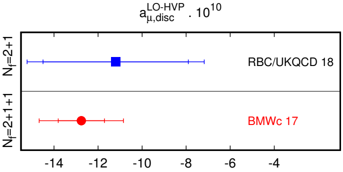
$a_\mu^{\text{LO-HVP}}$: flavor by flavor comparison

$$C_{\mu\nu}(x) \quad = \quad C^{ud}_{\mu\nu}(x) + C^{s}_{\mu\nu}(x) + C^{c}_{\mu\nu}(x) + C^{\text{disc}}_{\mu\nu}(x)$$





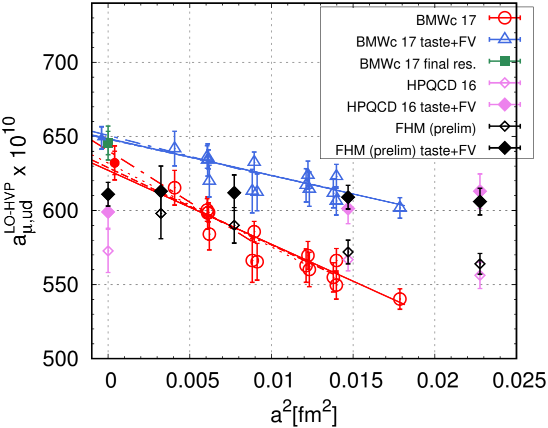




- $a^{\text{LO-HVP}}_{\mu,\,s,c,\text{disc}}$ already known with high enough precision for FNAL E989
- "disagreement" is on $a^{\text{LO-HVP}}_{\mu,\,ud}$

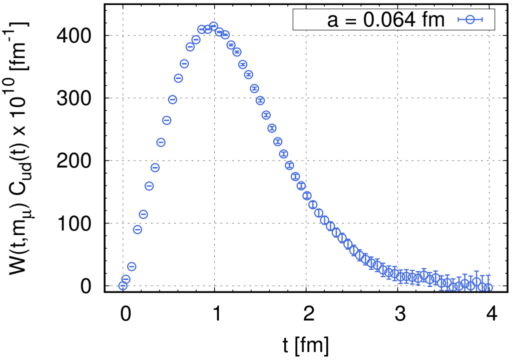
Staggered continuum extrapolation of $a_{\mu,ud}^{\text{LO-HVP}}$

- Goldstone has more massive "taste" partners that dilute Golsdtone contribution to $a_{\mu,ud}^{\text{LO-HVP}}$
- "Effective" pion mass larger at larger a, e.g. $M_\pi^{\text{RMS}} \simeq 310\,\text{MeV}$ for $a = 0.134\,\text{fm}$
- Effect dissappears in $a \to 0$ limit
- $a \to 0$ extrapolation includes $M_\pi^{\text{RMS}} \to M_\pi^{\text{PDG}}$ extrapolation and is quite pronounced



FNAL/HPQCD/MILC 16 & prelim already include large

Treatment of longer distances in $C_{ud}(t)$ differ



(BMWc 17)

- BMWc 17 and RBC/UKQCD 18 replace lattice data by average upper/lower bounds above $t_c \sim 3\,\mathrm{fm}$ where bounds agree within errors

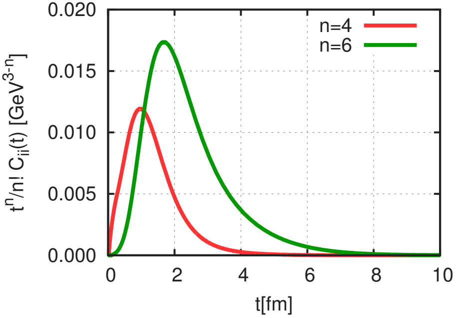
- FNAL/HPQCD/MILC models $C_{ud}(t)$ above $t_c \sim 1.5\,\mathrm{fm}$

$\rightarrow$ compare time moments that probe different distances $\leftrightarrow$ $\Pi(Q^2)$ derivatives at $Q^2 = 0$

Time moments

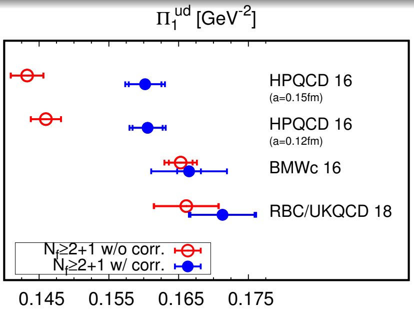
$$\Pi_n = \frac{1}{n!}\frac{\partial^n}{\partial Q^{2n}}\Pi(Q^2)|_{Q^2=0} = a\sum_t (-1)^n \frac{t^{2n+2}}{(2n+2)!}C(t)$$

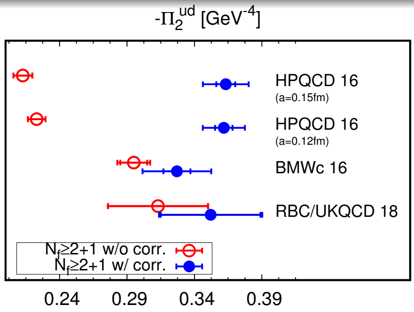
Larger n probe larger distances



$$\Pi_n(x) = \frac{5}{9}\Pi_n^{ud}(x) + \frac{1}{9}\Pi_n^{s}(x) + \frac{4}{9}\Pi_n^{c}(x) + \frac{1}{9}\Pi_n^{\text{disc}}(x)$$

Derivatives of $\Pi(Q^2)$ at $Q^2 = 0$: *ud* contribution

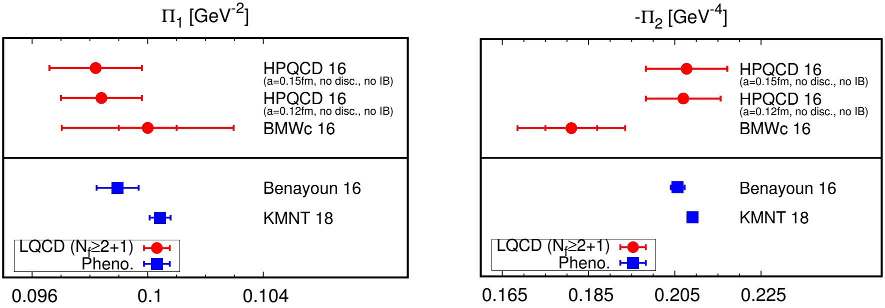




- In Padé picture (and probably generally) larger $\Pi_1 \to$ larger a_μ
- Larger $-\Pi_2 \to$ smaller a_μ
- HPQCD 16 has slightly smaller Π_1^{ud} and larger $-\Pi_2^{ud}$ than BMWc 16 and RBC/UKQCD 18 $\to$ combine to give smaller $a_{\mu,\,ud}^{\text{LO-HVP}}$
- Suggests that HPQCD 16 has smaller $C(t)$ for $t \sim 1\,\text{fm}$ but larger for $t \gtrsim 2\,\text{fm}$
- Difference comes from HPQCD 16's large corrections

Comparison of derivatives of $\Pi(Q^2)$ at $Q^2 = 0$

Add all flavor components and compare to phenomenology



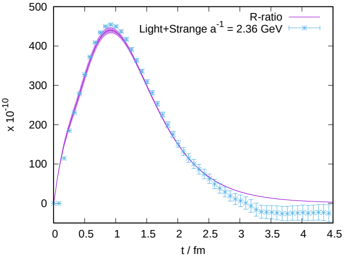
BMWc 16 has Π_1 comparable to phenomenology but smaller $-\Pi_2$

$\rightarrow$ suggests that BMWc (and RBC/UKQCD) has $C(t)$ slightly larger for $t \sim 1\,\mathrm{fm}$ and smaller for $t \gtrsim 2\,\mathrm{fm}$

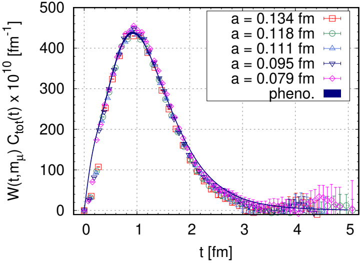
Euclidean time correlator: lattice vs phenomenology

$$C(t) = \frac{1}{2}\int_0^\infty ds\sqrt{s}\,\frac{R(s)}{3}\,e^{-\sqrt{s}|t|}\,, \qquad a_\ell^{\text{LO-HVP}}(Q^2 \le Q^2_{\text{max}}) = \left(\frac{\alpha}{\pi}\right)^2 \left(\frac{a}{m_\ell^2}\right) \sum_{t=0}^{T/2} W(tm_\ell\,, Q^2_{\text{max}}/m_\ell^2)\ \mathrm{Re}C(t)$$

Very rough comparison done at fixed lattice parameters



(RBC/UKQCD 18)



(BMWc 18)

Confirms suggestion that $C(t)$ in phenomenology is smaller for $t \sim 1\,\mathrm{fm}$ and larger for $t \gtrsim 2\,\mathrm{fm}$ than RBC/UKQCD and BMWc

Deficit in tail on finite lattice normal and corrected by treatment at large t and finite-volume corrections

Summary and conclusions

- Lattice computation of $a_\mu^{\text{LO-HVP}}$ has total error $\sim 2\% \gg\sim 0.4\%$ from phenomenology
- Some results are consistent with no new physics and phenomenology, others with phenomenology and new physics
- Difference comes from *ud* contribution and most probably from treatment of long-distance physics
- Comparison of *ud* time moments suggests:
  - larger intermediate-distance contribution in BMWc 17 & RBC/UKQCD 18
  - larger long-distance contribution in HPQCD 16, associated with model description
- Very useful to have first two moments (slope and curvature of $\Pi(Q^2)$) from all lattice collaborations
  - principally for *ud* and simulation per simulation
  - give separately raw-lattice contribution, long distance modeling and other corrections

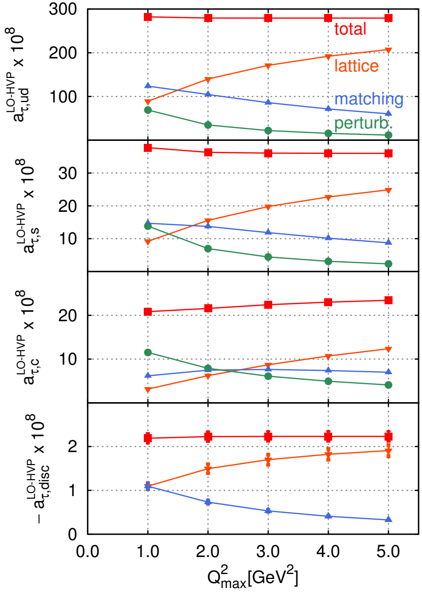
Summary and conclusions

- Detailed studies of long-distance contributions (see e.g. Georg & Aaron talks) are necessary
- Ensure adequate matching of lattice results to pQCD
- With current results, too early to make detailed comparisons with dispersive approach
  - all flavor and QED + SIB contributions must be included
  - continuum and infinite-volume limits must be taken (already done)
  - errors must be significantly reduced
- However, if independent, detailed agreement with comparable errors is shown, or differences understood, then combination of lattice and phenomenology (e.g. RBC/UKQCD 18) may deliver a reliable 0.2% $a_\mu^{\text{LO-HVP}}$
- Approaches such as Mellin-Barnes (see e.g. Eduardo's talk), which allow to maximize use of information in a model independent way, may be very helpful for that task

Matching to perturbation theory

Consider separation ($\ell = e,\ \mu,\ \tau$)

$$
\begin{aligned}
a_{\ell,f}^{\text{LO-HVP}} \quad = \quad & a_{\ell,f}^{\text{LO-HVP}}(Q \le Q_{\text{max}}) \\
& + \gamma_\ell(Q_{\text{max}})\ \hat{\Pi}^f(Q^2_{\text{max}}) \\
& + \Delta^{\text{pert}} a_{\ell,f}^{\text{LO-HVP}}(Q > Q_{\text{max}})
\end{aligned}
$$



- Compute $\Delta^{\text{pert}} a_{\ell,f}^{\text{LO-HVP}}(Q > Q_{\text{max}})$ using $R_{\text{pert}}(s)$ to $O(\alpha_s^4)$ from Harlander et al '03
- Not relevant for $\ell = e, \mu$ but important for τ
- Perfect matching of continuum lattice results for $Q^2_{\text{max}} \ge 2\,\text{GeV}^2$

  $\rightarrow$ control $\hat{\Pi}(Q^2)$ up to $Q^2 \rightarrow \infty$
- Get matching systematic from considering $Q^2_{\text{max}} = 2$ and $5\,\text{GeV}^2$

Time window: lattice + phenomenology

- $C(t)$ may be more precise in certain euclidean time ranges on lattice than in phenomenology

$\rightarrow$ combine lattice with phenomenology to reduce error (RBC/UKQCD 17 & 18)