

HVP Overview: The Status from the KEK workshop



Aida X. El-Khadra
(University of Illinois
and Fermilab)



Second Plenary Workshop of the Muon $g-2$ Theory Initiative, Helmholtz-Institut Mainz, 18-22 June 2018

Outline

- Introduction and Overview
- Experimental inputs
- Dispersive HVP
- Lattice HVP
- MUonE

First Workshop of the Muon $g-2$ Theory Initiative

3-6 June 2017 *Q Center*
US/Central timezone

Search

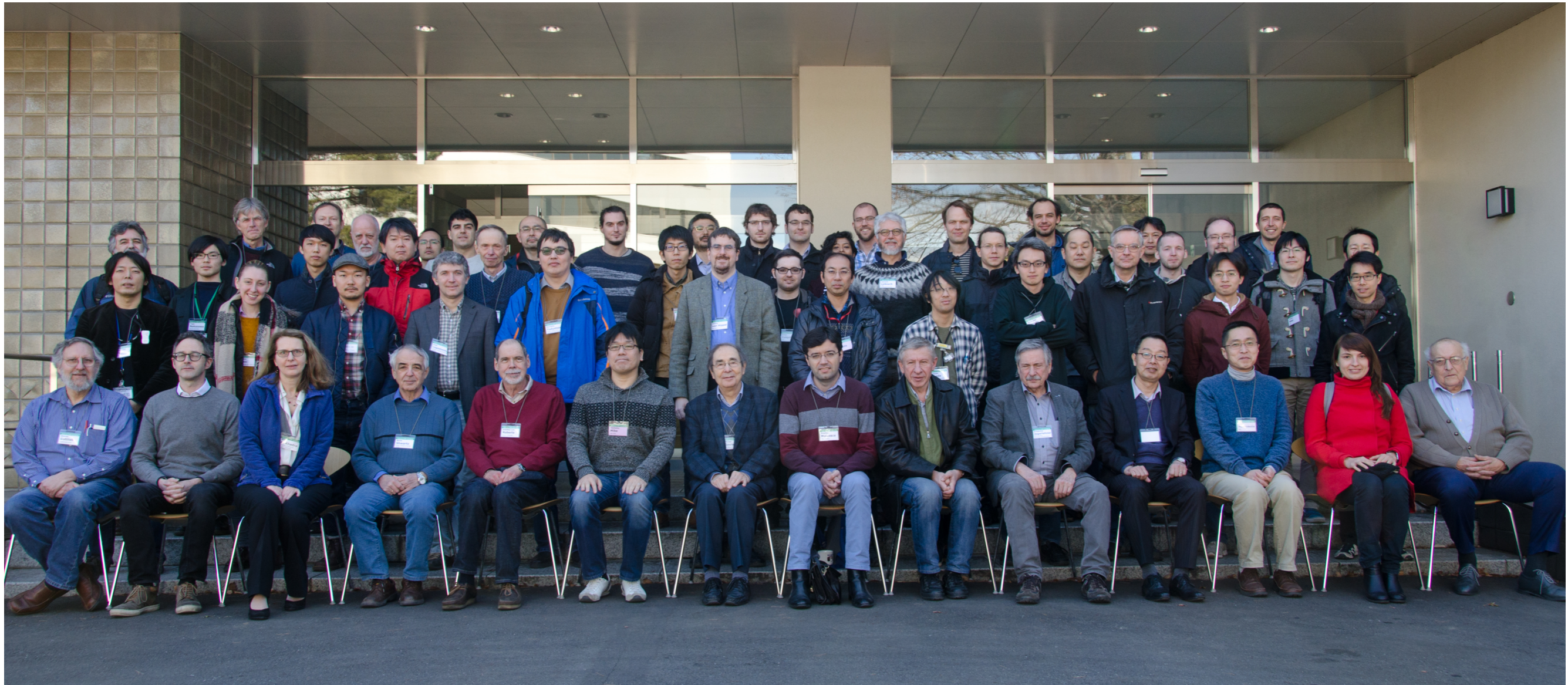


66 registered participants, 40 talks, 15 discussion sessions (525 minutes)

Workshop on hadronic vacuum polarization contributions to muon $g-2$

February 12-14, 2018
KEK, Tsukuba, Japan

<http://www-conf.kek.jp/muonHVPws/index.html>



70 registered participants, 28 talks, 6 discussion sessions (330 minutes)

KEK workshop

28 talks + 6 discussion sessions

- **joint session with KEK-PH meeting:**

P. Urquijo for Belle II (overview), S. Ganguly for Fermilab E989, T. Yamazaki for J-PARC E-34, H. Davoudiasl on BSM and $g-2$.

- **Inputs from $e+e-$ experiments:**

Y. Maeda for Belle II, M. Davier for BaBar, C. Redmer for BESIII, S. Serednyakov for SND, I. Logashenko for CMD-3, K. Todyshev for KEDR, S. Müller for KLOE, H. Czyż on MC generators

- **dispersive HVP:**

F. Jegerlehner, B. Malaescu for DHMZ, A. Keshavarzi for KNT, M. Benayoun on HLS model, M. Hoferichter (disp. constraints on pion vector form factor)

- **Lattice HVP:**

C. Lehner+A. Meyer+M. Bruno for RBC/UKQCD, G. von Hippel for Mainz, L. Lellouch +K.Miura for BMW, R. Van de Water for FNAL/MILC/HPQCD, E. Shintani for PACS

- **misc theory and new ideas:**

M. Nio on QED corrections (8th and 10th order), J. Bijnens (analytic results), M. Passera on MUonE, M. Marinkovic on Lattice for MUonE.

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D. Kawall for Fermilab E989, T. Mibe for J-PARC E-34

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S. Eidelman (review+outlook), E. Solodov (Novosibirsk), M. Davier (tensions), S. Tracz + H. Czyż on radiative corrections and MC generators

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updates from RBC/UKQCD, Mainz, FNAL/MILC/HPQCD, BMW, ETM + discussions on long distance effects, FV effects, QED+SIB, comparisons and cross checks

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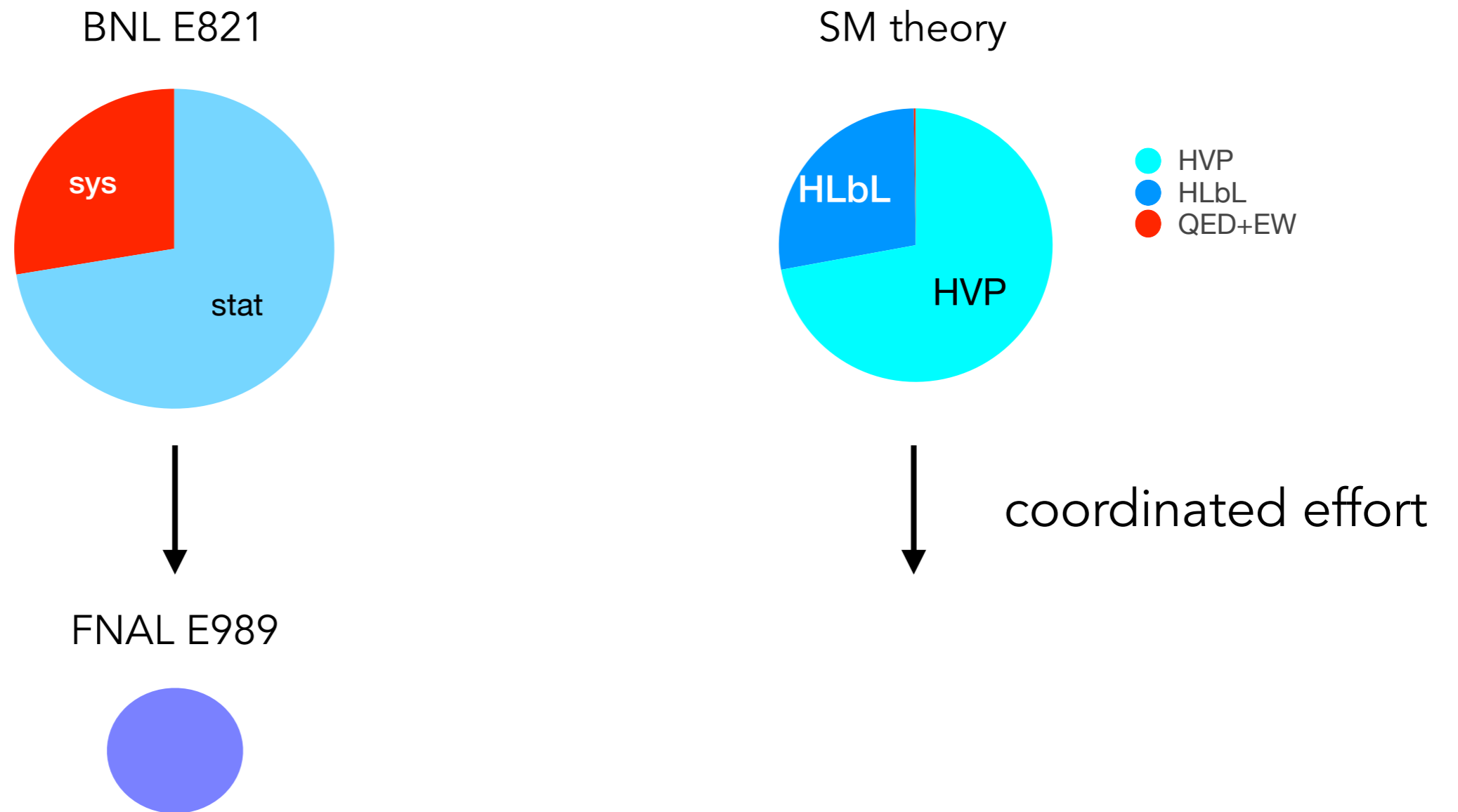
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- misc theory and new ideas:

analytical FV (A. Portelli), Hybrid approach (K. Schilcher), Mellin-Barnes (E. de Rafael), tau decays (M. Bruno), MUonE (C. Carloni, M. Marinkovic)

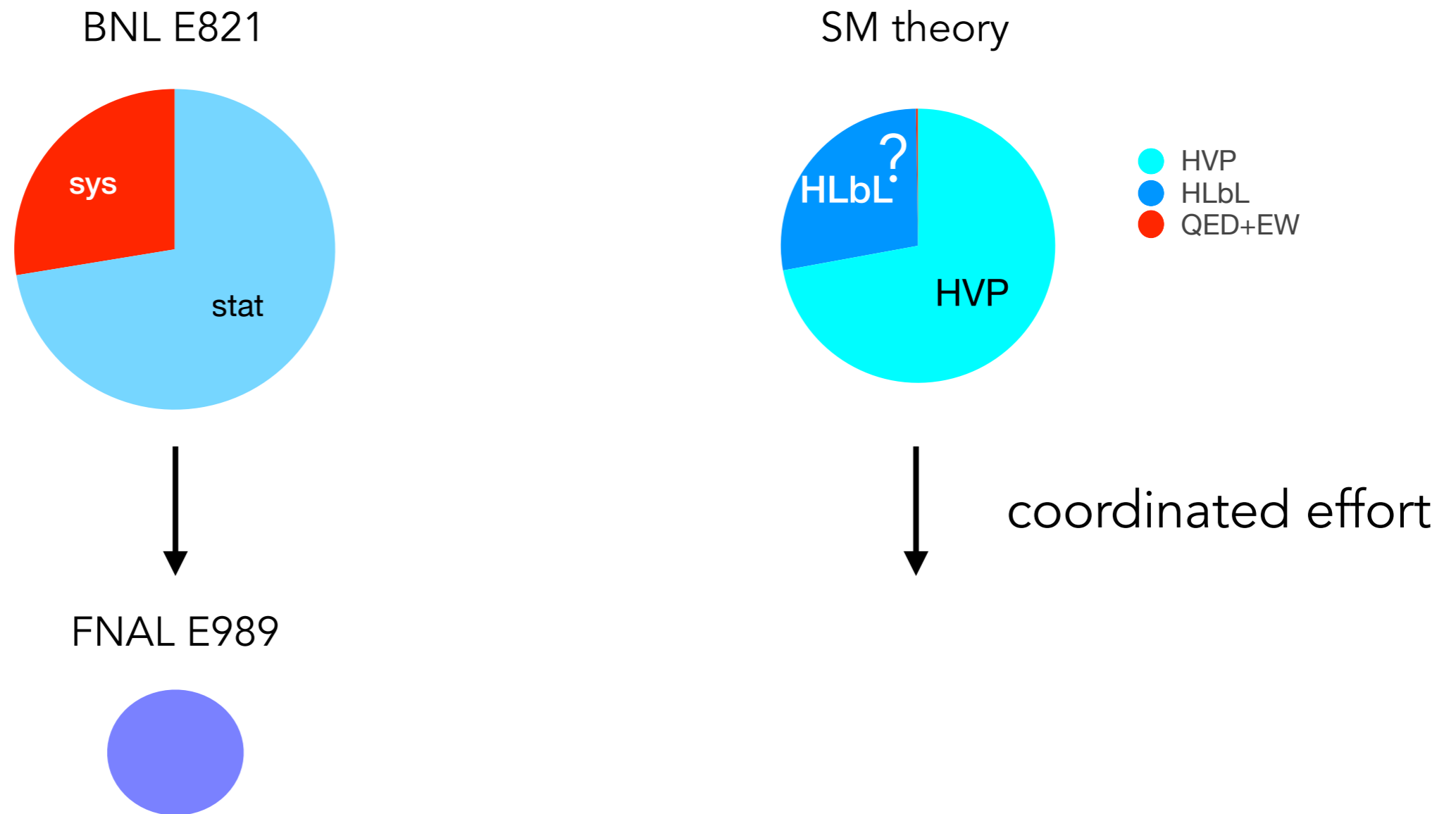
Introduction

Error budget comparison

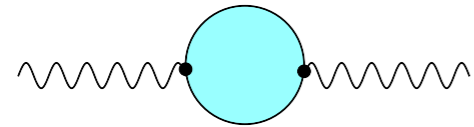
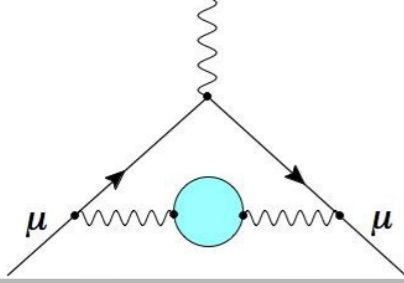


Introduction

Error budget comparison



Hadronic vacuum polarization



$$\hat{\Pi}(q^2) = \Pi(q^2) - \Pi(0)$$

$$\Pi_{\mu\nu} = \int d^4x e^{iqx} \langle j_\mu(x) j_\nu(0) \rangle = (q_\mu q_\nu - q^2 g_{\mu\nu}) \Pi(q^2)$$

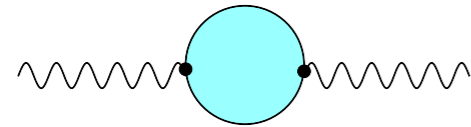
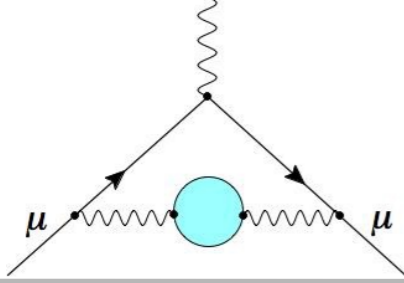
Leading order HVP correction:

$$a_\mu^{\text{HVP,LO}} = \left(\frac{\alpha}{\pi}\right)^2 \int dq^2 \omega(q^2) \hat{\Pi}(q^2)$$

- Use optical theorem and dispersion relation to rewrite the integral in terms of the hadronic e^+e^- cross section:

$$a_\mu^{\text{HVP,LO}} = \frac{m_\mu^2}{12\pi^3} \int ds \frac{\hat{K}(s)}{s} \sigma_{\text{exp}}(s)$$

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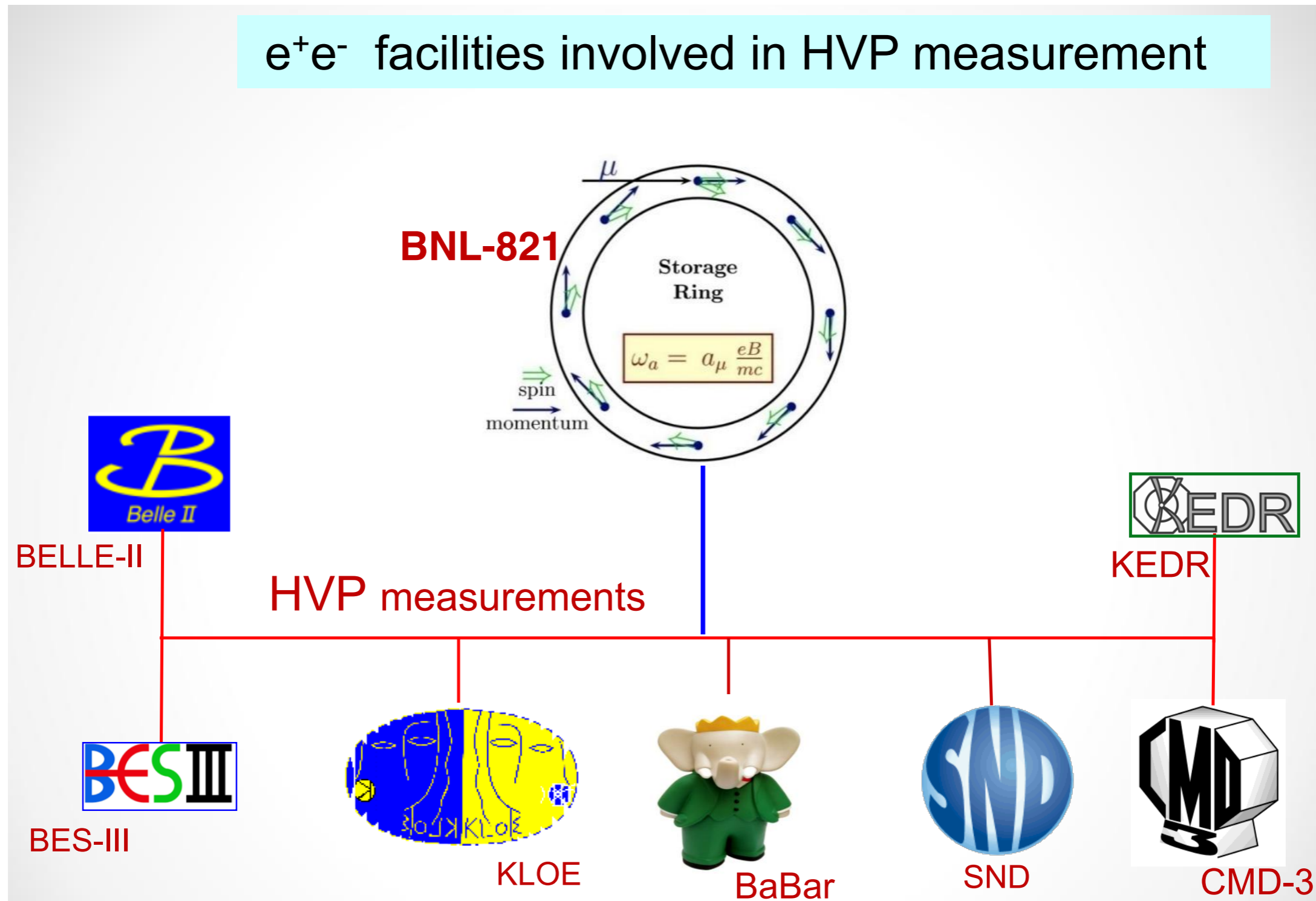
Dominant contributions from low energies

$\pi^+\pi^-$ channel: 73% of total $a_\mu^{\text{HVP,LO}}$

Experimental Inputs

S. Serednyakov (for SND) @ HVP KEK workshop

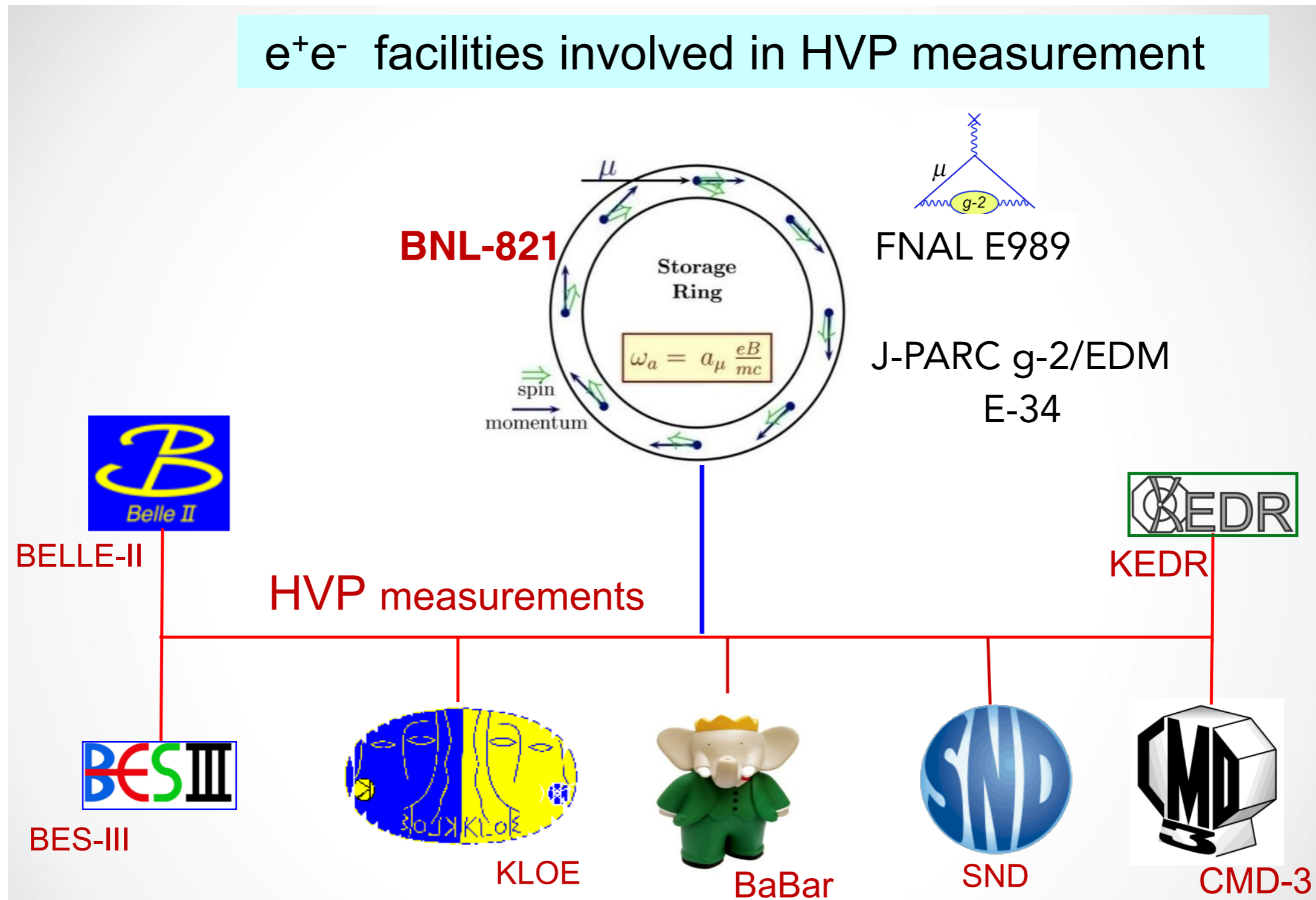
e^+e^- facilities involved in HVP measurement

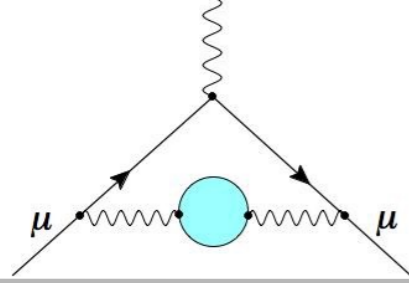


Experimental Inputs

S. Serednyakov (for SND) @ HVP KEK workshop

e^+e^- facilities involved in HVP measurement





Hadronic vacuum polarization

- ◆ Target: $\sim 0.2\%$ total error
- ◆ Dispersion relation + experimental data for $e^+e^- \rightarrow$ hadrons (and τ data)
 - current uncertainty $\sim 0.4-0.5\%$
 - can be improved with more precise experimental data
 - new experimental measurements expected/ongoing at BaBar, BES-III, Belle-II, CMD-3, SND, KEDR, KLOE,....
- ◆ Challenges:
 - below ~ 2 GeV: sum ~ 30 exclusive channels: $2\pi, 3\pi, 4\pi, 5\pi, 6\pi, 2K, 2K\pi, 2K2\pi, \eta\pi, \dots$ (use isospin relations for missing channels)
 - above ~ 1.8 GeV:
 - inclusive, pQCD (away from flavor thresholds)
 - + narrow resonances ($J/\psi, \Upsilon, \dots$)
 - Combine data from different experiments/measurements:
 - understanding correlations, sources of sys. error, tensions...
 - include FS radiative corrections

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Experimental Inputs

Y. Maeda (for Belle II) @ HVP KEK workshop

- Belle II plans to analyze $\pi\pi$ channel using ISR data
 - MC studies of trigger efficiency and backgrounds
-
- Several simulation studies are performed for the $ee \rightarrow \pi\pi\gamma$ mode, which shows
 - 100% L1 trigger efficiency for events with large angle ISR
 - BG level is found to be competitive to BaBar analysis with tentative event selection and PID
 - further studies
 - selection optimization
 - effect of beam background

Experimental Inputs

M. Davier (for BaBar) @ HVP KEK workshop

almost complete set of exclusive hadronic e^+e^- annihilation channels up to 2 GeV

$\pi^+\pi^-$

K^+K^-

$\pi^+\pi^-\pi^0$

$2(\pi^+\pi^-), K^+K^-\pi^+\pi^-, K^+K^-2\pi^0, 2(K^+K^-)$

$K_S^0 K^+ \pi^-, K^+K^-\pi^0, K^+K^-\eta$

$2(\pi^+\pi^-\pi^0), 2(\pi^+\pi^-\eta), K^+K^-\pi^+\pi^-\pi^0, K^+K^-\pi^+\pi^-\eta$

$3(\pi^+\pi^-), 2(\pi^+\pi^-\pi^0), 2(\pi^+\pi^-)K^+K^-$

$\Phi f^0(980)$

$\rho^-\bar{\rho}$

$\Lambda\bar{\Lambda}, \Lambda\bar{\Sigma}^0, \Sigma^0\bar{\Sigma}^0$

$K_S^0 K_L^0, K_S^0 K_L^0 \pi^+\pi^-, K_S^0 K_S^0 \pi^+\pi^-$

K^+K^- large Q^2

$K_S^0 K^+ \pi^-\pi^0, K_S^0 K^+ \pi^-\eta$

$K_S^0 K_L^0 \pi^0, K_S^0 K_L^0 \pi^0 \pi^0$

$\pi^+\pi^-2\pi^0$

$\eta\pi^+\pi^-$

$J/\psi (\mu^+\mu^-)$

$\pi^+\pi^-, \mu^+\mu^-$ LO ISR-FSR interference

in progress:

$\pi^+\pi^-$ new method + full data sample

not covered:

$\pi^+\pi^-3\pi^0, \pi^+\pi^-4\pi^0, \pi^+\pi^-\pi^0$ below 1.05 GeV, ≥ 7 hadrons

PRL 2009; PRD 2012

PRD 2013

PRD 2004

PRD 2007; PRD 2012; PRD 2012

PRD 2005; PRD 2008

PRD 2007

PRD 2006

PRD 2006; PRD 2007

PRD 2006,

PRD 2007

PRD 2014

PRD 2015

PRD 2017

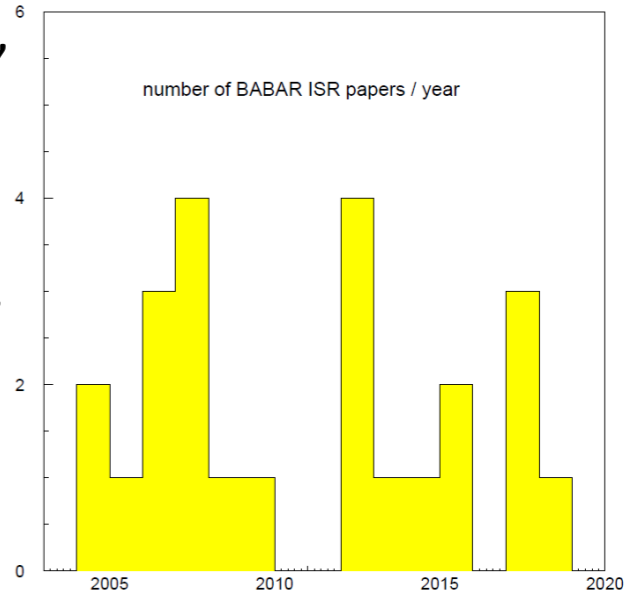
PRD 2017

PRD 2017

PRD 2018

PRD 2004

PRD 2015



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PRD 2007; PRD 2012; PRD 2012

$K_S^0 K^+ \pi^-, K^+K^-\pi^0, K^+K^-\eta$

PRD 2005; PRD 2008

$2(\pi^+\pi^-)\pi^0, 2(\pi^+\pi^-)\eta, K^+K^-\pi^+\pi^-\pi^0, K^+K^-\pi^+\pi^-\eta$

PRD 2007

$3(\pi^+\pi^-), 2(\pi^+\pi^-\pi^0), 2(\pi^+\pi^-)K^+K^-$

PRD 2006

$\Phi f^0(980)$

PRD 2006; PRD 2007

$p\bar{p}$

PRD 2006,

$\Lambda\bar{\Lambda}, \Lambda\bar{\Sigma}^0, \Sigma^0\bar{\Sigma}^0$

PRD 2007

$K_S^0 K_L^0, K_S^0 K_L^0 \pi^+\pi^-, K_S^0 K_S^0 \pi^+\pi^-$

PRD 2014

K^+K^- large Q^2

PRD 2015

$K_S^0 K^+ \pi^-\pi^0, K_S^0 K^+ \pi^-\eta$

PRD 2017

$K_S^0 K_L^0 \pi^0, K_S^0 K_L^0 \pi^0 \pi^0$

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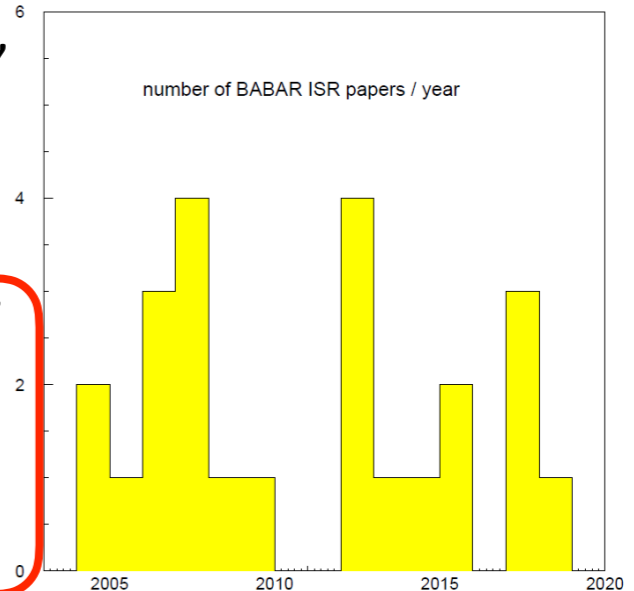
in progress:

$\pi^+\pi^-$ new method + full data sample

by the end of 2018?

not covered:

$\pi^+\pi^-3\pi^0, \pi^+\pi^-4\pi^0, \pi^+\pi^-\pi^0$ below 1.05 GeV, ≥ 7 hadrons



Experimental Inputs

C. Redmer (for BESIII) @ HVP KEK workshop

- Hadronic cross section measurements at BESIII

- Scan, tagged and untagged ISR methods

- Competitive accuracy

- $\pi^+\pi^-$ result confirms $a_\mu^{\text{theo,SM}} - a_\mu^{\text{exp}} > 3\sigma$

- Preliminary results on $e^+e^- \rightarrow \pi^+\pi^-\pi^0$, $e^+e^- \rightarrow \pi^+\pi^-\pi^0\pi^0$ and $e^+e^- \rightarrow \pi^+\pi^-3\pi^0$

- Measurement of R value ongoing, 3% accuracy targeted

- Pion form factor to be evaluated in additional mass regions from ISR and scan data

- Additional exclusive final states in preparation

Experimental Inputs

S. Serednyakov (for SND) @ HVP KEK workshop

Conclusions

1. **SND** detector is taking data at the **VEPP-2000** e^+e^- collider in the energy range **0.3-2.0** GeV. New data on e^+e^- cross section to hadrons are obtained :

----- $e^+e^- \rightarrow \pi^+\pi^-\pi^0\eta,$

----- $e^+e^- \rightarrow \omega\pi^0\eta,$

----- $e^+e^- \rightarrow K^+K^-,$

----- $e^+e^- \rightarrow \pi^0\gamma$

----- $e^+e^- \rightarrow \eta K^+K^-,$

----- $e^+e^- \rightarrow K_S K_L \pi^0,$

----- $e^+e^- \rightarrow \eta \pi^+\pi^-,$

2. The $e^+e^- \rightarrow p\bar{p}+n\bar{n}$ step-like cross section does not lead to a step in the total cross section

3. The contribution of newly measured to the total hadronic cross section is estimated

Experimental Inputs

I. Logashenko (for CMD-3) @ HVP KEK workshop

At VEPP-2000 we do **exclusive** measurement of $\sigma(e^+e^- \rightarrow \text{hadrons})$.

- 2 charged

$$e^+e^- \rightarrow \pi^+\pi^-, K^+K^-, K_S K_L, p\bar{p}$$

published
in progress

- 2 charged + γ 's

$$e^+e^- \rightarrow \pi^+\pi^-\pi^0, \pi^+\pi^-\eta, K^+K^-\pi^0, K^+K^-\eta, K_S K_L \pi^0, \pi^+\pi^-\pi^0\eta, \\ \pi^+\pi^-\pi^0\pi^0, \pi^+\pi^-\pi^0\pi^0\pi^0, \pi^+\pi^-\pi^0\pi^0\pi^0\pi^0$$

- 4 charged

$$e^+e^- \rightarrow \pi^+\pi^-\pi^+\pi^-, K^+K^-\pi^+\pi^-, K_S K^*$$

- 4 charged + γ 's

$$e^+e^- \rightarrow \pi^+\pi^-\pi^+\pi^-\pi^0, \pi^+\pi^-\eta, \pi^+\pi^-\omega, \pi^+\pi^-\pi^+\pi^-\pi^0\pi^0, K^+K^-\eta, K^+K^-\omega$$

- 6 charged

$$e^+e^- \rightarrow \pi^+\pi^-\pi^+\pi^-\pi^+\pi^-$$

- γ 's only

$$e^+e^- \rightarrow \pi^0\gamma, \eta\gamma, \pi^0\pi^0\gamma, \pi^0\eta\gamma, \pi^0\pi^0\pi^0\gamma, \pi^0\pi^0\eta\gamma$$

- other

$$e^+e^- \rightarrow n\bar{n}, \pi^0 e^+ e^-, \eta e^+ e^-$$

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published
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- 2 charged + γ 's

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- In 2013-2016 the collider and the CMD-3 detector have been upgraded and the data taking was resumed in 2017 and about 65 1/pb were collected so far.
- Data analysis of exclusive modes of $e^+e^- \rightarrow \text{hadrons}$ is in progress. Many results have been published.

- Other

$$e^+e^- \rightarrow n\bar{n}, \pi^0 e^+e^-, \eta e^+e^-$$

Experimental Inputs

K. Todyshev (for KEDR)
@ HVP KEK workshop

■ J/ψ results:

$$\Gamma_{ee}(J/\psi) = 5.550 \pm 0.056 \pm 0.089 \text{ keV}$$

$$\Gamma_{ee}(J/\psi) \times \mathcal{B}_{\text{hadrons}}(J/\psi) = 4.884 \pm 0.048 \pm 0.078 \text{ keV}$$

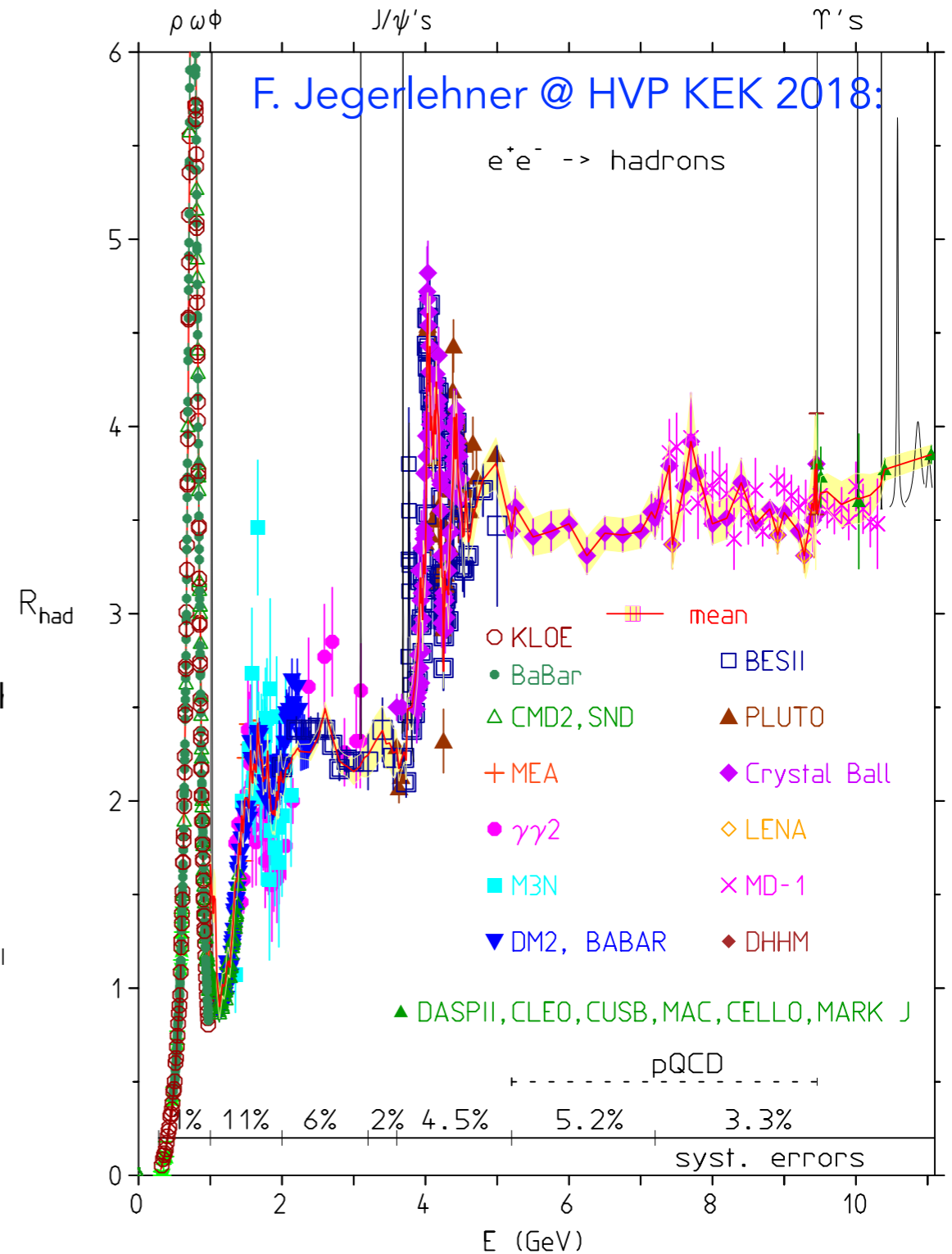
$$\Gamma_{ee}(J/\psi) \times \mathcal{B}_{ee}(J/\psi) = 0.3331 \pm 0.0066 \pm 0.0040 \text{ keV}$$

■ $\psi(2S)$ results:

$$\Gamma_{ee} = 2.282 \pm 0.015 \pm 0.042 \text{ keV}$$

$$\Gamma_{ee} \times \Gamma_{\mu\mu} / \Gamma = 19.3 \pm 0.3 \pm 0.5 \text{ eV}$$

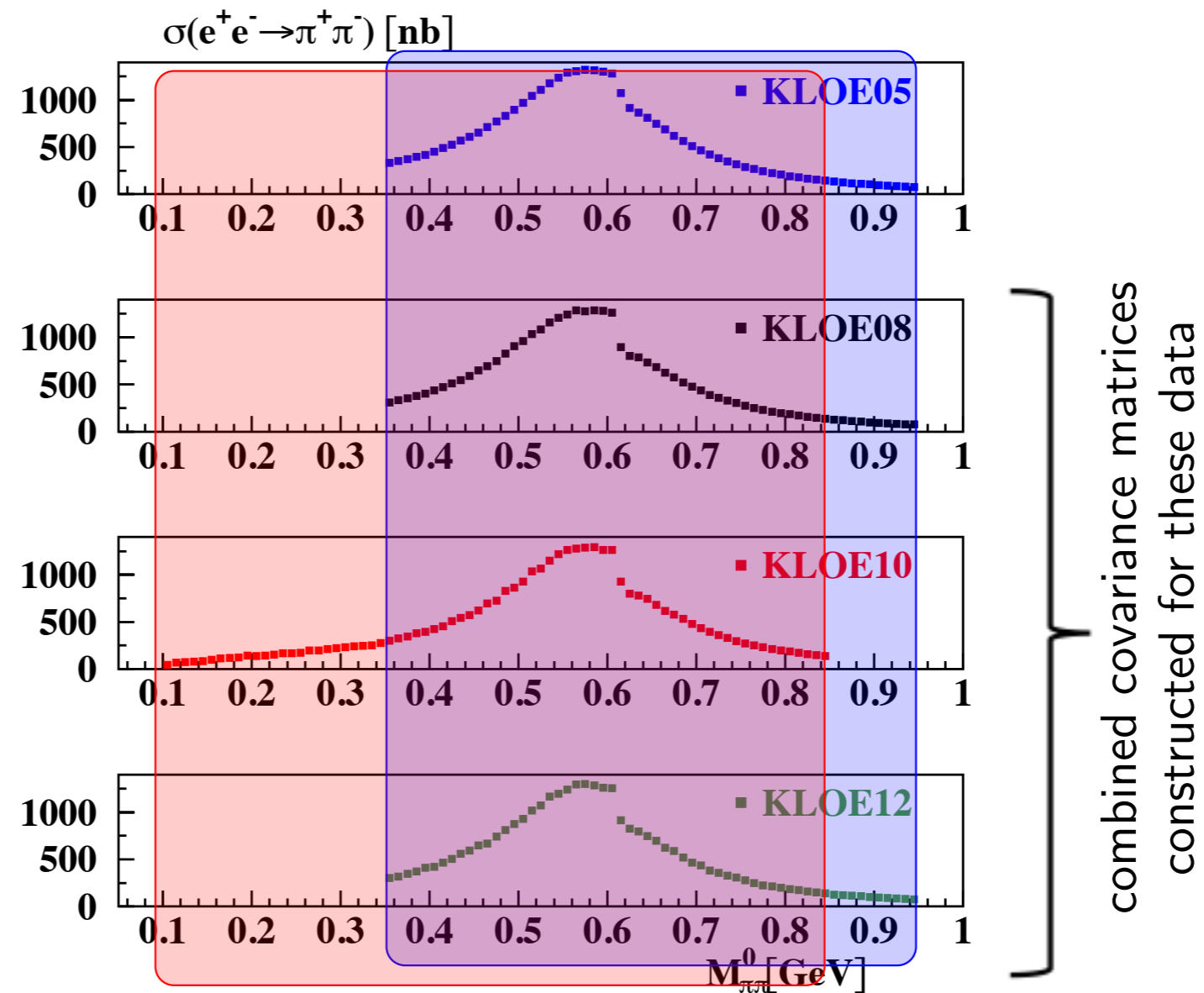
- We have determined the values of R at thirteen points of the center-of-mass energy between 1.84 and 3.05 GeV. The achieved accuracy is about or better than 3.9% at most of energy points with a systematic uncertainty less than 2.4%.
- We measured the values of R at seven points of the center-of-mass energy between 3.12 and 3.72 GeV. The total achieved accuracy is about or better than 3.3% at most of energy points with a systematic uncertainty of about 2.1%.
- R measurement in the energy range 3.077 – 3.72 GeV after detector repair and upgrade: analysis in progress.
- We plan to measure R value in the energy range 5-7 GeV



Experimental Inputs

S. Müller (for KLOE) @ HVP KEK workshop

The KLOE data sets

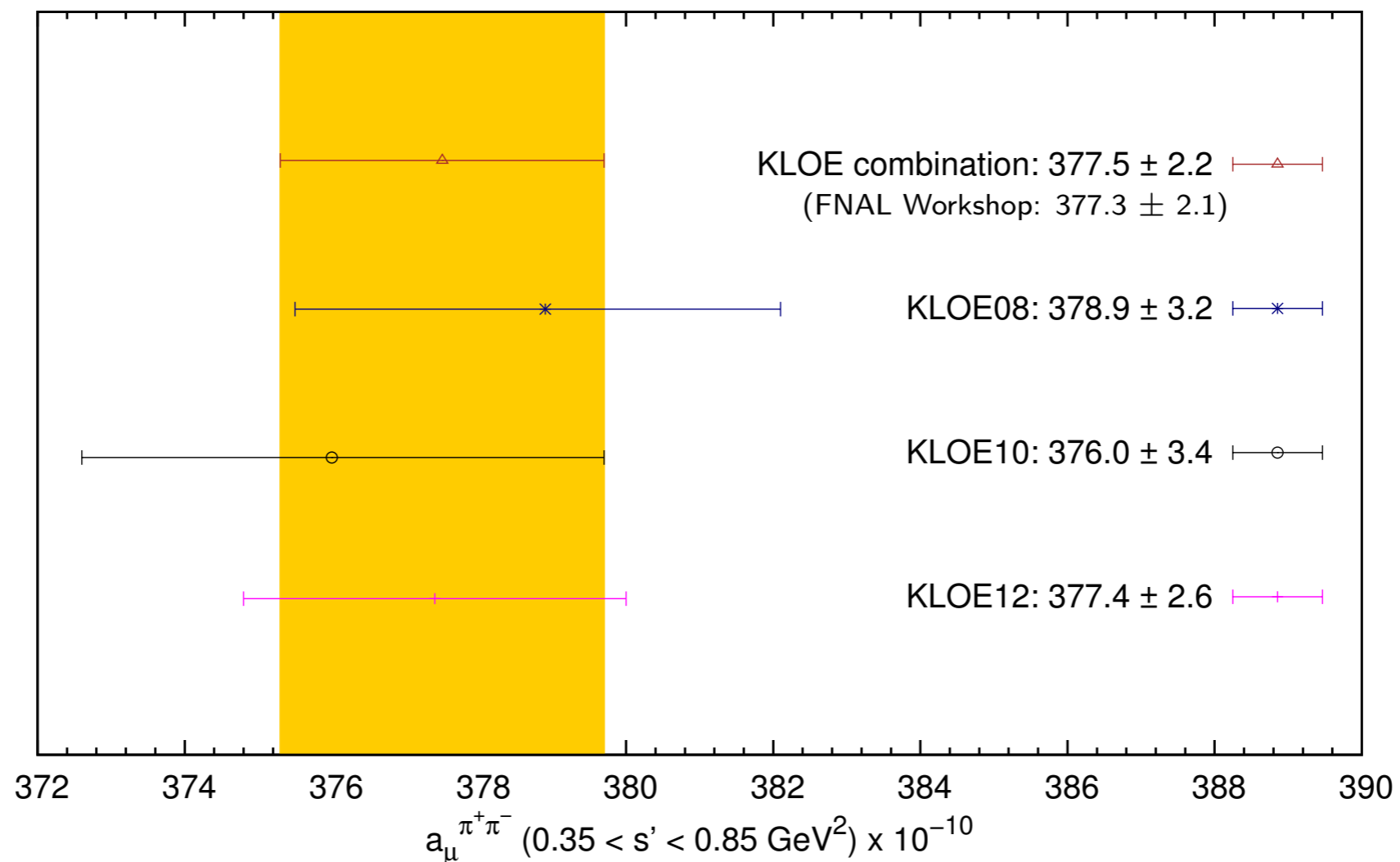


Experimental Inputs

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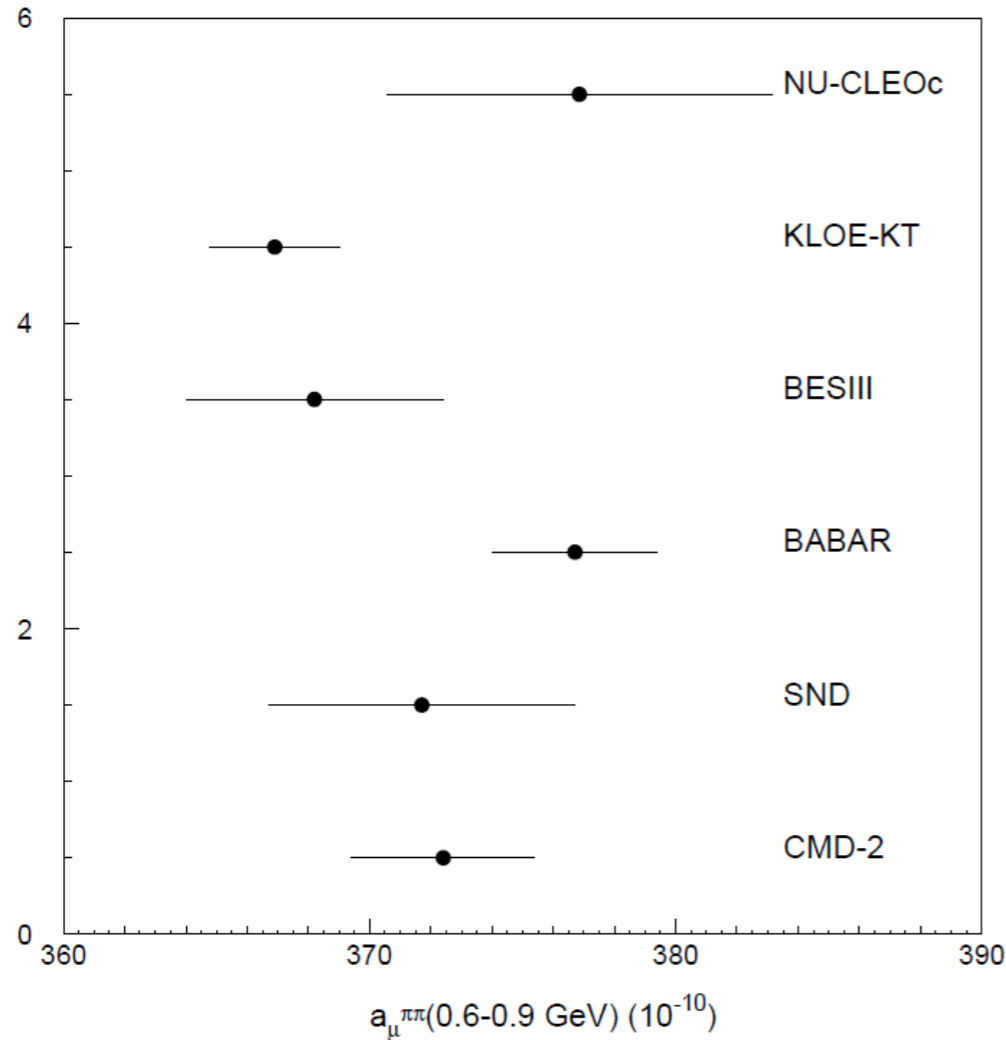
Combination of KLOE data sets

Estimates of $\Delta a_\mu^{\pi\pi}$ in the range $0.35 \leq s' \leq 0.85 \text{ GeV}^2$ for the different KLOE analyses and the prel. combination:

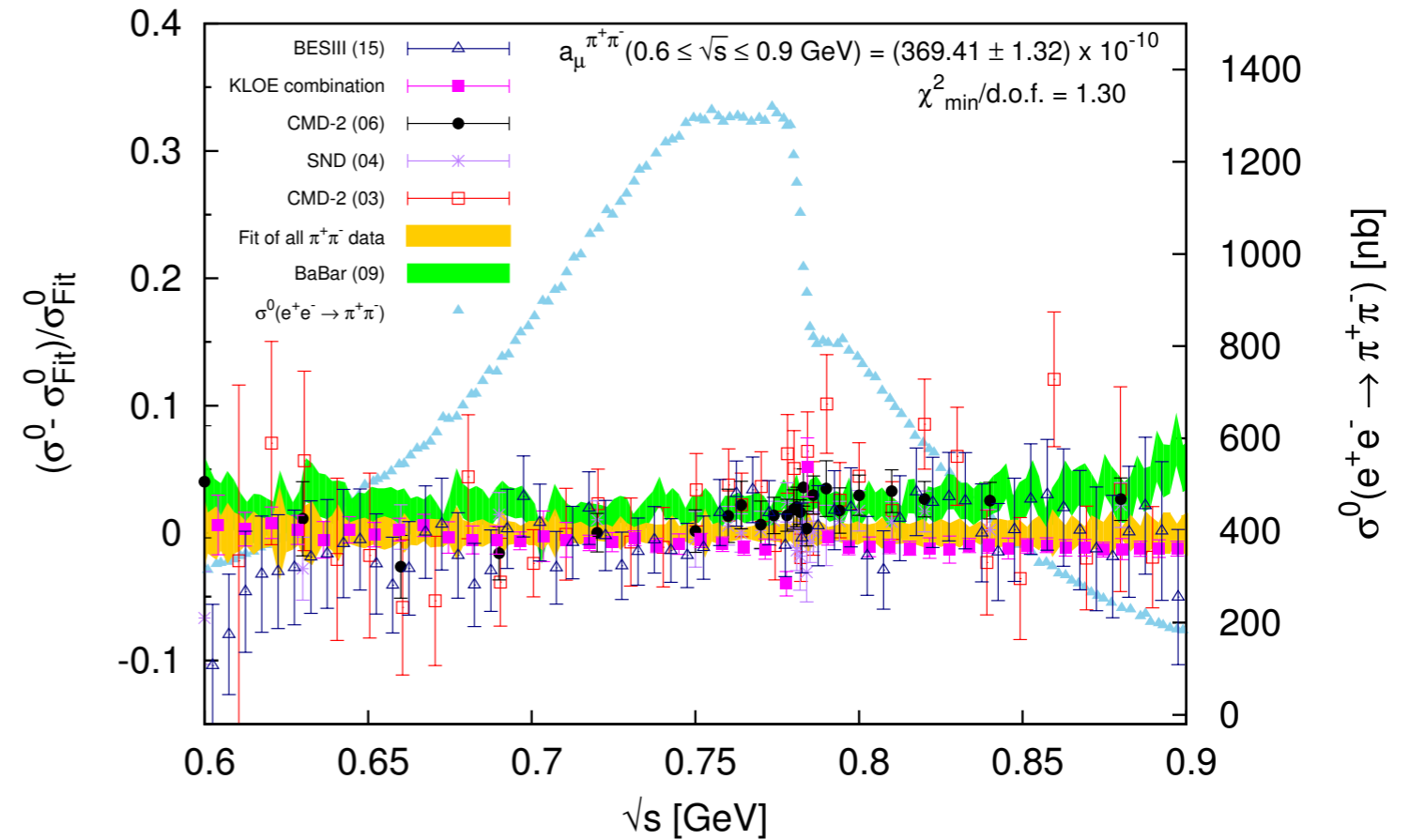


Experimental Inputs

M. Davier @ HVP KEK 2018



A. Keshavarzi @ HVP KEK 2018



Tension in $\pi\pi$ channel between different data sets

Experimental Inputs

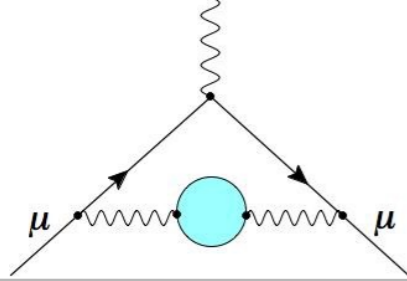
H. Czyż @ HVP KEK 2018 workshop

- ⇒ PHOKHARA and EKHARA in brief
- ⇒ Recent developments in PHOKHARA and EKHARA
 - ⇒ χ_{c_i} production: PHOKHARA and EKHARA
 - ⇒ Models: $\mathcal{L}_{\gamma\gamma P}$, $\mathcal{L}_{\gamma V}$, $\mathcal{L}_{V\gamma P}$, $\mathcal{L}_{VV P}$
 - ⇒ PHOKHARA: $e^+e^- \rightarrow P\gamma(\gamma)$
 - ⇒ EKHARA: $e^+e^- \rightarrow e^+e^-P$
 - ⇒ $a_\mu(P)$
- ⇒ Radiative corrections in EKHARA
- ⇒ Radiative corrections in PHOKHARA
 - ⇒ update on NLO corrections by Szymon Tracz

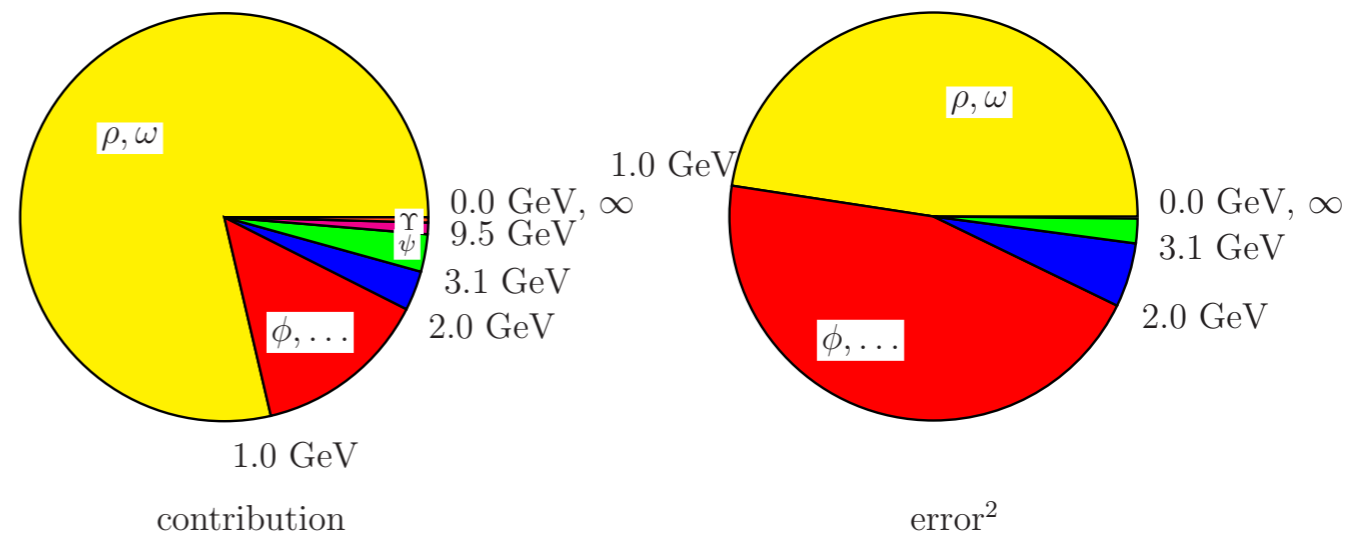
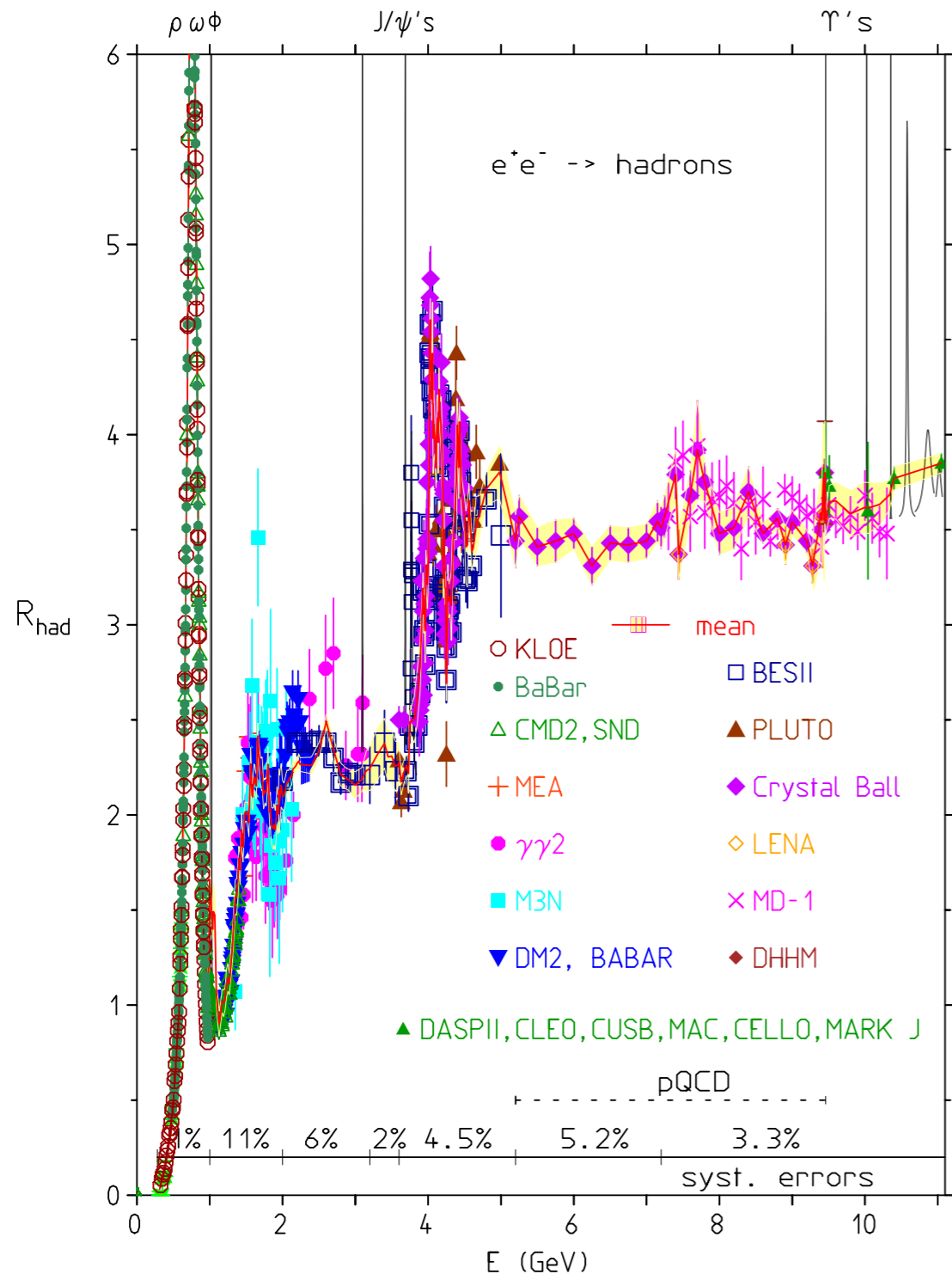
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- Lattice HVP
- MUonE

Hadronic vacuum polarization

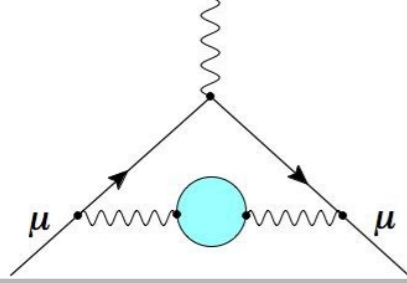


F. Jegerlehner @ HVP KEK 2018:



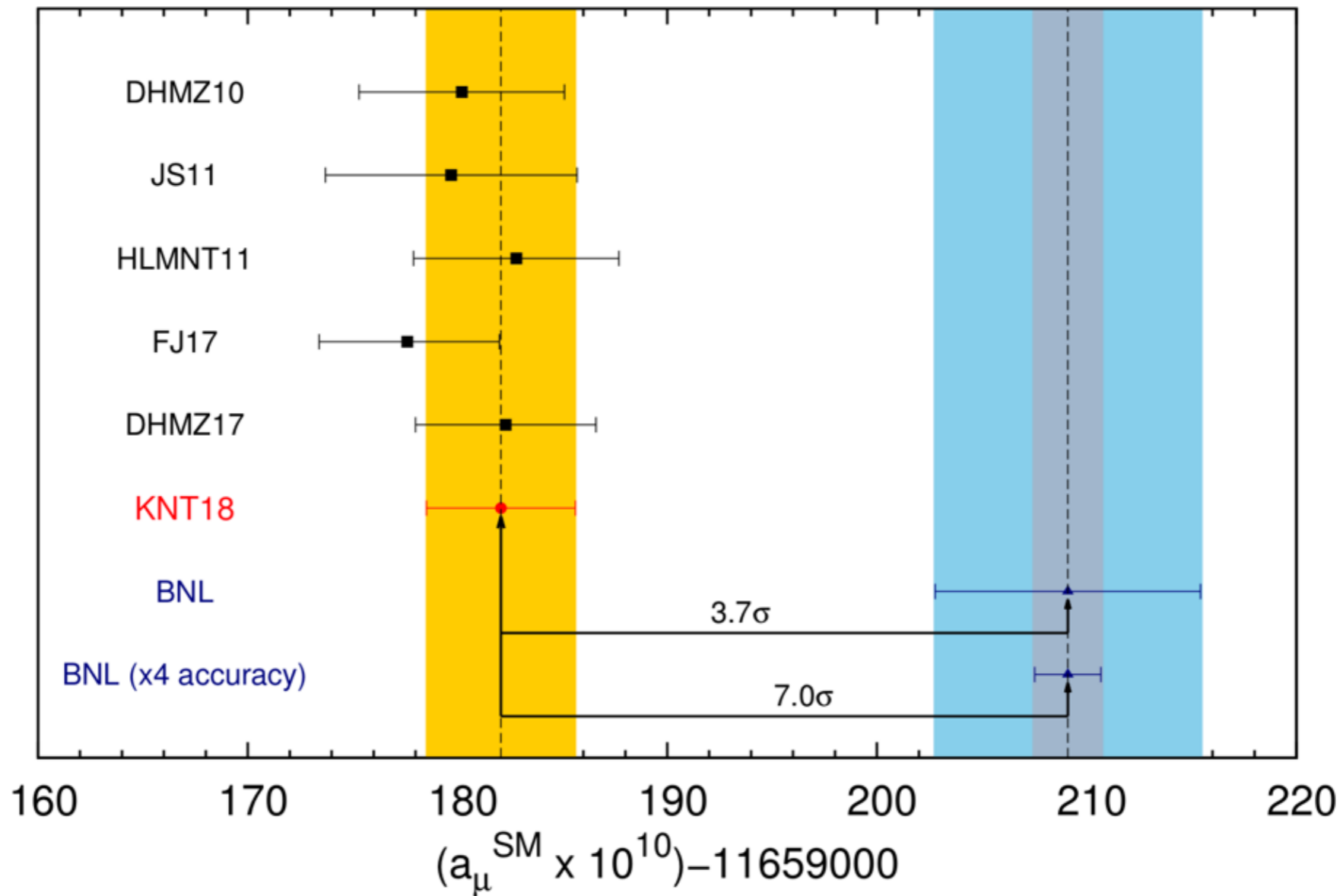
$$a_{\mu}^{\text{had}(1)} = (686.99 \pm 4.21)[687.19 \pm 3.48] 10^{-10}$$

e^+e^- -data based [incl. τ]

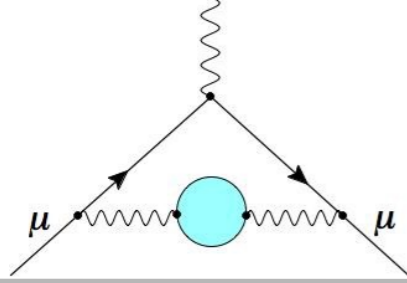


Hadronic vacuum polarization

A. Keshwarzi @ HVP KEK 2018 (arXiv:1802.02995):



M. Benayoun @ HVP KEK 2018: BHLS218: 4.5 σ



Hadronic vacuum polarization

M. Hoferichter @ HVP KEK 2018:

Dispersive constraints on the pion vector form factor

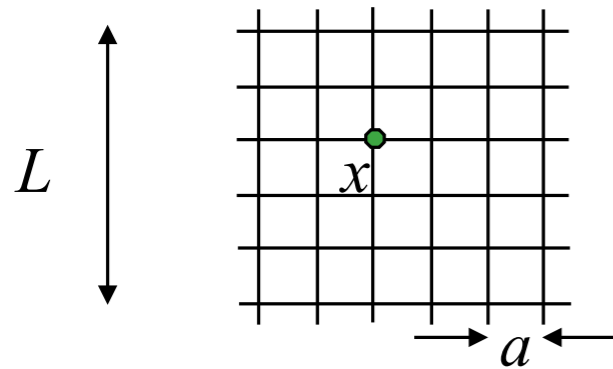
- Overview of constraints from **analyticity** and **unitarity**
- Role of **radiative corrections**
 - define $F_{\pi}^V(s)$ in QCD (**without VP**)
 - with $\pi\pi$ channel described dispersively, no issues with $\rho-\gamma$ mixing
 - radiative corrections to ω **pole parameters** at the level of PDG uncertainties
- Some preliminary fit results
- Outlook
 - Error analysis: both experimental uncertainties and systematics of dispersive fit
 - Combination of data sets in global fit
 - Space-like data, ω parameters
 - Goal: **independent way to assess HVP uncertainty in $\pi\pi$ channel**, accounting for global constraints from analyticity and unitarity

Outline

- Introduction and Overview
- Experimental inputs
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- Lattice HVP
- MUonE

Lattice QCD Introduction

$$\mathcal{L}_{\text{QCD}} = \sum_f \bar{\psi}_f (\not{D} + m_f) \psi_f + \frac{1}{4} \text{tr} F_{\mu\nu} F^{\mu\nu}$$



- ◆ discrete Euclidean space-time (spacing a)
derivatives \rightarrow difference operators, etc...
- ◆ finite spatial volume (L)
- ◆ finite time extent (T)

adjustable parameters

- ❖ lattice spacing: $a \rightarrow 0$
- ❖ finite volume, time: $L \rightarrow \infty, T > L$
- ❖ quark masses (m_f): $M_{H,\text{lat}} = M_{H,\text{exp}}$
 $m_f \rightarrow m_{f,\text{phys}}$
 tune using hadron masses
 extrapolations/interpolations



m_{ud}

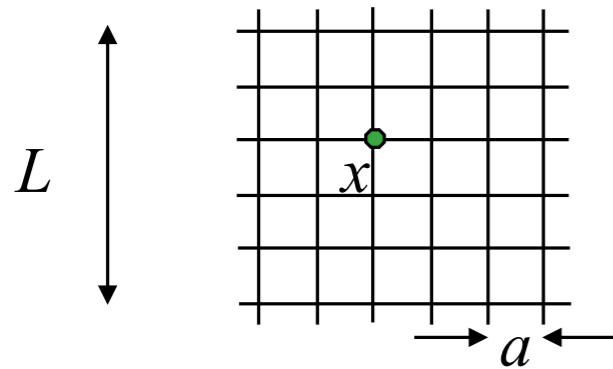
m_s

m_c

m_b

Lattice QCD Introduction

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- ◆ discrete Euclidean space-time (spacing a)
derivatives \rightarrow difference operators, etc...
- ◆ finite spatial volume (L)
- ◆ finite time extent (T)

Integrals are evaluated numerically using monte carlo methods.

adjustable parameters

- ❖ lattice spacing: $a \rightarrow 0$
- ❖ finite volume, time: $L \rightarrow \infty, T > L$
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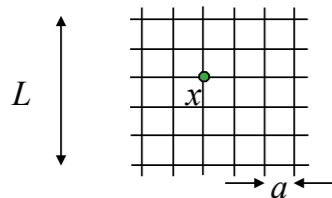


m_{ud}

m_s

m_c

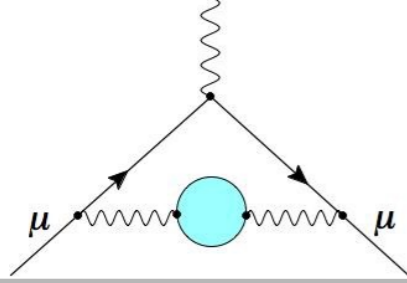
m_b



Lattice QCD Introduction

The State of the Art

- ☆ Lattice QCD calculations of simple quantities (with at most one stable meson in initial/final state) that **quantitatively account for all systematic effects** (discretization, finite volume, renormalization,...) , in some cases with
 - sub percent precision.
 - total errors that are commensurate (or smaller) than corresponding experimental uncertainties.
- ☆ Scope of LQCD calculations is increasing due to continual development of new methods:
 - nucleons and other baryons
 - nonleptonic decays ($K \rightarrow \pi\pi, \dots$)
 - resonances, scattering, long-distance effects, ...
 - QED effects
 - radiative decay rates ...



Hadronic vacuum polarization

Leading order HVP correction:
$$a_{\mu}^{\text{HVP,LO}} = \left(\frac{\alpha}{\pi}\right)^2 \int dq^2 \omega(q^2) \hat{\Pi}(q^2)$$

- Calculate a_{μ}^{HVP} in Lattice QCD:

- ◆ Calculate $\hat{\Pi}(q^2)$ and evaluate the integral

(Blum, PRL 03, Lautrup et al, 71)

- ◆ Time-momentum representation:

reorder the integrations and compute
$$C(t) = \frac{1}{3} \sum_{i,x} \langle j_i(x,t) j_i(0,0) \rangle$$

$$a_{\mu}^{\text{HVP}} = \left(\frac{\alpha}{\pi}\right)^2 \int dt \tilde{\omega}(t) C(t)$$

(Bernecker & Meyer, EPJ 12)

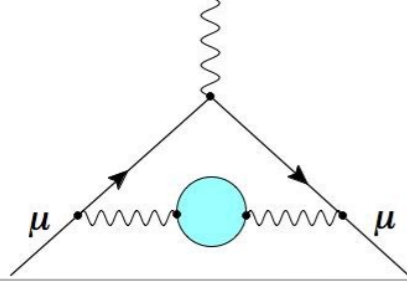
- ◆ Time-moments:

Taylor expand
$$\hat{\Pi}(q^2) = \sum_k q^{2k} \Pi_k$$

(Chakraborty et al, PRD 14)

and compute Taylor coefficients from time moments:

$$C_{2n} = a \sum_t t^{2n} C(t)$$



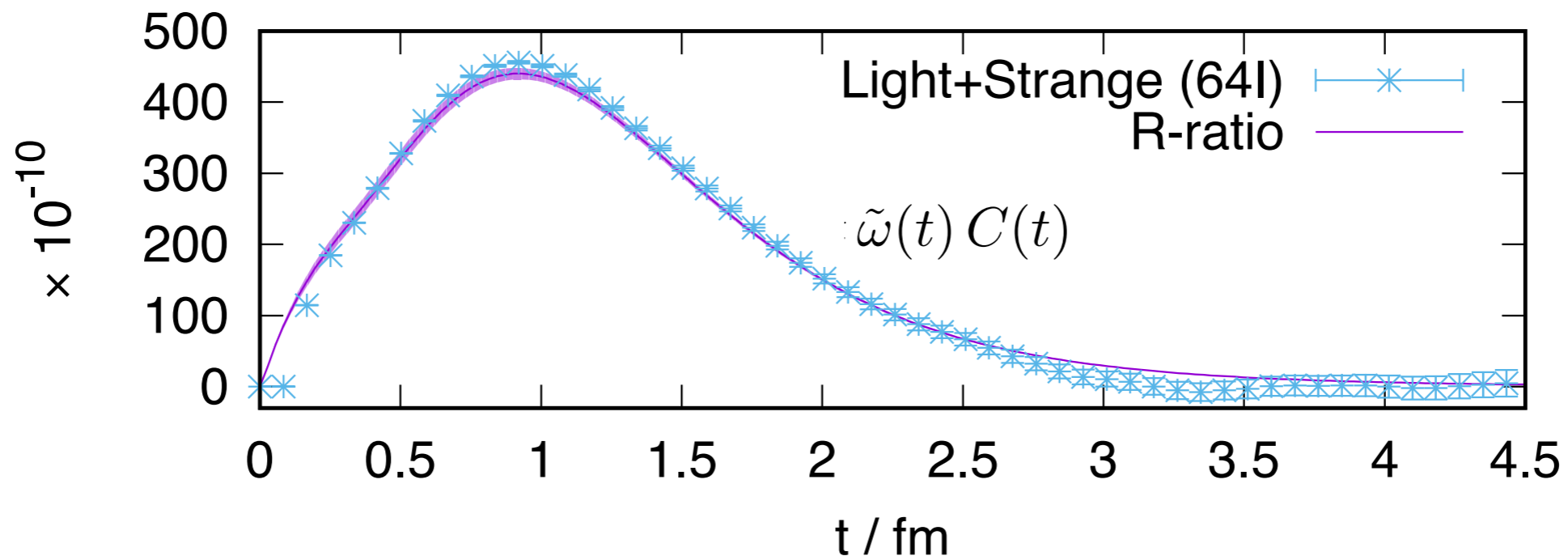
Hadronic vacuum polarization

$$C(t) = \frac{1}{3} \sum_{i,x} \langle j_i(x,t) j_i(0,0) \rangle$$

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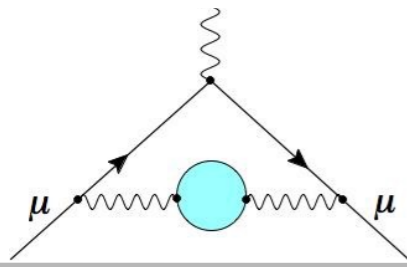
(Bernecker & Meyer, EPJ 12)

C. Lehner @ HVP KEK 2018 (for RBC/UKQCD):



Errors on the lattice data increase rapidly with Euclidean time:

- sensitive to long-distance and finite volume corrections
use EFT methods
- control statistical noise at long distances through fits, including $\pi\pi$ correlation functions, ...



Hadronic vacuum polarization

G. von Hippel @ HVP KEK (for Mainz):

The time-momentum representation – large-time correlator

Large-time modelling of the vector correlator:

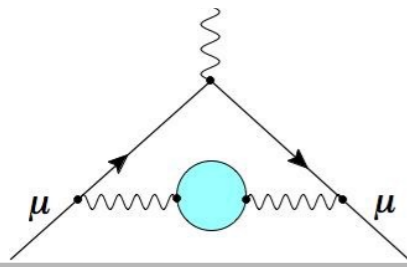
- Extract naive ground-state mass from fit to a smeared correlator, model $G(t)$ as single exponential at large t ;
- Use the large-time approximation

$$G_{\infty}^{\rho\rho}(t) = \frac{1}{48\pi^2} \int_0^{\infty} d\omega \omega^2 \left(1 - \frac{4m_{\pi}^2}{\omega^2}\right)^{\frac{3}{2}} |F_{\pi}(\omega)|^2 e^{-\omega t}$$

NEW!

extracting F_{π} using the Lüscher method → B. Hörz, F. Erben (PhD)

In both cases finite-volume effects can be studied by comparing infinite-volume $G_{\infty}^{\rho\rho}$ with finite-volume $G_L^{\rho\rho}$ reconstructed by summing finite-volume matrix elements using Lüscher method in reverse.



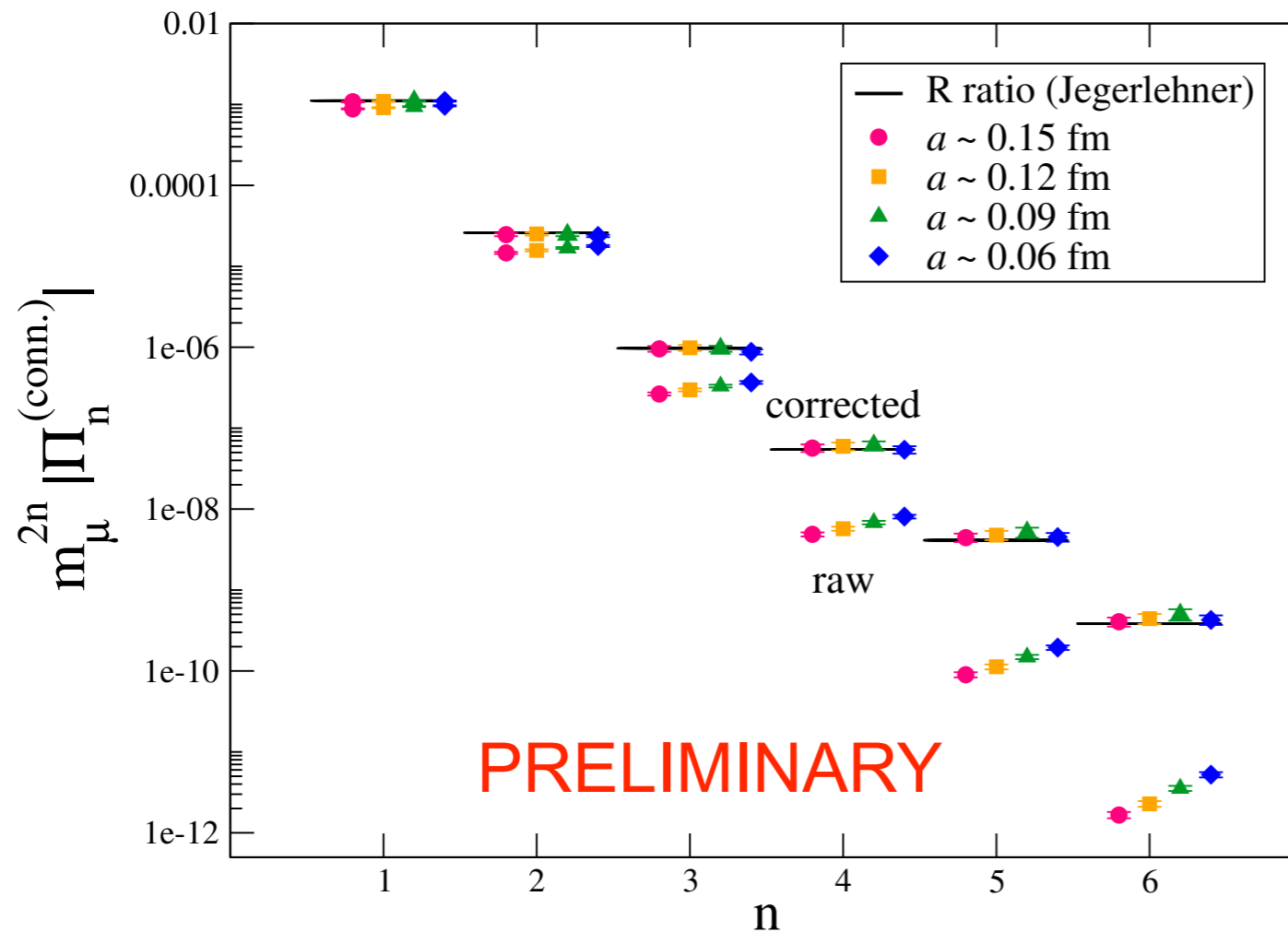
Hadronic vacuum polarization

- ◆ Target: $\sim 0.2\%$ total error
- ◆ Complete lattice QCD results by several groups.
A complete LQCD result ...
 - is based on physical mass ensembles
 - includes disconnected contributions
 - includes QED and strong isospin breaking corrections ($m_u \neq m_d$)
 - includes finite volume corrections, continuum extrapolation
- ◆ current uncertainties at $\sim 2\%$ level
 - Statistical errors grow at large Euclidean times
 - noise reduction methods
 - include two-pion channels into analysis
- ◆ Compare intermediate quantities (Taylor coefficients,...) with R-ratio data.

Compare Taylor coefficients to R-ratio data

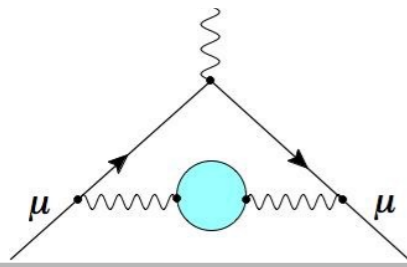
R. Van de Water @ HVP KEK 2018 (for FNAL/MILC/HPQCD):

Compare Taylor coefficients before and after (finite volume, discretization,..) corrections with R-ratio data:



- Corrections are calculated in EFT.
- First comparison performed by RVdW on HPQCD initial's data set **after** HPQCD paper was posted.

Lowest moments make the largest contributions to a_μ .



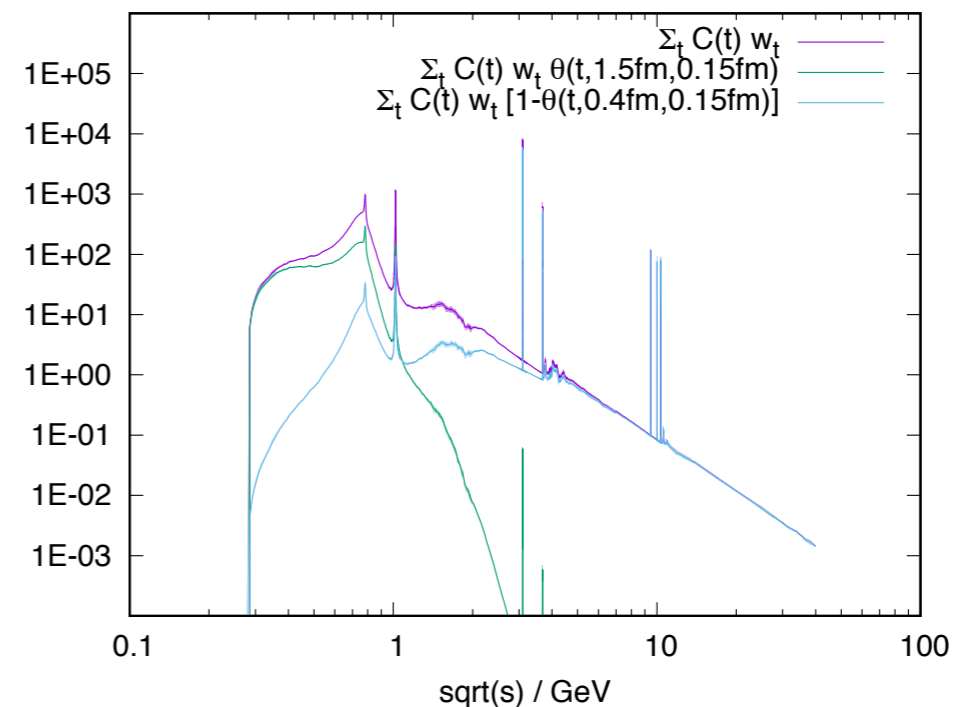
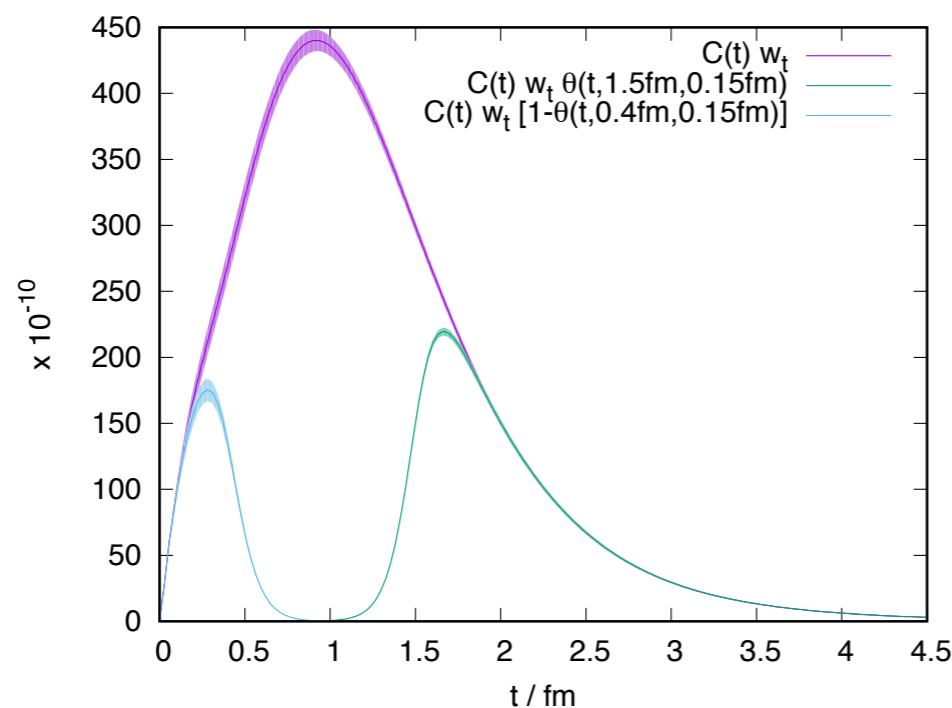
Hadronic vacuum polarization

Hybrid method: combine LQCD with R-ratio data

C. Lehner @ HVP KEK 2018 (from T. Blum et al, arXiv:1801.07224)

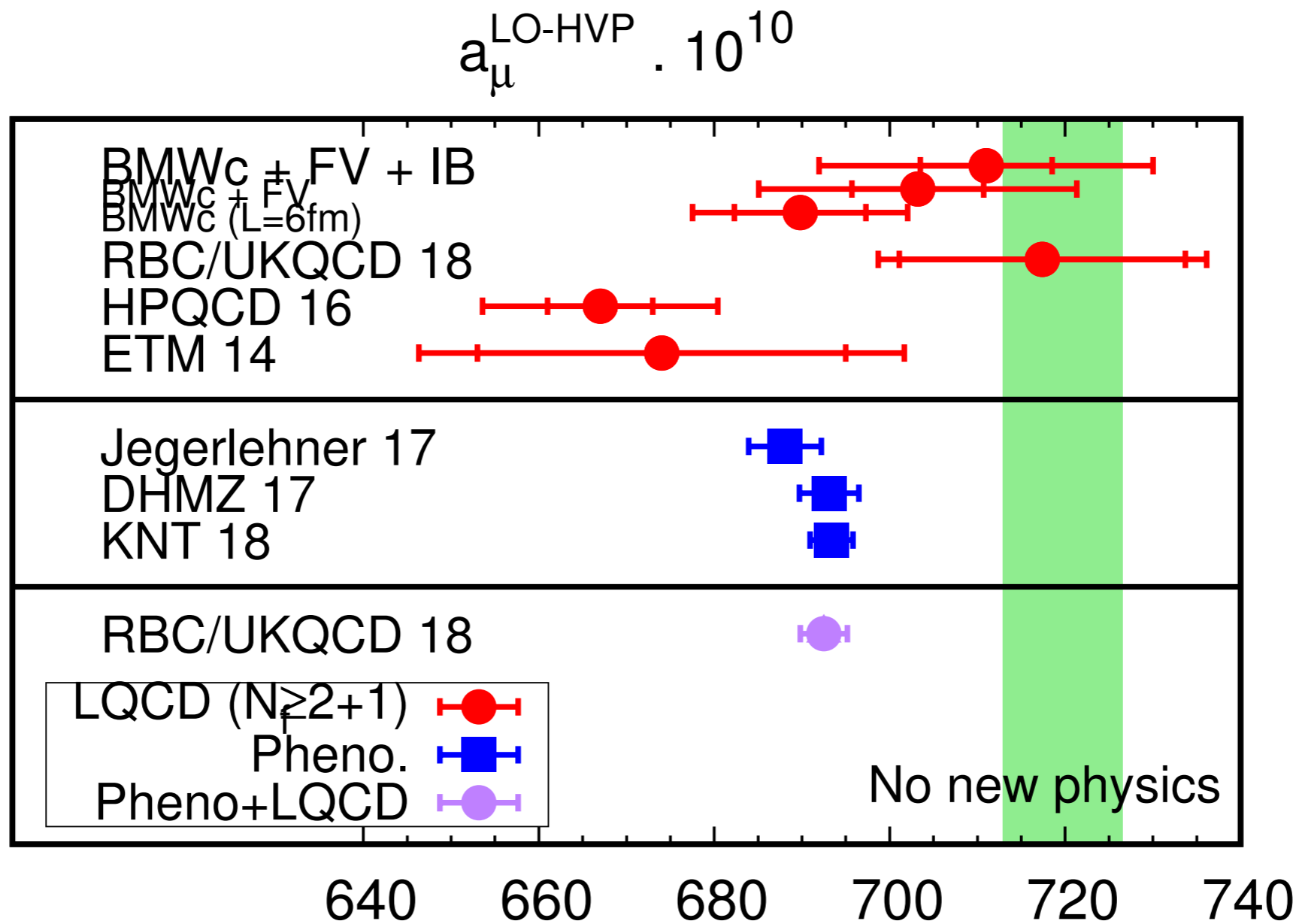
Direct LQCD calculations of HVP are still less precise than dispersive methods. But comparisons between R-ratio and lattice data are already useful.

- Convert R-ratio data to Euclidean correlation function (via the dispersive integral).
- Compare lattice/R-ratio data (after adding all the corrections and extrapolating to continuum, infinite volume).
- Use R-ratio data where LQCD errors are large and vice versa.



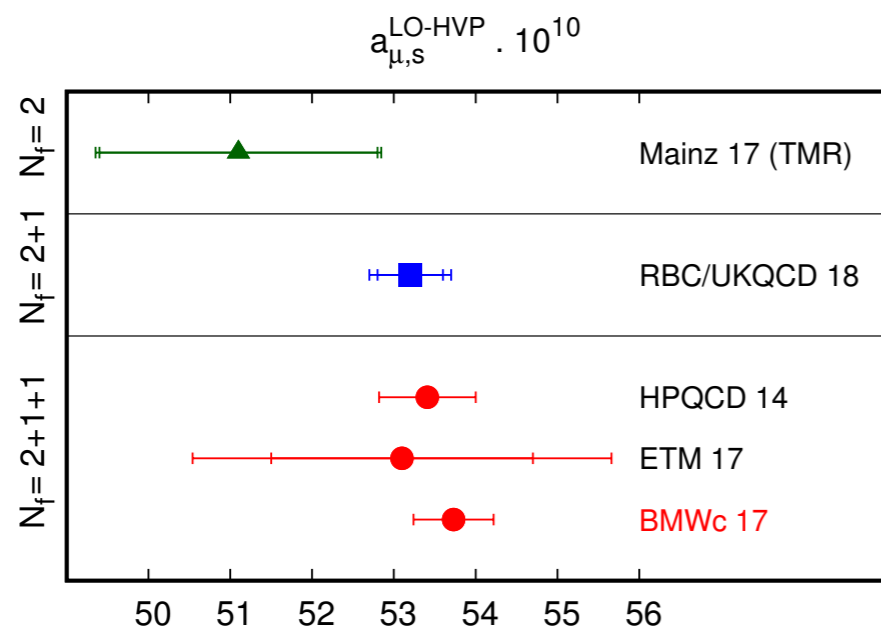
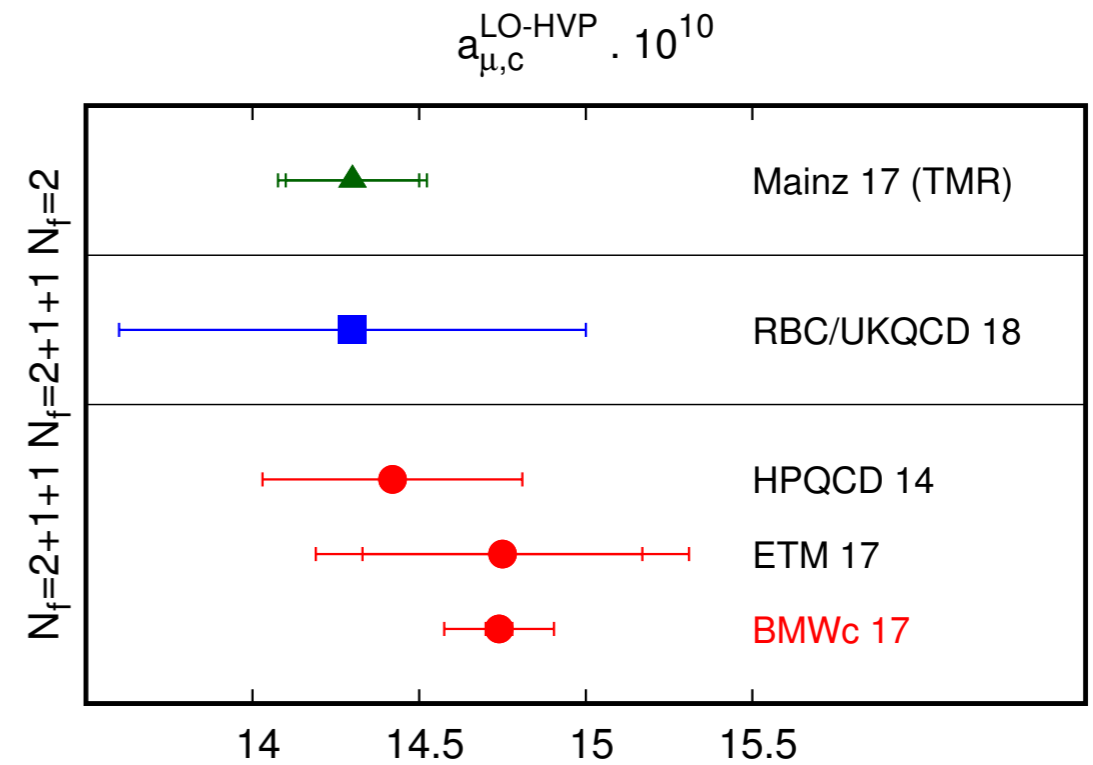
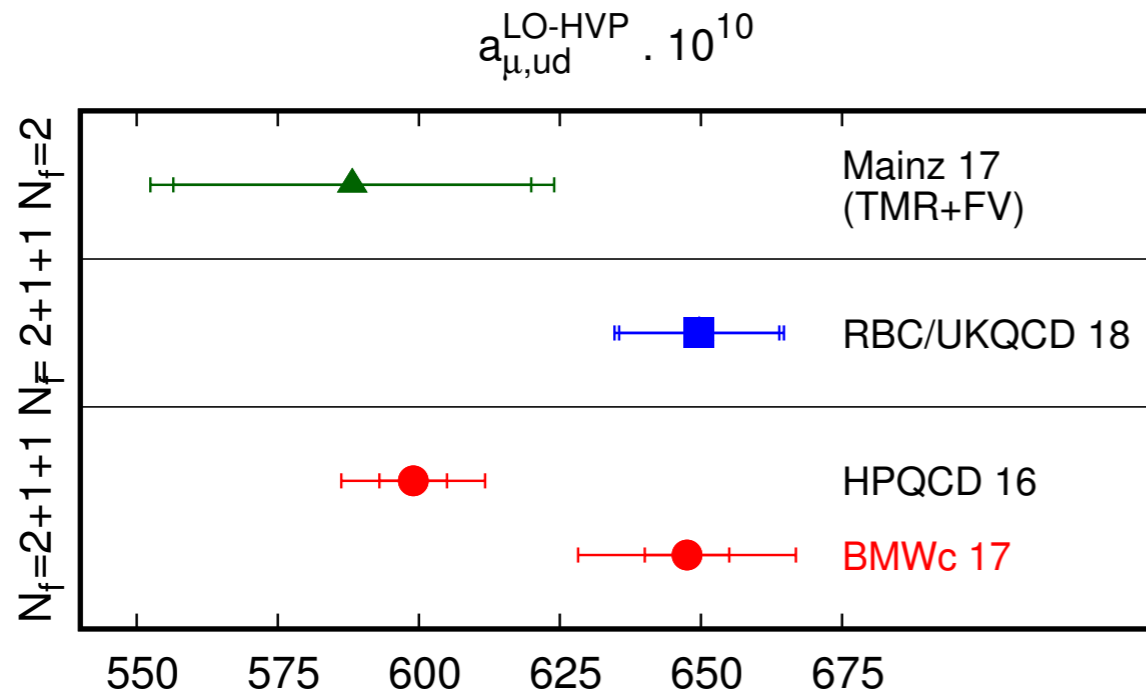
Summary of recent HVP results

L. Lellouch @ HVP KEK 2018 (for BMW collaboration)

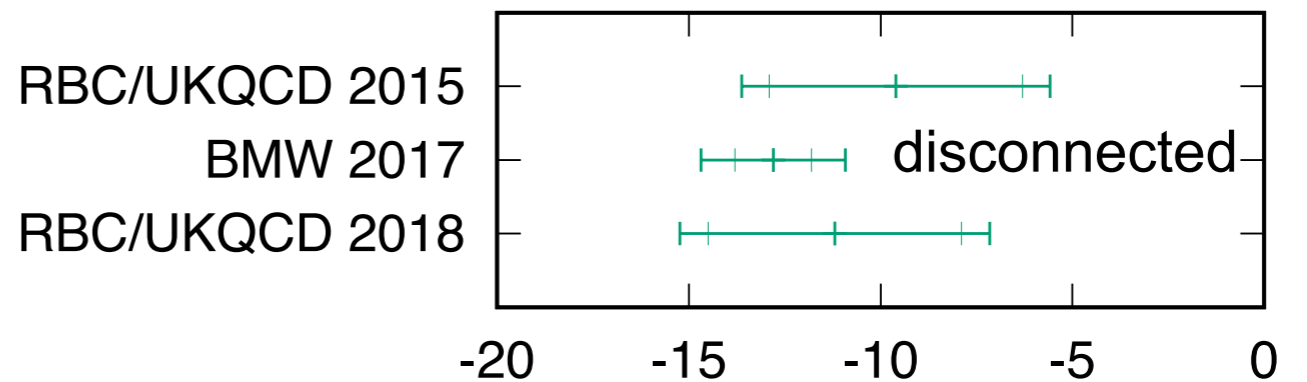


Detailed comparison of lattice HVP results

L. Lellouch @ HVP KEK 2018 (for BMW collaboration)

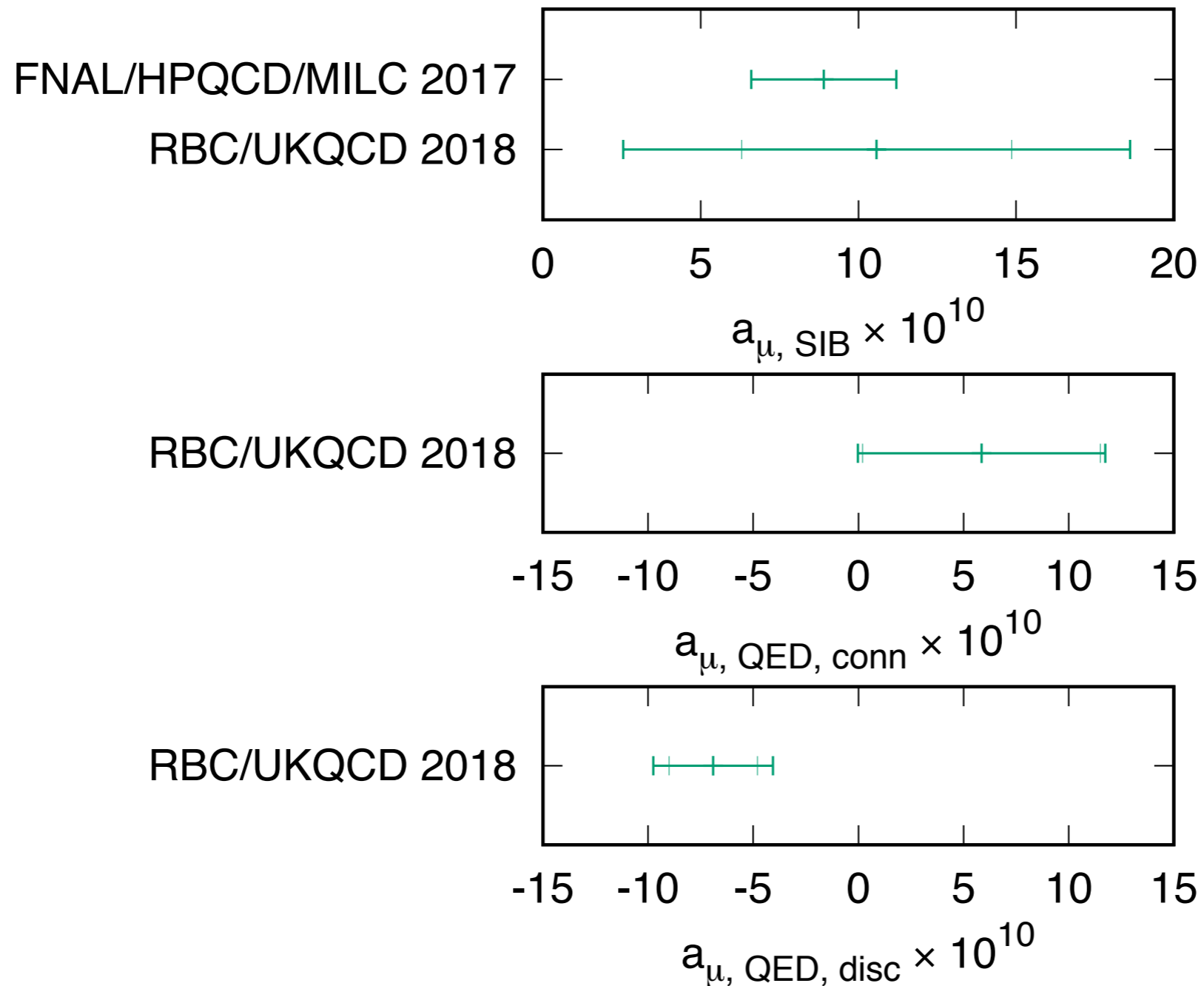


C. Lehner @ HVP KEK 2018 (for RBC/UKQCD)



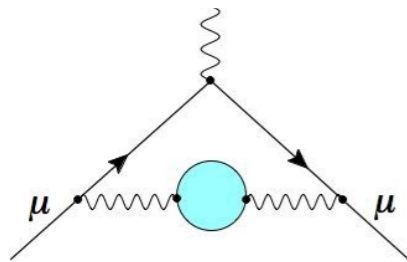
Detailed comparison of lattice HVP results

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Outline

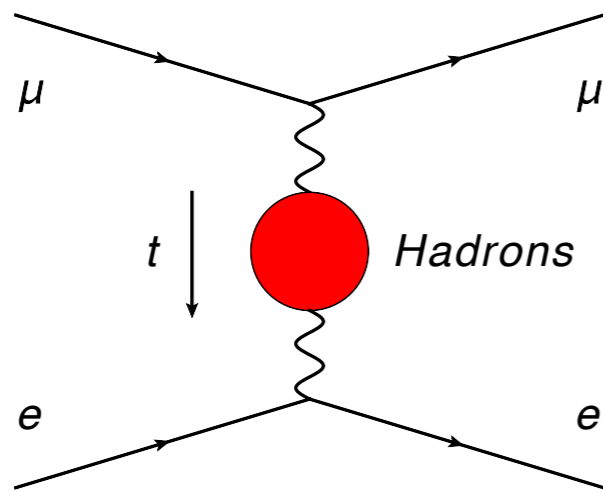
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Hadronic vacuum polarization

μ -e elastic scattering to measure a_μ^{HVP}

M. Passera @ HVP KEK 2018 (A. Abbiendi et al, arXiv:1609.08987, EPJC 2017)



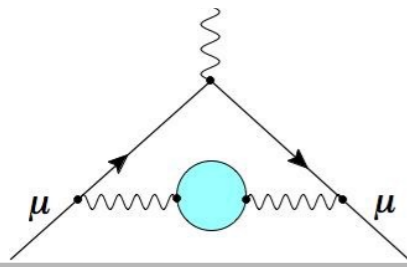
$$a_\mu^{\text{HLO}} = \frac{\alpha}{\pi} \int_0^1 dx (1-x) \Delta\alpha_{\text{had}}[t(x)]$$

$$t(x) = \frac{x^2 m_\mu^2}{x-1} < 0$$

$\Delta\alpha_{\text{had}}(t)$ is the hadronic contribution to the running of α in the **space-like** region. It can be extracted from scattering data!



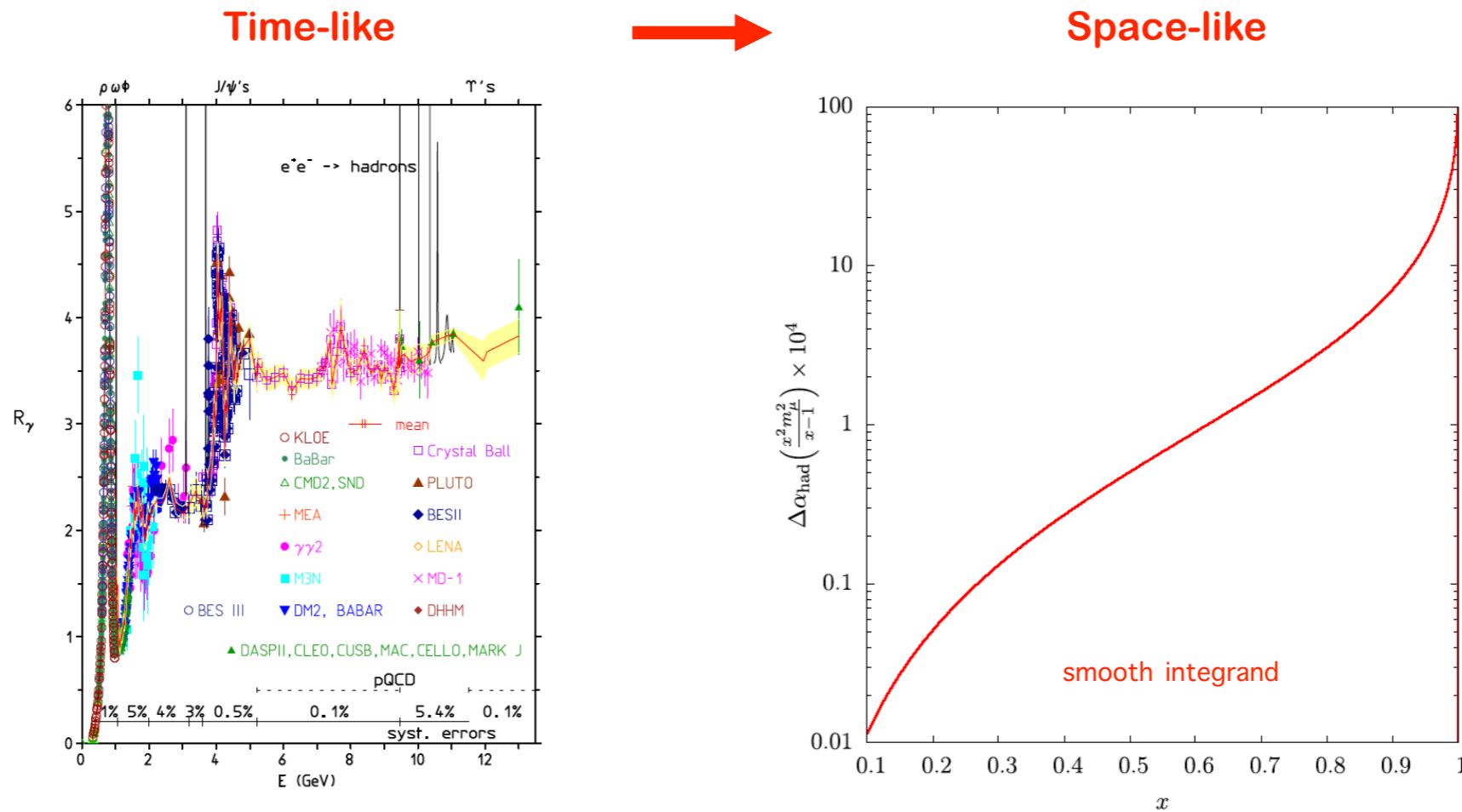
- use CERN M2 muon beam (150 GeV)
- test detector prototype in August 2018
- LOI planned for 2018-2019
- Physics beyond colliders program @ CERN



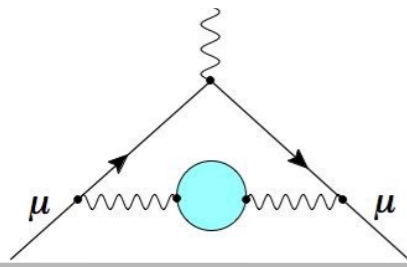
Hadronic vacuum polarization

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M. Passera @ HVP KEK 2018 (A. Abbiendi et al, arXiv:1609.08987, EPJC 2017)



- requires NNLO QED calculation, ...
- complement region not accessible to experiment with LQCD calculation (M. Marinkovic @ HVP KEK 2018)



Hadronic vacuum polarization

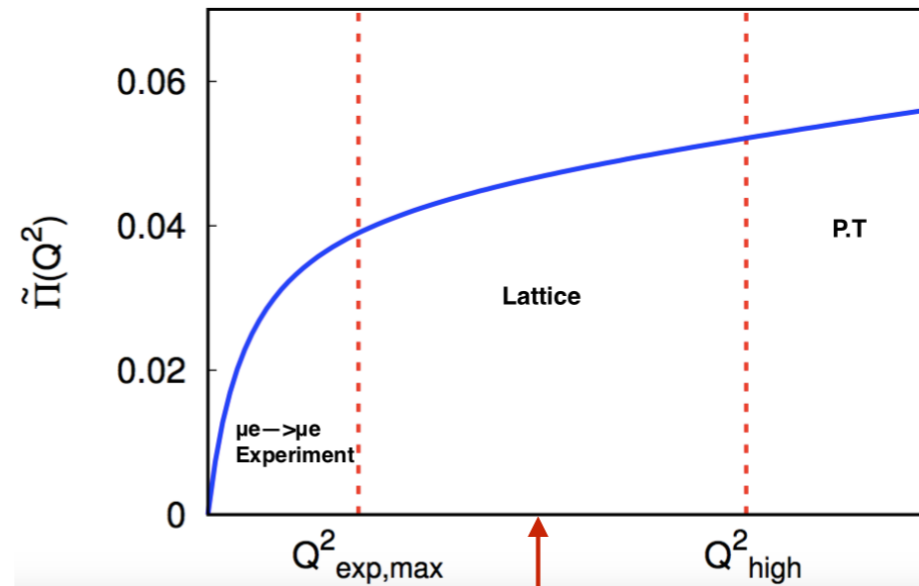
μ -e elastic scattering to measure a_{μ}^{HVP}

M. Marinkovic @ HVP KEK 2018:

- complement region not accessible to experiment with LQCD calculation

Hybrid method

Phys. Rev. D 90, 074508 (2014),
[Golterman, Maltman, Peris]



- Low momentum region
- ➔ Experiment (NLO, NNLO, radiative corrections ...)

- Vary low and high Q^2 cut

- ➔ continuum limit: $a \rightarrow 0$
- ➔ infinite volume limit: $V \rightarrow \infty$
- ➔ physical quark masses
- ➔ isospin breaking corrections ($m_u \neq m_d$ and $\alpha_{em} \neq 0$)

strategy proposed for the hybrid determination of the total HVP ($u+d+s+c+b$)

Summary and Outlook

- ★ Experimental inputs:

new recent results by BaBar, KLOE, BESIII, SND, CMD-3, KEDR, and more expected soon

- ★ dispersive HVP determinations

improved precision, but suffer from tension between experimental inputs for $\pi\pi$ channel

- ★ LQCD calculations

many groups, new methods, a lot of activity
need to understand tensions between LQCD results:
compare intermediate quantities (Taylor coefficients)

- ★ room for new ideas

HVP sessions @ Mainz workshop

- **Inputs from e+e- experiments:**

S. Eidelman (review+outlook), E. Solodov (Novosibirsk), M. Davier (tensions), S. Tracz + H. Czyż on radiative corrections and MC generators

- **dispersive HVP:**

B. Malaescu for DHMZ, A. Keshavarzi for KNT, M. Benayoun on HLS model, M. Hoferichter (disp. constraints on pion vector form factor), Malaescu + Teubner on error propagation

- **Lattice HVP:**

C. Lehner (RBC/UKQCD), H. Meyer (Mainz), R. Van de Water (FNAL/MILC/HPQCD), K. Miura (BMW), S. Simula (ETM)

+ discussions on:

long distance effects (von Hippel), FV effects (Giusti), QED+SIB (Gülpers), comparisons and cross checks (Lellouch)

- **misc theory and new ideas:**

analytical FV (A. Portelli), Hybrid approach (K. Schilcher), Mellin-Barnes (E. de Rafael), tau decays (M. Bruno), MUonE (C. Carloni, M. Marinkovic)



Thank you!

Appendix

Introduction

SM contribution	$10^{11} \times (\text{value} \pm \text{error})$	Refs and notes
QED (5 loops)	116584718.951 ± 0.080	[Ayoma et al, 2012, Laporta'17]
EW (2 loops)	153.6 ± 1.0	[Gnendiger et al, 2013]
HVP (LO)	6923 ± 42	[DHMZ'11, see also HLMNT'11, JS'11,...]
HVP (NLO)	-98.4 ± 1.0	[Hagiwara et al, 2011]
HVP (NNLO)	12.4 ± 0.1	[Kurz et al, 2014]
HLbL	105 ± 26	[Prades et al, 2014] "Glasgow consensus"
HLbL (NLO)	3 ± 2	[Colangelo et al, 2014]
Total	116591803 ± 49	[Davier et al, 2011]
Experiment	116592089 ± 63	[Bennet et al, 2006]
Diff (Exp. - SM):	286 ± 80	

The difference is large: $\sim 2 \times$ (EW contribution)

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