Pion-pole contribution to hadronic light-by-light scattering in the muon $(g-2)_\mu$

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in collab. with M. Hoferichter, B. Kubis, S. Leupold and S. P. Schneider

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Outline

Hadronic light-by-light (HLbL) scattering

Pion-pole contribution to a_μ

Pion transition form factor

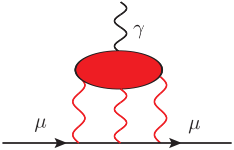
- Dispersion relations (DR)
- Double-spectral representation
- Matching to the asymptotic behavior

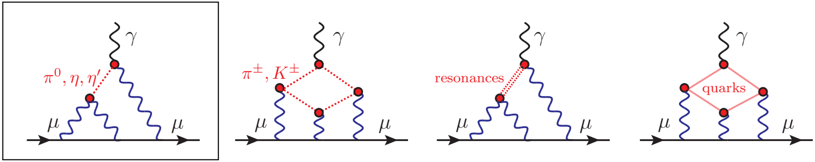
Numerical results

Conclusions and outlook

Hadronic light-by-light scattering

- Estimates based on hadronic models so far except for lattice
- New model-independent initiatives: employ DR to relate dominant contributions to observables like form factors Colangelo et al., 2014, Pauk, Vanderhaeghen, 2014





- π^0-pole term is the largest individual contribution to HLbL

Hadronic light-by-light scattering

A general master formula for the complete HLbL contributions:

Colangelo et al., 2015

$$a_\mu^{\text{HLbL}} = \frac{2\alpha^3}{3\pi^2} \int_0^\infty \mathrm{d}Q_1 \int_0^\infty \mathrm{d}Q_2 \int_{-1}^1 \mathrm{d}\tau \sqrt{1-\tau^2} Q_1^3 Q_2^3 \sum_{i=1}^{12} T_i(Q_1, Q_2, \tau) \bar{\Pi}_i(Q_1, Q_2, \tau)$$

- $T_i(Q_1, Q_2, \tau)$: kernel functions
- $\bar{\Pi}_i(Q_1, Q_2, \tau)$: hadronic scalar functions

The pion pole easily identified with the hadronic functions $\bar{\Pi}_i$:

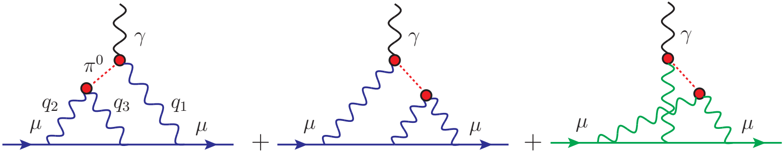
$$\bar{\Pi}_1^{\pi^0\text{-pole}}(Q_1, Q_2, \tau) = -\frac{F_{\pi^0\gamma^*\gamma^*}(-Q_1^2, -Q_2^2) F_{\pi^0\gamma^*\gamma^*}\big(-(Q_1+Q_2)^2, 0\big)}{(Q_1+Q_2)^2 + M_{\pi^0}^2}$$

$$\bar{\Pi}_2^{\pi^0\text{-pole}}(Q_1, Q_2, \tau) = -\frac{F_{\pi^0\gamma^*\gamma^*}\big(-Q_1^2, -(Q_1+Q_2)^2\big) F_{\pi^0\gamma^*\gamma^*}(-Q_2^2, 0)}{Q_2^2 + M_{\pi^0}^2}$$

Pion-pole contribution to a_μ

A 3 dimensional representation for the pion-pole contribution:

Jegerlehner, Nyffeler, 2009



$$a_\mu^{\pi^0\text{-pole}} = \left(\frac{\alpha}{\pi}\right)^3 \int_0^\infty \mathrm{d}Q_1 \int_0^\infty \mathrm{d}Q_2 \int_{-1}^1 \mathrm{d}\tau$$

$$\times \Big[w_1(Q_1,Q_2,\tau)\, F_{\pi^0\gamma^*\gamma^*}(-Q_1^2, -(Q_1+Q_2)^2)\, F_{\pi^0\gamma^*\gamma^*}(-Q_2^2, 0)$$

$$+ w_2(Q_1,Q_2,\tau)\, F_{\pi^0\gamma^*\gamma^*}(-Q_1^2, -Q_2^2)\, F_{\pi^0\gamma^*\gamma^*}(-(Q_1+Q_2)^2, 0) \Big]$$

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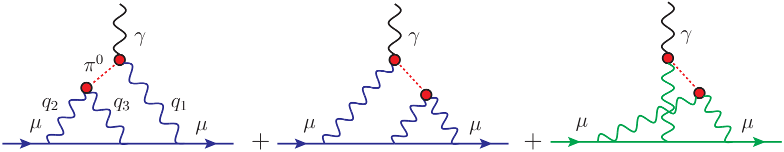

$$a_\mu^{\pi^0\text{-pole}} = \left(\frac{\alpha}{\pi}\right)^3 \int_0^\infty dQ_1 \int_0^\infty dQ_2 \int_{-1}^{1} d\tau$$

$$\times \Big[w_1(Q_1, Q_2, \tau)\, F_{\pi^0\gamma^*\gamma^*}(-Q_1^2, -(Q_1+Q_2)^2)\, F_{\pi^0\gamma^*\gamma^*}(-Q_2^2, 0)$$

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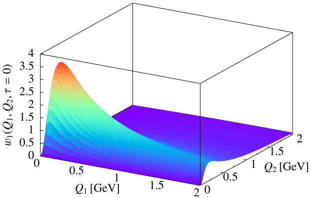
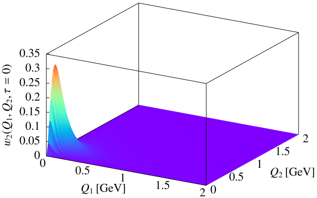
- $w_1(Q_1, Q_2, \tau)$ & $w_2(Q_1, Q_2, \tau)$: weight functions; $\tau = \cos\theta$
- $F_{\pi^0\gamma^*\gamma^*}(-Q_1^2, -Q_2^2)$: on-shell space-like pion transition form factors

Pion-pole contribution to a_μ

The weight functions $w_{1/2}(Q_1, Q_2, \tau)$ as functions of Q_1 and Q_2:

Nyffeler, 2016



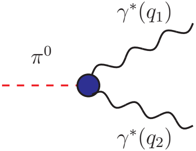


- concentrated in $Q_i \leq 0.5$ GeV.
- Pion-pole contribution arises dominantly from the low-energy region
- The region where the pion transition form factor is model-independently & precisely determined from DR

Pion transition form factor

- Defined by the matrix element of two electromagnetic currents $j_\mu(x)$

$$i\int \mathrm{d}^4x\, e^{iq_1\cdot x}\, \langle 0 \big| T\left\{j_\mu(x)j_\nu(0)\right\}\big| \pi^0(q_1+q_2)\rangle$$
$$= -\epsilon_{\mu\nu\rho\sigma}\, q_1^\rho q_2^\sigma F_{\pi^0\gamma^*\gamma^*}(q_1^2, q_2^2)$$



- Normalization fixed by the Adler–Bell–Jackiw anomaly:

$$F_{\pi^0\gamma^*\gamma^*}(0,0) = \frac{1}{4\pi^2 F_\pi} \equiv F_{\pi\gamma\gamma}$$

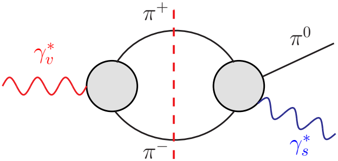
$F_\pi = 92.28(9)\,\mathrm{MeV}$: pion decay constant C. Patrignani et al., 2016

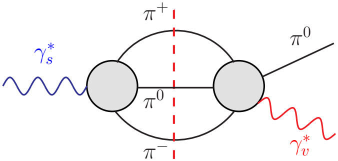
- Bose symmetry and isospin decomposition:

$$F_{\pi^0\gamma^*\gamma^*}(q_1^2, q_2^2) = F_{\pi^0\gamma^*\gamma^*}(q_2^2, q_1^2) = F_{vs}(q_1^2, q_2^2) + F_{vs}(q_2^2, q_1^2)$$

Pion transition form factor

Dispersive reconstruction from the lowest-lying hadronic intermediate states:





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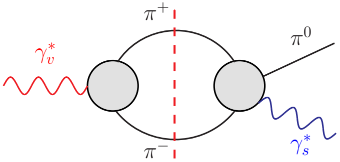

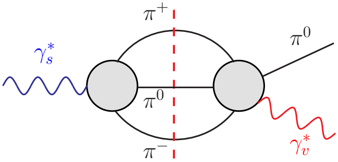

Isovector photon: 2 pions

- $\gamma_v^* \to \pi^+\pi^- \to \gamma_s^*\pi^0$
- $\text{disc} \propto$ pion vector form factor $\times\ \gamma_s^* \to 3\pi$ amplitude

Isoscalar photon: 3 pions

- $\gamma_s^* \to \pi^+\pi^-\pi^0 \to \gamma_v^*\pi^0$
- Dominated by resonances $\omega,\ \phi,\ \omega',\ \&\,\omega''$

Pion transition form factor

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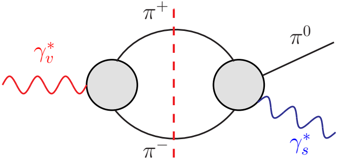

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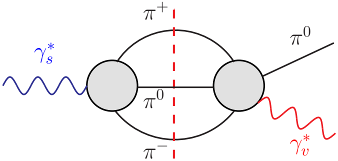

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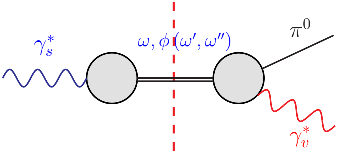

Pion transition form factor

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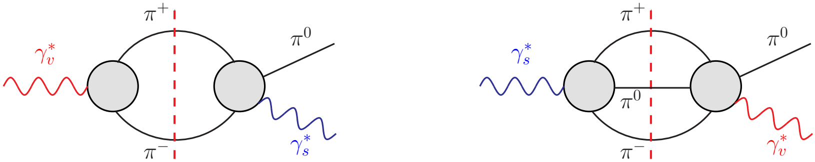
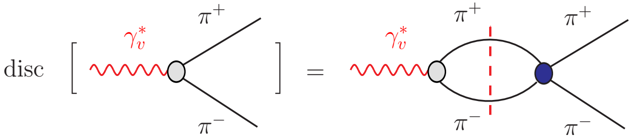


Building blocks of the dispersive treatment:

- Pion vector form factor $F_\pi^V(s)$
- Partial wave amplitude $f_1(s, q^2)$ for the $\gamma_s^*(q) \to \pi^+\pi^-\pi^0$ reaction

Pion transition form factor

Pion vector form factor $F_\pi^V(s)$:

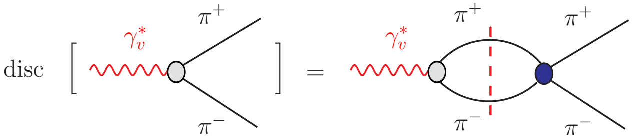


disc $F_\pi^V(s) = 2i\,\mathrm{Im}\,F_\pi^V(s) = 2iF_\pi^V(s)\,\sin\delta_1^1(s)\,e^{-i\delta_1^1(s)}\theta(s-4M_\pi^2)$

Watson's final-state theorem: phase of $F_\pi^V(s)$ is given by $\delta_1^1(s)$ Watson, 1954

Pion transition form factor

Pion vector form factor $F_\pi^V(s)$:



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Watson's final-state theorem: phase of $F_\pi^V(s)$ is given by $\delta_1^1(s)$ Watson, 1954

Solution:

$$F_\pi^V(s) = P(s)\Omega(s), \quad \Omega(s) = \exp\left\{\frac{s}{\pi}\int_{4M_\pi^2}^\infty \mathrm{d}s'\frac{\delta_1^1(s')}{s'(s'-s)}\right\}$$

- $\Omega(s)$ is the Omnès function Omnès, 1958
- $P(s)$ polynomial, $P(0) = 1$ from charge conservation
- $\pi\pi$ P-wave phase shift $\delta_1^1(s)$ from Roy equations

Pion transition form factor

The $\gamma_s^*(q) \to \pi^+\pi^-\pi^0$ decay amplitude $\mathcal{M}(s,t,u;q^2)$:

$$\mathcal{M}(s,t,u;q^2) = i\epsilon_{\mu\nu\rho\sigma} n^\mu p_+^\nu p_-^\rho p_0^\sigma \mathcal{F}(s,t,u;q^2)$$

Decompose into single-variable functions:

$$\mathcal{F}(s,t,u;q^2) = \mathcal{F}(s,q^2) + \mathcal{F}(t,q^2) + \mathcal{F}(u,q^2)$$

Normalization from the Wess–Zumino–Witten anomaly:

$$\mathcal{F}(0,0,0;0) = \frac{1}{4\pi^2 F_\pi^3} \equiv F_{3\pi}$$

Discontinuity equation:

$$\text{disc}\,\mathcal{F}(s,q^2) = 2i(\mathcal{F}(s,q^2) + \hat{\mathcal{F}}(s,q^2))\theta(s-4M_\pi^2)\sin\delta_1^1(s)\,e^{-i\delta_1^1(s)}$$

- $\mathcal{F}(s,q^2)$: right-hand cut
- $\hat{\mathcal{F}}(s,q^2)$: left-hand cut; angular averages of $\mathcal{F}(t,q^2)$ & $\mathcal{F}(u,q^2)$

Pion transition form factor

A once-subtracted dispersive solution to the discontinuity equation:

Hoferichter et al., 2014

$$\mathcal{F}(s,q^2) = a(q^2)\Omega(s)\Big\{1 + \frac{s}{\pi}\int_{4M_\pi^2}^{\infty} \mathrm{d}s' \frac{\hat{\mathcal{F}}(s',q^2)\sin\delta_1^1(s')}{s'(s'-s)|\Omega(s')|}\Big\}$$

$a(q^2)$ fit to different $e^+e^- \to 3\pi$ cross-section data with parameterization:

$$a(q^2) = \frac{F_{3\pi}}{3} + \frac{q^2}{\pi}\int_{s_{\rm thr}}^{\infty} \mathrm{d}s' \frac{\mathrm{Im}\,\mathcal{A}(s')}{s'(s'-q^2)} + C_n(q^2)$$

Pion transition form factor

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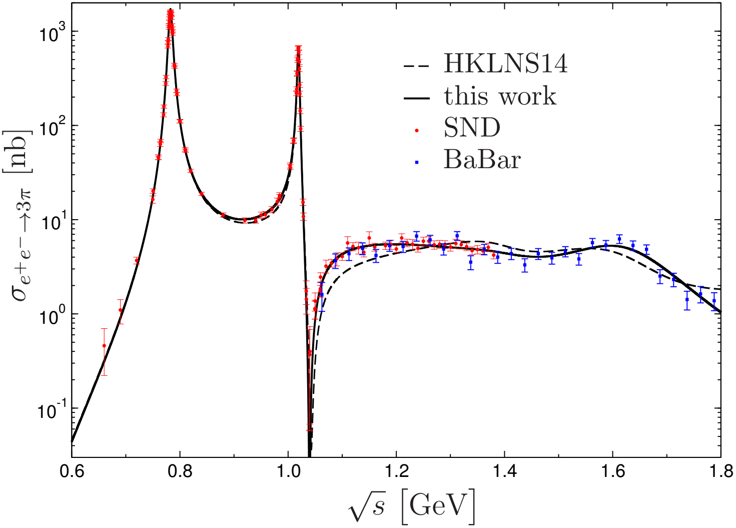
$$\mathcal{A}(q^2) = \sum_V \frac{c_V}{M_V^2 - q^2 - i\sqrt{q^2}\,\Gamma_V(q^2)}, \qquad V = \omega, \phi, \omega', \omega''$$

$$C_n(q^2) = \sum_{i=1}^{n} c_i\left(z(q^2)^i - z(0)^i\right), \qquad z(q^2) = \frac{\sqrt{s_{\mathrm{inel}} - s_1} - \sqrt{s_{\mathrm{inel}} - q^2}}{\sqrt{s_{\mathrm{inel}} - s_1} + \sqrt{s_{\mathrm{inel}} - q^2}}$$

- S-wave cusp eliminated, asymptotic behavior of $C_n(q^2)$ controlled
- Exact implementation of $\gamma_s^*(q) \to 3\pi$ anomaly

Pion transition form factor

6 (7) parameters c_ω, c_ϕ, $c_{\omega'}$, $c_{\omega''}$, c_1, c_2 & (c_3) fit to $e^+e^- \to 3\pi$ data:



- Substantially improved above the ϕ peak

Pion transition form factor

$F_{vs}(q_1^2, q_2^2)$ fulfills a dispersion relation: Hoferichter et al., 2014

$$F_{vs}(q_1^2, q_2^2) = \frac{1}{12\pi^2} \int_{4M_\pi^2}^{\infty} \mathrm{d}x \, \frac{q_\pi^3(x) \big(F_\pi^V(x)\big)^* f_1(x, q_2^2)}{x^{1/2}(x - q_1^2)},$$

$q_\pi(s) = \sqrt{s/4 - M_\pi^2}$, $f_1(s, q^2) = \mathcal{F}(s, q^2) + \hat{\mathcal{F}}(s, q^2)$: $\gamma_s^*(q) \to 3\pi$ P-wave

Go to doubly-space-like kinematics by writing another dispersion relation:

$$F_{vs}(-Q_1^2, q_2^2) = \frac{1}{\pi} \int_{s_{\mathrm{thr}}}^{s_{\mathrm{is}}} \mathrm{d}y \, \frac{\mathrm{Im}\, F_{vs}(-Q_1^2, y)}{y - q_2^2}$$

Double-spectral representation of the form factor:

$$F_{\pi^0\gamma^*\gamma^*}^{\mathrm{disp}}(-Q_1^2, -Q_2^2) = \frac{1}{\pi^2} \int_{4M_\pi^2}^{s_{\mathrm{iv}}} \mathrm{d}x \int_{s_{\mathrm{thr}}}^{s_{\mathrm{is}}} \frac{\rho^{\mathrm{disp}}(x, y)\, \mathrm{d}y}{(x + Q_1^2)(y + Q_2^2)},$$

$$\rho^{\mathrm{disp}}(x, y) = \frac{q_\pi^3(x)}{12\pi\sqrt{x}} \mathrm{Im}\left[\big(F_\pi^V(x)\big)^* f_1(x, y)\right] + [x \leftrightarrow y]$$

Pion transition form factor

Effective pole term:

$F^{\text{disp}}_{\pi^0\gamma^*\gamma^*}(q_1^2, q_2^2)$ fulfills the chiral anomaly $F_{\pi\gamma\gamma}$ by around 90%

$\Rightarrow$ Introduce an effective pole term

$$F^{\text{eff}}_{\pi^0\gamma^*\gamma^*}(q_1^2, q_2^2) = \frac{g_{\text{eff}}}{4\pi^2 F_\pi} \frac{M_{\text{eff}}^4}{(M_{\text{eff}}^2 - q_1^2)(M_{\text{eff}}^2 - q_2^2)}$$

g_{eff} fixed by fulfilling the chiral anomaly

$g_{\text{eff}} \sim 10\%$, $\Rightarrow$ small

M_{eff} fit to singly-virtual data excluding BaBar above $5\,\text{GeV}^2$ Gronberg et al., 1998, Aubert et al., 2009, Uehara et al., 2012

$M_{\text{eff}} \sim 1.5$–$2\,\text{GeV}$, $\Rightarrow$ reasonable

Pion transition form factor

Asymptotically, $F_{\pi^0\gamma^*\gamma^*}$ should fulfill — Brodsky, Lepage, 1979-1981

$$F_{\pi^0\gamma^*\gamma^*}(q_1^2, q_2^2) = -\frac{2F_\pi}{3}\int_0^1 \mathrm{d}x \frac{\phi_\pi(x)}{xq_1^2 + (1-x)q_2^2} + \mathcal{O}\left(\frac{1}{q_i^4}\right),$$

Pion distribution amplitude $\phi_\pi(x) = 6x(1-x) + \cdots$

Brodsky–Lepage (BL) limit:

$$\lim_{Q^2\to\infty} F_{\pi^0\gamma^*\gamma}(-Q^2, 0) = \frac{2\,F_\pi}{Q^2}$$

Operator product expansion (OPE): — Nesterenko, Radyushkin, 1983, Novikov et al., 1984, Manohar, 1990

$$\lim_{Q^2\to\infty} F_{\pi^0\gamma^*\gamma^*}(-Q^2, -Q^2) = \frac{2\,F_\pi}{3\,Q^2}$$

Pion transition form factor

Rewrite the asymptotic form into a double-spectral representation:

$$F^{\text{asym}}_{\pi^0\gamma^*\gamma^*}(q_1^2, q_2^2) = \frac{1}{\pi^2}\int_0^\infty \int_0^\infty \mathrm{d}x\mathrm{d}y \frac{\rho^{\text{asym}}(x,y)}{(x-q_1^2)(y-q_2^2)},$$

$$\rho^{\text{asym}}(x,y) = -2\pi^2 F_\pi xy\delta''(x-y)$$

Decomposition of the pion-transition form factor:

$$F_{\pi^0\gamma^*\gamma^*}(q_1^2, q_2^2) = \frac{1}{\pi^2}\int_0^{s_{\text{m}}} \mathrm{d}x \int_0^{s_{\text{m}}} \mathrm{d}y \frac{\rho^{\text{disp}}(x,y)}{(x-q_1^2)(y-q_2^2)} + \frac{1}{\pi^2}\int_{s_{\text{m}}}^\infty \mathrm{d}x \int_{s_{\text{m}}}^\infty \mathrm{d}y \frac{\rho^{\text{asym}}(x,y)}{(x-q_1^2)(y-q_2^2)}$$
$$+ \frac{1}{\pi^2}\int_0^{s_{\text{m}}} \mathrm{d}x \int_{s_{\text{m}}}^\infty \mathrm{d}y \frac{\rho(x,y)}{(x-q_1^2)(y-q_2^2)} + \frac{1}{\pi^2}\int_{s_{\text{m}}}^\infty \mathrm{d}x \int_0^{s_{\text{m}}} \mathrm{d}y \frac{\rho(x,y)}{(x-q_1^2)(y-q_2^2)}$$

- s_{m}: continuum threshold
- $\rho(x,y)$ not known rigorously, $\rho^{\text{asym}}(x,y)$ applied in mixed regions vanishes

  $\Rightarrow$ All constraints can be fulfilled discarding mixed regions

Pion transition form factor

This defines the asymptotic contribution

$$F^{\text{asym}}_{\pi^0\gamma^*\gamma^*}(q_1^2, q_2^2) = 2F_\pi \int_{s_\text{m}}^{\infty} \mathrm{d}x \frac{q_1^2 q_2^2}{(x - q_1^2)^2 (x - q_2^2)^2}$$

- Does not contribute for the singly-virtual kinematics
- Restores the asympotics for singly/doubly-virtual kinematics

The final representation:

$$\boxed{F_{\pi^0\gamma^*\gamma^*}(q_1^2, q_2^2) = F^{\text{disp}}_{\pi^0\gamma^*\gamma^*}(q_1^2, q_2^2) + F^{\text{eff}}_{\pi^0\gamma^*\gamma^*}(q_1^2, q_2^2) + F^{\text{asym}}_{\pi^0\gamma^*\gamma^*}(q_1^2, q_2^2)}$$

- Reconstructed from all lowest-lying singularities
- Fulfills the asymptotic constraints at $\mathcal{O}\big(1/Q^2\big)$
- Full freedom to account for the tension of BaBar/Belle space-like data

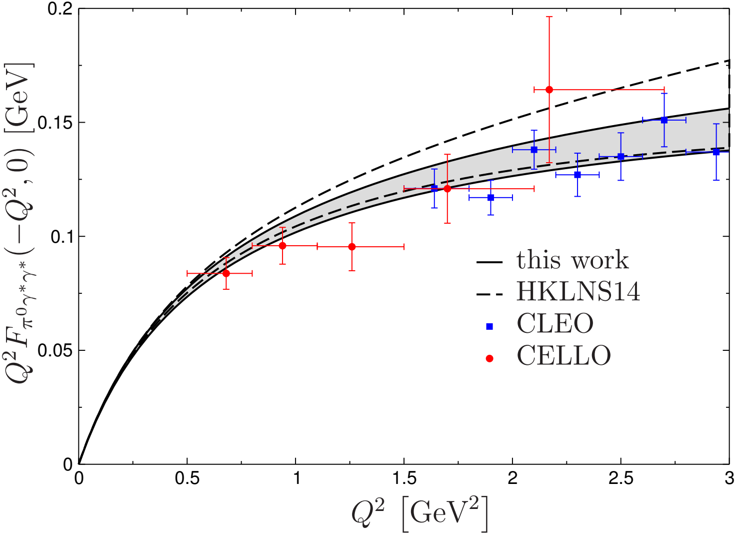
Numerical results

Uncertainty estimates:

- The uncertainty in $F_{\pi\gamma\gamma}$ at 1.4% from PrimEx — Larin et al., 2011
  - Varying the coupling g_{eff}
- Dispersive uncertainties estimated by
  - Varying the cutoffs between 1.8 and $2.5\,\text{GeV}$
  - Different $\pi\pi$ phase shifts — Caprini et al., 2012, García-Martín et al., 2011, Schneider et al., 2012
  - Different representations of $F_\pi^V(s)$
- BL limit uncertainty by $^{+20}_{-10}\%$ — Aubert et al., 2009, Uehara et al., 2012
  - Varying the mass parameter M_{eff}
  - Completely covers 3σ band
- Asymptotic part $s_{\text{m}} = 1.7(3)\,\text{GeV}^2$ — Khodjamirian, 1999, Agaev et al., 2011, Mikhailov et al., 2016
  - Expected from light-cone sum rules

Numerical results

Singly-virtual space-like transition form factor in $Q^2 F_{\pi^0\gamma^*\gamma}(-Q^2, 0)$:



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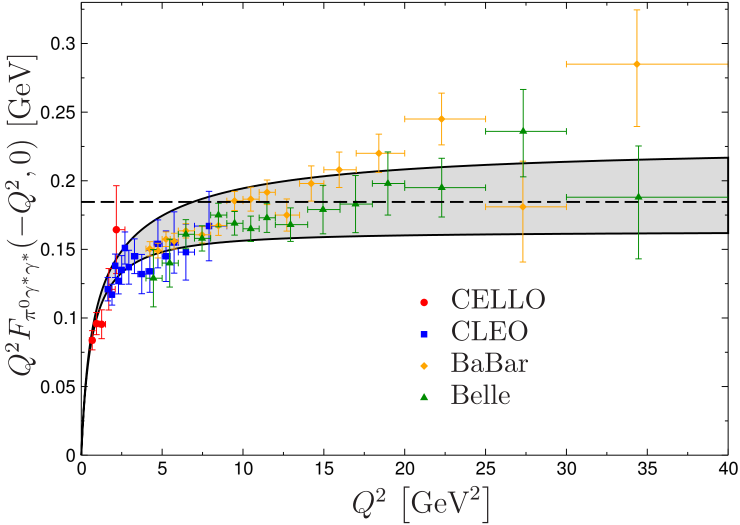
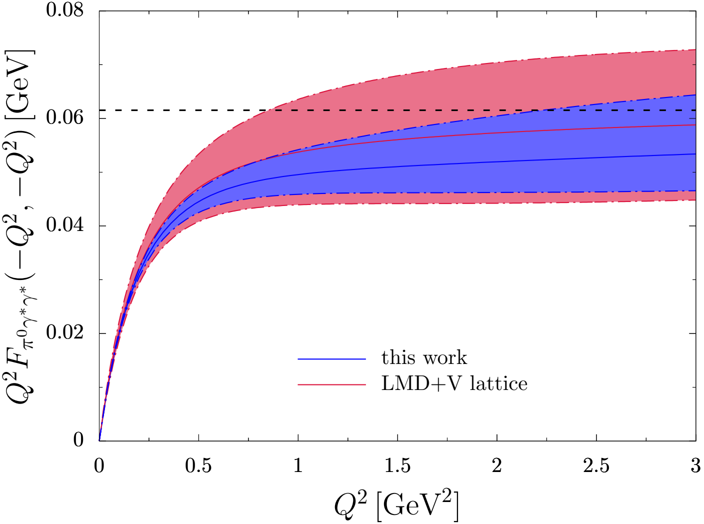


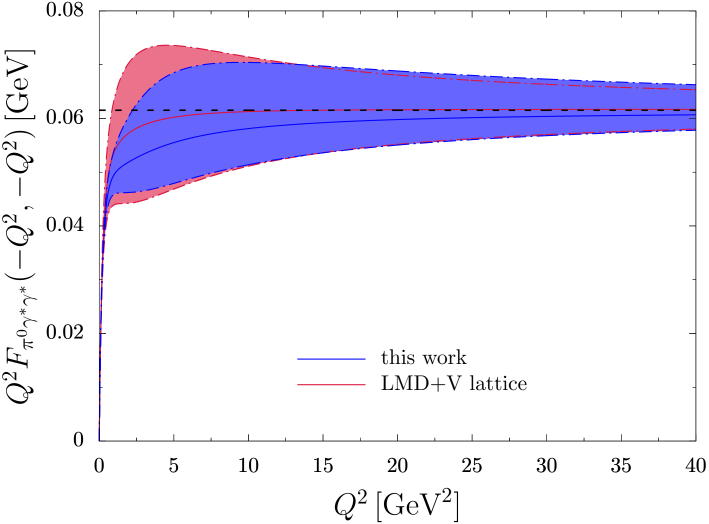
Numerical results

Diagonal form factor in $Q^2 F_{\pi^0\gamma^*\gamma^*}(-Q^2, -Q^2)$ in comparison to LMD+V fit to lattice: Gérardin et al., 2016



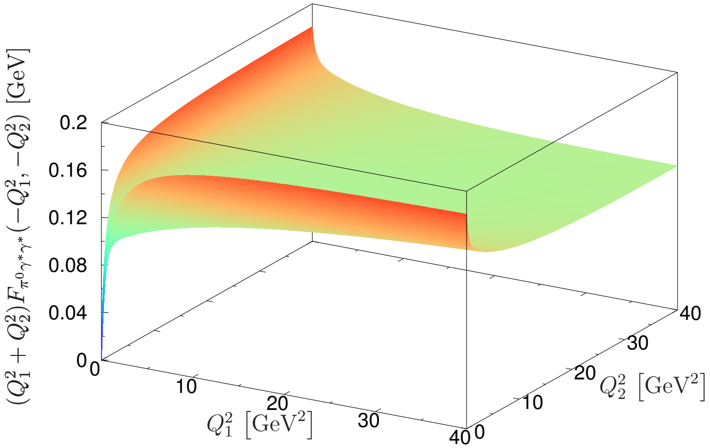
Numerical results

Diagonal form factor in $Q^2 F_{\pi^0\gamma^*\gamma^*}(-Q^2, -Q^2)$ in comparison to LMD+V fit to lattice: Gérardin et al., 2016



Numerical results

$(Q_1^2 + Q_2^2) F_{\pi^0\gamma^*\gamma^*}(-Q_1^2, -Q_2^2)$ as a function of Q_1^2 and Q_2^2:



- $1/Q_i^2$ behavior in the entire domain of space-like virtualities

  $\Rightarrow$ Hard to obtain in resonance models

Numerical results

Pion-pole contribution to a_μ from the final representation:

$$\boxed{\begin{aligned} a_\mu^{\pi^0\text{-pole}} &= 62.6(1.7)_{F_{\pi\gamma\gamma}}(1.1)_{\text{disp}}\binom{2.2}{1.4}_{\text{BL}}(0.5)_{\text{asym}} \times 10^{-11} \\ &= 62.6^{+3.0}_{-2.5} \times 10^{-11} \end{aligned}}$$

- First complete data-driven determination
- Fully controlled uncertainty estimates

The slope parameter:

$$\begin{aligned} a_\pi &= \frac{M_{\pi^0}^2}{F_{\pi\gamma\gamma}} \frac{\partial}{\partial q^2} F_{\pi^0\gamma^*\gamma^*}(q^2, 0)\Big|_{q^2=0} \\ &= 31.5(2)_{F_{\pi\gamma\gamma}}(8)_{\text{disp}}(3)_{\text{BL}} \times 10^{-3} = 31.5(9) \times 10^{-3} \end{aligned}$$

- In comparison to $a_\pi = 30.7(6) \times 10^{-3}$ (HKLNS14) Hoferichter et al., 2014

  $\Rightarrow$ Larger value expected from matching

Conclusions and outlook

- Dispersive reconstruction of the pion transition form factor
  - Incorporated all the lowest-lying singularities
  - Matched to perturbative QCD
- Data-driven determination of $a_\mu^{\pi^0\text{-pole}}$ with carefully estimated improvable uncertainties
  - Preliminary updated 0.85% $F_{\pi\gamma\gamma}$ uncertainty from PrimEx-II
  - Dispersive inputs may be consolidated with COMPASS, BESIII
  - BL limit may be clarified from BELLE II
  - Doubly virtual form factor comparing to lattice QCD
- Applications to η and η' transition form factors
- $\cdots$

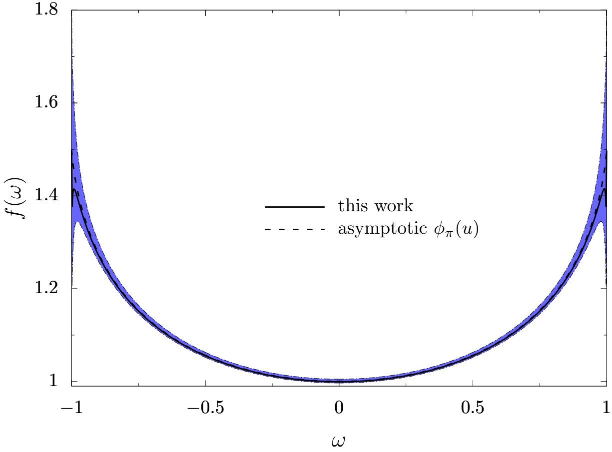
Much obliged for your attention!

"$g-2$ is not an experiment: it is a way of life."

John Adams (CERN Director General 1971 - 1980)

Backup

$f(\omega)$ versus ω:



$$F_{\pi^0\gamma^*\gamma^*}(q_1^2, q_2^2) = -\frac{4F_\pi}{3}\frac{f(\omega)}{(q_1^2+q_2^2)}, \quad f(\omega) = \int_0^1 \mathrm{d}u \frac{\phi_\pi(u)}{u(1-\omega)+(1-u)(1+\omega)}$$