The muon $g-2$: a new data-based analysis

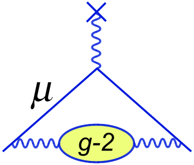
Alex Keshavarzi

with Daisuke Nomura, Thomas Teubner (KNT18)

[arXiv:1802.02995, **accepted for publication in Phys. Rev. D (in press)**]

Muon $g-2$ Theory Initiative Workshop, JGU Mainz

20th June 2018

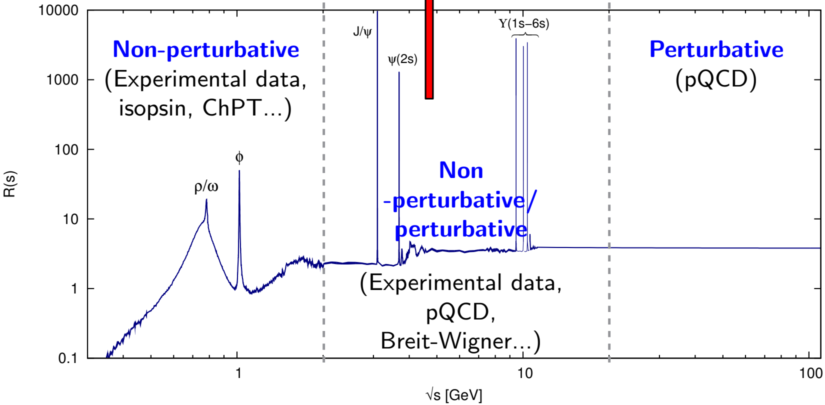


UNIVERSITY OF LIVERPOOL

LIVERPOOL THEORETICAL PHYSICS

Hadronic cross section input

$$a_\mu^{\mathrm{had,\,LO\,VP}} = \frac{\alpha^2}{3\pi^2}\int_{s_{th}}^{\infty} \frac{\mathrm{d}s}{s} R(s)K(s), \text{ where } R(s) = \frac{\sigma^0_{\mathrm{had},\gamma}(s)}{4\pi\alpha^2/3s}$$



Must build full hadronic cross section/R-ratio...

Building the hadronic R-ratio

$m_\pi \leq \sqrt{s} \leq 2$ GeV

- Input experimental hadronic cross section data*
- Combine all available data in exclusive hadronic final states $(\pi^+\pi^-, K^+K^-, ...)$
- Sum ~ 35 exclusive channels
- Detailed data analysis
- Robust treatment of experimental errors
- Estimate missing data input (isospin relations, ChPT...)

$2 \leq \sqrt{s} \leq 11.2$ GeV

- Can use experimental inclusive R data* or pQCD
- Must use data at quark flavour thresholds
- Combine all available R data
- Robust treatment of experimental errors
- Include narrow resonances

$11.2 \leq \sqrt{s} < \infty$ GeV

- Calculate R using pQCD (`rhad`)

*$\sigma_{\rm had}$ experiments

- KLOE
- BaBar
- SND
- CMD-(2/3)
- KEDR
- BESIII

Question: for reliable precision, how are data correlated and how should those correlations be implemented?

Data combination: setup

⇒ Re-bin data into *clusters*

→ Scan cluster sizes for preferred solution (error, χ^2, check by sight...)

⇒ Correlated data beginning to dominate full data compilation...

→ Non-trivial, energy dependent influence on both mean value and error estimate

KNT18 prescription

- Construct full covariance matrices for each channel & entire compilation
  ⇒ Framework available for inclusion of any and all inter-experimental correlations
- If experiment does not provide matrices...
  → Statistics occupy diagonal elements only
  → Systematics are 100% correlated
- If experiment does provide matrices...
  → Use all information provided
- Use correlations to full capacity

Data combination consideration

Question:

What are the main points of concern when combining experimental data to evaluate $a_\mu^{\text{had, VP}}$?

$\Rightarrow$ When combining data...

- $\rightarrow$ ...how to best combine large amounts of data from different experiments
- $\rightarrow$ ...the correct implementation of correlated uncertainties (statistical and systematic)
- $\rightarrow$ ...finding a solution that is free from bias

d'Agostini bias [Nucl.Instrum.Meth. A346 (1994) 306-311]

$$x_1 = 0.9 \pm \delta x_1 \qquad x_2 = 1.1 \pm \delta x_2 \qquad C_{\text{sys}} = \begin{pmatrix} p^2 x_1^2 & p^2 x_1 x_2 \\ p^2 x_2 x_1 & p^2 x_2^2 \end{pmatrix}$$

$\Rightarrow \bar{x} \simeq 0.98$ (systematic bias)

(Normalisation uncertainties defined by data)

Effect worsened with full, iterative data combination

Data combination consideration

Question:

What are the main points of concern when combining experimental data to evaluate $a_\mu^{\text{had, VP}}$?

$\Rightarrow$ When combining data...

- $\rightarrow$ ...how to best combine large amounts of data from different experiments
- $\rightarrow$ ...the correct implementation of correlated uncertainties (statistical and systematic)
- $\rightarrow$ ...finding a solution that is free from bias

Fixed matrix method [R. D. Ball et al. [NNPDF Collaboration], JHEP 1005 (2010) 075.]

$$x_1 = 0.9 \pm \delta x_1 \qquad x_2 = 1.1 \pm \delta x_2 \qquad C_{\text{sys}} = \begin{pmatrix} p^2\bar{x}^2 & p^2\bar{x}^2 \\ p^2\bar{x}^2 & p^2\bar{x}^2 \end{pmatrix} \quad \Rightarrow \bar{x} = 1.00 \text{ (systematic bias)}$$

(Normalisation uncertainties defined by estimator)

Redefinition repeated at each stage of iterative data combination

Linear χ^2 minimisation [KNT18: arXiv:1802.02995, PRD (in press)]

⇒ Clusters are defined to have linear cross section

→ Fix covariance matrix with linear interpolants at each iteration (extrapolate at boundary)

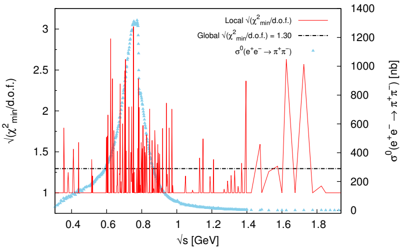
$$\chi^2 = \sum_{i=1}^{N_{\rm tot}} \sum_{j=1}^{N_{\rm tot}} \left(R_i^{(m)} - \mathcal{R}_m^i\right) \mathbf{C}^{-1}\left(i^{(m)}, j^{(n)}\right) \left(R_j^{(n)} - \mathcal{R}_n^j\right)$$

⇒ Through correlations and linearisation, result is the minimised solution of all available uncertainty information

→ ... through a method that has been shown to avoid d'Agostini bias

⇒ The flexibly of the fit to vary due to the energy dependent, correlated uncertainties benefits the combination

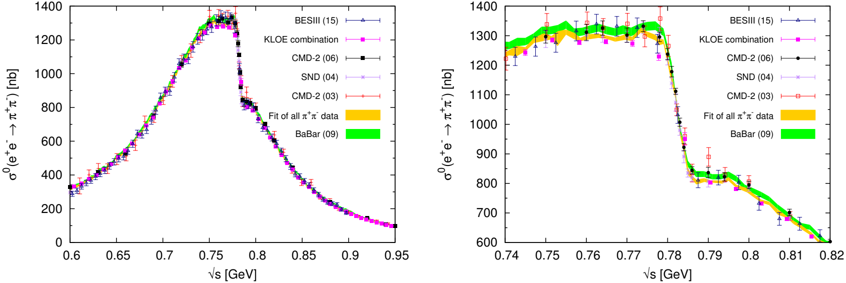
→ ... and any data tensions are reflected in a local and global $\chi^2_{\rm min}/{\rm d.o.f.}$ error inflation



$\pi^+\pi^-$ channel [KNT18: arXiv:1802.02995, PRD (in press)]

$\Rightarrow$ $\pi^+\pi^-$ accounts for over 70% of $a_\mu^{\mathrm{had,\,LO\,VP}}$

$\rightarrow$ Combines 30 measurements totalling 999 data points



$\Rightarrow$ Correlated & experimentally corrected $\sigma^0_{\pi\pi(\gamma)}$ data now entirely dominant

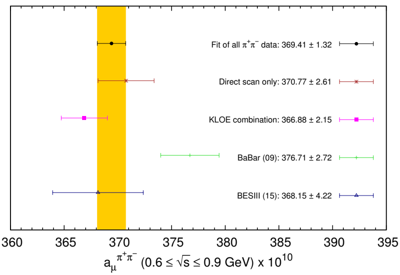
$$a_\mu^{\pi^+\pi^-}[0.305 \le \sqrt{s} \le 1.937\ \text{GeV}] = 502.97 \pm 1.14_{\mathrm{stat}} \pm 1.59_{\mathrm{sys}} \pm 0.06_{\mathrm{vp}} \pm 0.14_{\mathrm{fsr}}$$
$$= 502.97 \pm 1.97_{\mathrm{tot}} \quad \text{HLMNT11: } 505.77 \pm 3.09$$

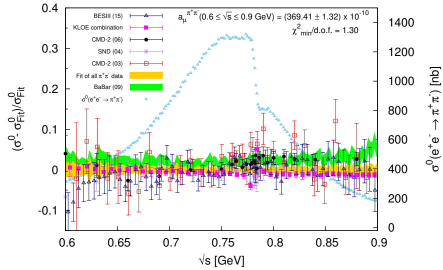
$\Rightarrow$ 15% local $\chi^2_{\mathrm{min}}/\mathrm{d.o.f.}$ error inflation due to tensions in clustered data

$\pi^+\pi^-$ channel [KNT18: arXiv:1802.02995, PRD (in press)]

$\Rightarrow$ Tension exists between BaBar data and all other data in the dominant ρ region.

$\rightarrow$ Agreement between other radiative return measurements and direct scan data largely compensates this.





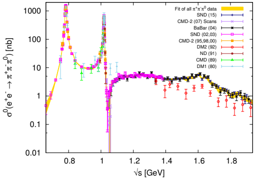
Compared to $a_\mu^{\pi^+\pi^-} = 502.97 \pm 1.97$: $\Rightarrow a_\mu^{\pi^+\pi^-}$ (BaBar data only) $= 513.2 \pm 3.8$.

Simple weighted average of all data $\Rightarrow a_\mu^{\pi^+\pi^-}$ (Weighted average) $= 509.1 \pm 2.9$.
(i.e. - no correlations in determination of mean value)

BaBar data dominate when no correlations are taken into account for the mean value
Highlights importance of fully incorporating all available correlated uncertainties

Other notable exclusive channels [KNT18: arXiv:1802.02995, PRD (in press)]

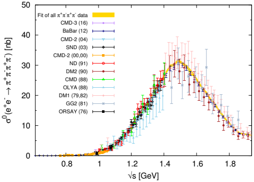
$\pi^+\pi^-\pi^0$



HLMNT11: 47.51 ± 0.99

KNT18: 47.92 ± 0.89

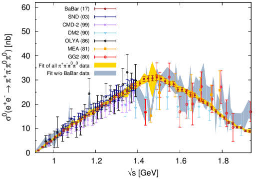
$\pi^+\pi^-\pi^+\pi^-$



HLMNT11: 14.65 ± 0.47

KNT18: 14.87 ± 0.20

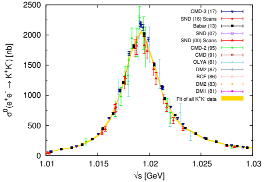
$\pi^+\pi^-\pi^0\pi^0$



HLMNT11: 20.37 ± 1.26

KNT18: 19.39 ± 0.78

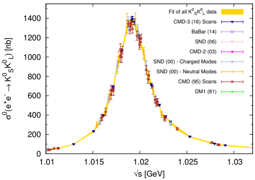
K^+K^-



HLMNT11: 22.15 ± 0.46

KNT18: 23.03 ± 0.22

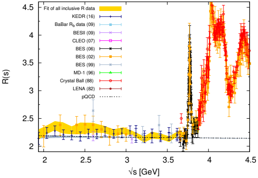
$K_S^0 K_L^0$



HLMNT11: 13.33 ± 0.16

KNT18: 13.04 ± 0.19

Inclusive ($\sqrt{s} > 2$ GeV)



HLMNT11: 41.40 ± 0.87

KNT18: 41.27 ± 0.62

KNT18 $a_\mu^{\text{had, VP}}$ update [KNT18: arXiv:1802.02995, PRD (in press)]

$$\text{HLMNT(11)}: \; 694.91 \pm 4.27$$
$$\downarrow$$

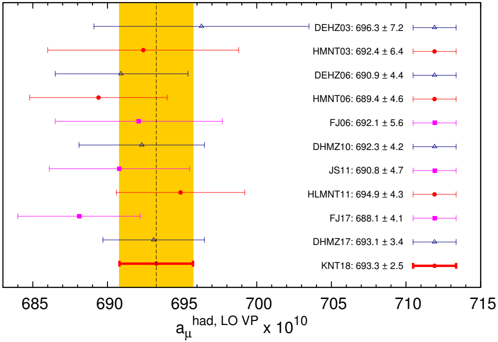
This work: $a_\mu^{\text{had, LO VP}} = 693.27 \pm 1.19_{\text{stat}} \pm 2.01_{\text{sys}} \pm 0.22_{\text{vp}} \pm 0.71_{\text{fsr}}$

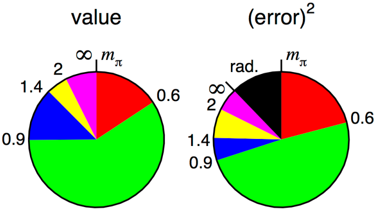
$$= 693.27 \pm 2.34_{\text{exp}} \pm 0.74_{\text{rad}}$$

$$= 693.27 \pm 2.46_{\text{tot}}$$

$$a_\mu^{\text{had, NLO VP}} = -9.82 \pm 0.04_{\text{tot}}$$

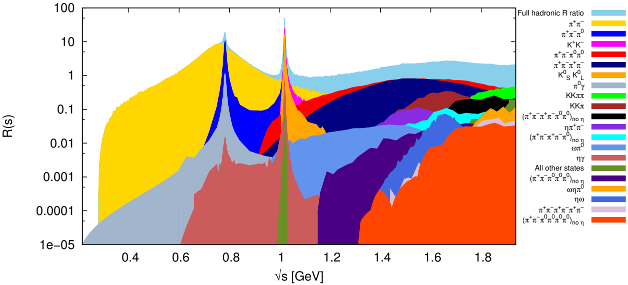
$\Rightarrow$ Accuracy better then 0.4%
(uncertainties include all available correlations and local χ^2 inflation)





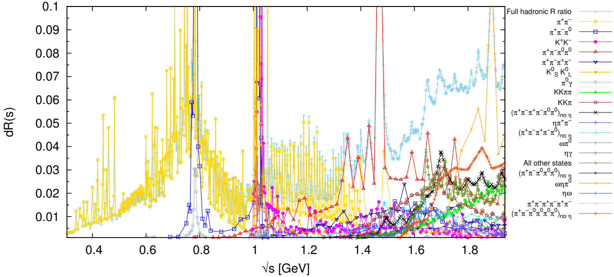
$\Rightarrow$ 2π dominance

Contributions below 2GeV [KNT18: arXiv:1802.02995, PRD (in press)]

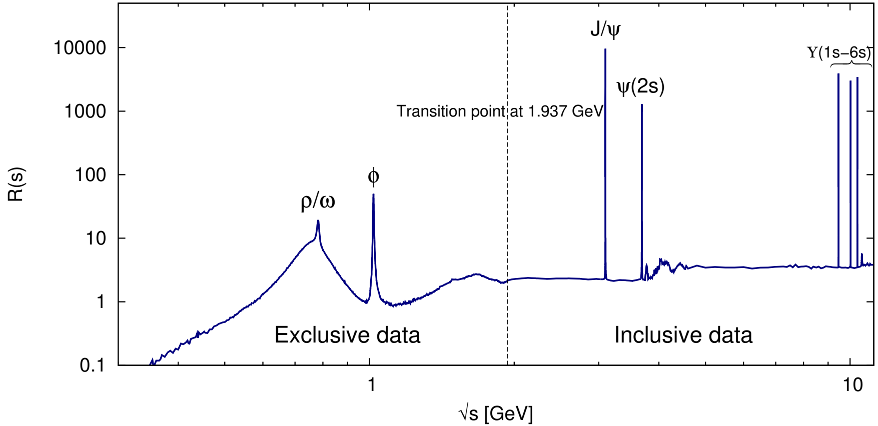


→ Dominance of 2π below 0.9 GeV evident for both cross section and uncertainty

→ Large improvement to cross section and uncertainty from new 4π data



$R(s)$ for $m_\pi \leq \sqrt{s} \leq 11.2$ GeV [KNT18: arXiv:1802.02995, PRD (in press)]



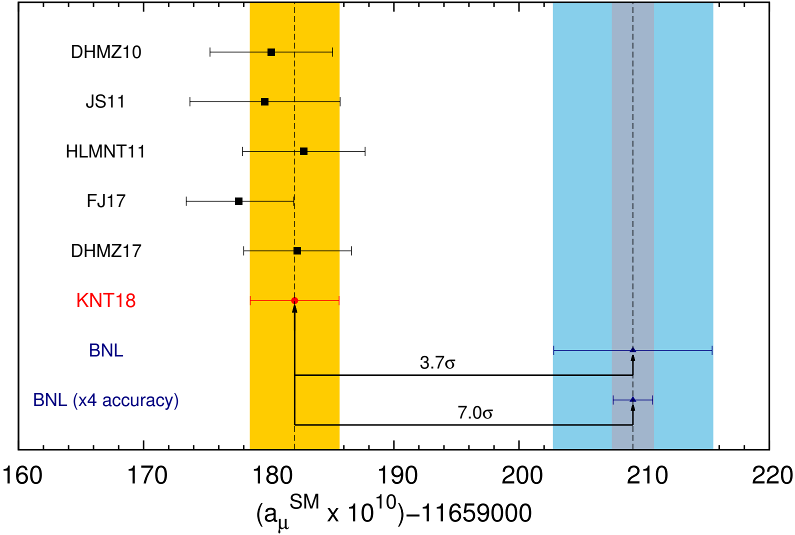
⇒ Full KNT18 compilation data set for hadronic R-ratio now available...

$\Longrightarrow$ **...complete with full covariance matrix**

KNT18 a_μ^{SM} update [KNT18: arXiv:1802.02995, PRD (in press)]

	2011		2017	
QED	11658471.81 (0.02)	$\longrightarrow$	11658471.90 (0.01)	[arXiv:1712.06060]
EW	15.40 (0.20)	$\longrightarrow$	15.36 (0.10)	[Phys. Rev. D 88 (2013) 053005]
LO HLbL	10.50 (2.60)	$\longrightarrow$	9.80 **(2.60)**	[EPJ Web Conf. 118 (2016) 01016]
NLO HLbL			0.30 (0.20)	[Phys. Lett. B 735 (2014) 90]
	HLMNT11		**KNT18**	
LO HVP	694.91 (4.27)	$\longrightarrow$	693.27 **(2.46)**	this work
NLO HVP	-9.84 (0.07)	$\longrightarrow$	-9.82 (0.04)	this work
NNLO HVP			1.24 (0.01)	[Phys. Lett. B 734 (2014) 144]
Theory total	11659182.80 (4.94)	$\longrightarrow$	11659182.05 (3.56)	this work
Experiment			11659209.10 (6.33)	world avg
Exp - Theory	26.1 (8.0)	$\longrightarrow$	27.1 (7.3)	this work
Δa_μ	3.3σ	$\longrightarrow$	3.7σ	this work

KNT18 $a_\mu^{\rm SM}$ update [KNT18: arXiv:1802.02995, PRD (in press)]



Conclusions

⇒ Accuracy of $a_\mu^{\rm SM}$ limited by hadronic contributions

⇒ Hadronic VP contirbutions can be determined from dispersion relations and hadronic cross section

⇒ Must build hadronic R-ratio from experimental data

⇒ New data combination method + new data yields improvements in all channels due to increased fit flexibility

⇒ Correlations have large effect on mean value and uncertainty and all available information should be correctly incorporated

⇒ $a_\mu^{\rm had,LOVP}$ accuracy better than 0.4%

⇒ Improvement in HVP yields $g-2$ discrepancy of 3.7σ

⇒ Overall HVP uncertainty now better than HLbL uncertainty

Thank you

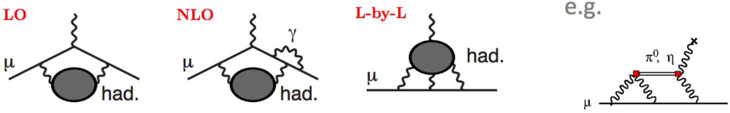
Extra Slides

Hadronic contributions

⇒ Uncertainty on a_μ^{SM} dominated by hadronic contributions

→ Non-perturbative, low energy region of hadronic resonances

$$a_\mu^{\mathrm{had}} = a_\mu^{\mathrm{had,VP\ LO}} + a_\mu^{\mathrm{had,VP\ NLO}} + a_\mu^{\mathrm{had,Light-by-Light}}$$



⇒ LbL contributions $\left(\mathcal{O}(\alpha^3)\right)$, so far only *fully* determined using model calculations

→ Difficult to quantify/control uncertainties from models

→ Huge progress from lattice and dispersive approaches

→ So far, no indication of unpleasant surprises

→ But, big improvements expected in near future Phys. Rev. D 94 (2016) 053006.

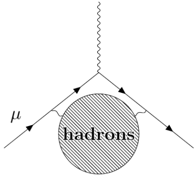
⇒ LO LbL, updated 'Glasgow consensus' estimate: $a_\mu^{\mathrm{had,\,LO\,LbL}} = (9.8 \pm 2.6) \times 10^{-10}$

→ NLO LbL estimated to be $a_\mu^{\mathrm{had,\,NLO\,LbL}} = (0.3 \pm 0.2) \times 10^{-10}$ Phys. Lett. B 735 (2014) 90.

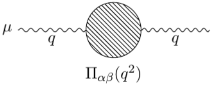
$$a_\mu^{\mathrm{had,\,LbL}} = (10.1 \pm 2.6) \times 10^{-10}$$

$a_\mu^{\mathrm{had,VP}}$: theoretical setup

⇒ We want to calculate the leading order hadronic vacuum polarisation (HVP) contribution

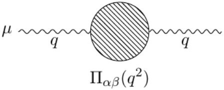


1) Feynman rules for HVP insertion to photon propagator:



$$= \frac{-ig^{\mu\alpha}}{(q^2-i\varepsilon)}(-ie)i\Pi_{\alpha\beta}(q^2)(-ie)\frac{-ig^{\beta\nu}}{(q^2-i\varepsilon)}$$

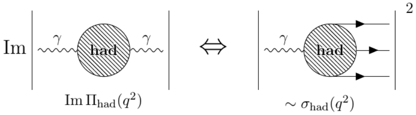
2) Employ analyticity:



$$= \frac{ie^2 g_{\mu\nu}}{(q^2-i\varepsilon)^2}\frac{q^4}{\pi}\int_{s_{th}}^{\infty} \mathrm{d}s \frac{\mathrm{Im}\,\Pi(s)}{s(s-q^2-i\varepsilon)}$$

3) Insert to vertex correction, solve for a_μ: $a_\mu^{\mathrm{had,\,LO\,VP}} = \dfrac{\alpha}{\pi^2}\displaystyle\int_{s_{th}}^{\infty}\frac{\mathrm{d}s}{s}\mathrm{Im}\,\Pi_{\mathrm{had}}(s)K(s)$

4) Utilise optical theorem:



5) Arrive at equation for $a_\mu^{\mathrm{had,\,LO\,VP}}$:

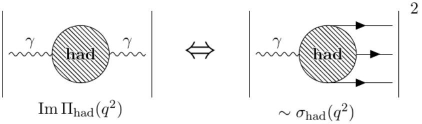
$$a_\mu^{\mathrm{had,\,LO\,VP}} = \frac{1}{4\pi^3}\int_{s_{th}}^{\infty}\mathrm{d}s\,\sigma^0_{\mathrm{had},\gamma}(s)K(s)$$

$\sigma^0_{\mathrm{had},\gamma}$ = bare cross section, FSR included

⇒ Similar dispersion integrals for NLO and NNLO HVP

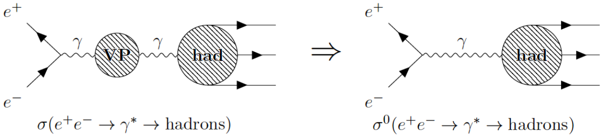
$\sigma^0_{\mathrm{had},\gamma}$: vacuum polarisation corrections

⇒ Reconsider the optical theorem: Im



⇒ Photon VP corresponds to higher order contributions to $a_\mu^{\mathrm{had,\,VP}}$

→ Must subtract VP:



⇒ Fully updated, self-consistent VP routine: [vp_knt_v3_0], available for distribution

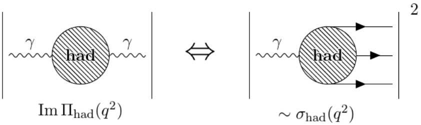
→ Cross sections undressed with full photon propagator (must include imaginary part), $\sigma^0_{\mathrm{had}}(s) = \sigma_{\mathrm{had}}(s)|1 - \Pi(s)|^2$

⇒ If correcting data, apply corresponding radiative correction uncertainty

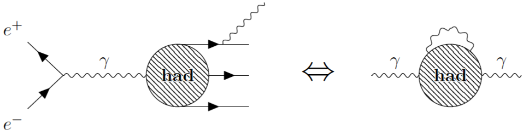
→ Take $\frac{1}{3}$ of total correction per channel as conservative extra uncertainty

$\sigma^0_{\mathrm{had},\gamma}$: final state radiation corrections

$\Rightarrow$ Reconsider the optical theorem: Im



$\Rightarrow$ Photon FSR formally higher order corrections to $a_\mu^{\mathrm{had,\,VP}}$



$\Rightarrow$ Cannot be unambiguously separated, not accounted for in HO contributions

$\rightarrow$ Must be included as part of 1PI hadronic blobs

$\Rightarrow$ Experiment may cut/miss photon FSR $\rightarrow$ Must be added back

$\Rightarrow$ For $\pi^+\pi^-$, sQED approximation [Eur. Phys. J. C 24 (2002) 51, Eur. Phys. J. C 28 (2003) 261]

$\Rightarrow$ For higher multiplicity states, difficult to estimate correction

$\therefore$ Apply conservative uncertainty

Need new, more developed tools to increase precision here

(e.g. - CARLOMAT 3.1 [Eur.Phys.J. C77 (2017) no.4, 254]?)

Systematic bias and use of the data/covariance matrix

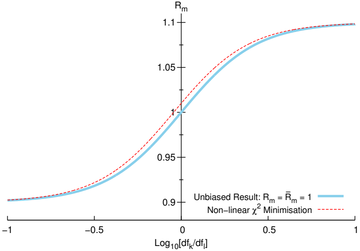
⇒ Data is re-binned using an adaptive clustering algorithm

⇒ Iterative fit of covariance matrix as defined by data → D'Agostini bias

[Nucl.Instrum.Meth. A346 (1994) 306-311]

HLMNT11

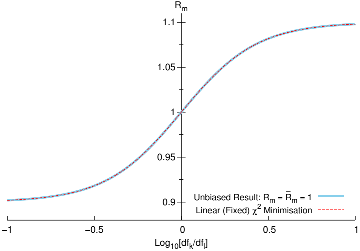
⇒ Non-linear χ^2 minimisation fitting nuisance parameters
→ Penalty trick bias



⟹

KNT18

⇒ Fix the covariance matrix in an iterative χ^2 minimisation
→ Free from bias



Allows for increased fit flexibility and full use of energy dependent, correlated uncertainties

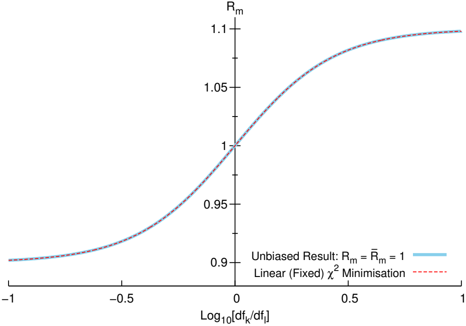
Fixing the covariance matrix [JHEP 1005 (2010) 075, Eur.Phys.J. C75 (2015), 613]

⇒ Apply a procedure to fix the covariance matrix

$$\mathbf{C}_I\left(i^{(m)},j^{(n)}\right)=\mathsf{C}^{\mathsf{stat}}\left(i^{(m)},j^{(n)}\right)+\frac{\mathsf{C}^{\mathsf{sys}}\left(i^{(m)},j^{,n)}\right)}{R_i^{(m)}R_j^{(n)}}R_mR_n \quad ,$$

in an iterative χ^2 minimisation method that, to our best knowledge, is free from bias

- ⇒ Fixing with theory value regulates influence
- ⇒ Can be shown from toy models to be free from bias
- ⇒ Swift convergence
- ⇒ Comparison with past results shows HLMNT11 estimates are largely unaffected



Allows for increased fit flexibility and full use of energy dependent, correlated uncertainties

Properties of a covariance matrix

Any covariance matrix, $\mathcal{C}_{ij}$, of dimension $n \times n$ must satisfy the following requirements:

- As the diagonal elements of any covariance matrix are populated by the corresponding variances, all the diagonal elements of the matrix are positive. Therefore, the trace of the covariance matrix must also be positive

$$\mathsf{Trace}(\mathcal{C}_{ij}) = \sum_{i=1}^{n} \sigma_{ii} = \sum_{i=1}^{n} \mathsf{Var}_i > 0$$

- It is a symmetric matrix, $\mathcal{C}_{ij} = \mathcal{C}_{ji}$, and is, therefore, equal to its transpose, $\mathcal{C}_{ij} = \mathcal{C}_{ij}^T$
- The covariance matrix is a positive, semi-definite matrix,

$$\mathbf{a}^T \mathcal{C}\ \mathbf{a} \geq 0\ ;\ \mathbf{a} \in \mathbf{R^n},$$

  where $\mathbf{a}$ is an eigenvector of the covariance matrix $\mathcal{C}$
- Therefore, the corresponding eigenvalues $\lambda_{\mathbf{a}}$ of the covariance matrix must be real and positive and the distinct eigenvectors are orthogonal

$$\mathbf{b}\ \mathcal{C}\ \mathbf{a} = \lambda_{\mathbf{a}}(\mathbf{b} \cdot \mathbf{a}) = \mathbf{a}\ \mathcal{C}\ \mathbf{b} = \lambda_{\mathbf{b}}(\mathbf{a} \cdot \mathbf{b})$$

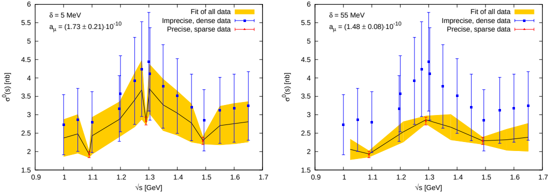
$$\therefore \text{ if } \lambda_{\mathbf{a}} \neq \lambda_{\mathbf{b}} \Rightarrow (\mathbf{a} \cdot \mathbf{b}) = 0$$

- The determinant of the covariance matrix is positive: $\mathsf{Det}(\mathcal{C}_{ij}) \geq 0$

Clustering data

⇒ Re-bin data into *clusters*

Better representation of data combination through adaptive clustering algorithm



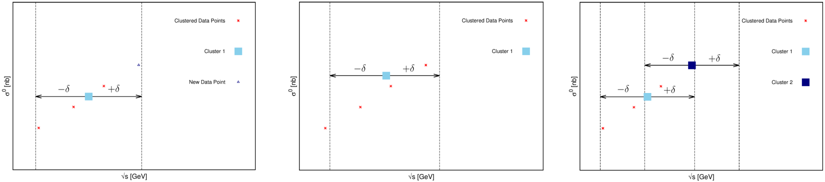
→ More and more data ⇒ risk of over clustering

⇒ loss of information on resonance

→ Scan cluster sizes for optimum solution (error, χ^2, check by sight...)

⇒ Scanning/sampling by varying bin widths

→ Clustering algorithm now adaptive to points at cluster boundaries



Correlation and covariance matrices

⇒ Correlated data beginning to dominate full data compilation...

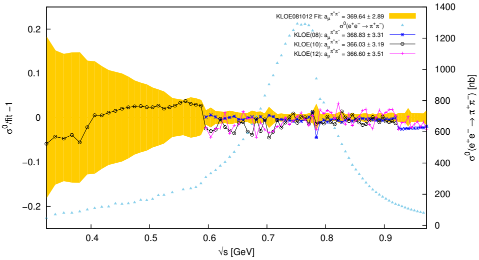
→ Non-trivial, energy dependent influence on both mean value and error estimate

KNT18 prescription

- Construct full covariance matrices for each channel & entire compilation
  ⇒ Framework available for inclusion of any and all inter-experimental correlations
- If experiment does not provide matrices...
  → Statistics occupy diagonal elements only
  → Systematics are 100% correlated
- If experiment does provide matrices...
  → Matrices **must** satisfy properties of a covariance matrix

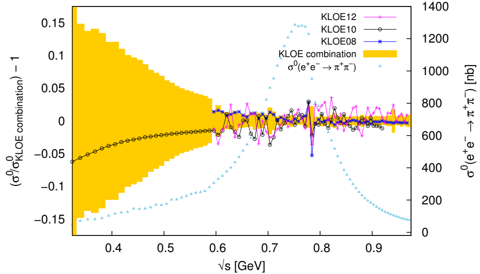
e.g. - KLOE $\pi^+\pi^-\gamma(\gamma)$ combination covariance matrices update

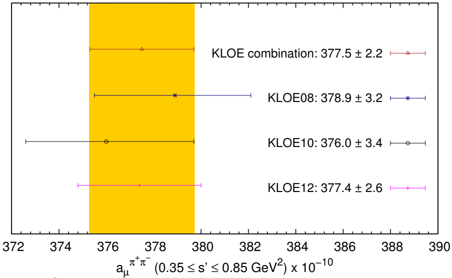
$\Longrightarrow$ Originally, NOT a positive semi-definite matrix



The resulting KLOE $\pi^+\pi^-\gamma(\gamma)$ combination [JHEP 1803 (2018) 173.]

⇒ Combination of KLOE08, KLOE10 and KLOE12 gives 85 distinct bins between $0.1 \leq s \leq 0.95$ GeV^2





→ Covariance matrix now correctly constructed

⇒ a positive semi-definite matrix

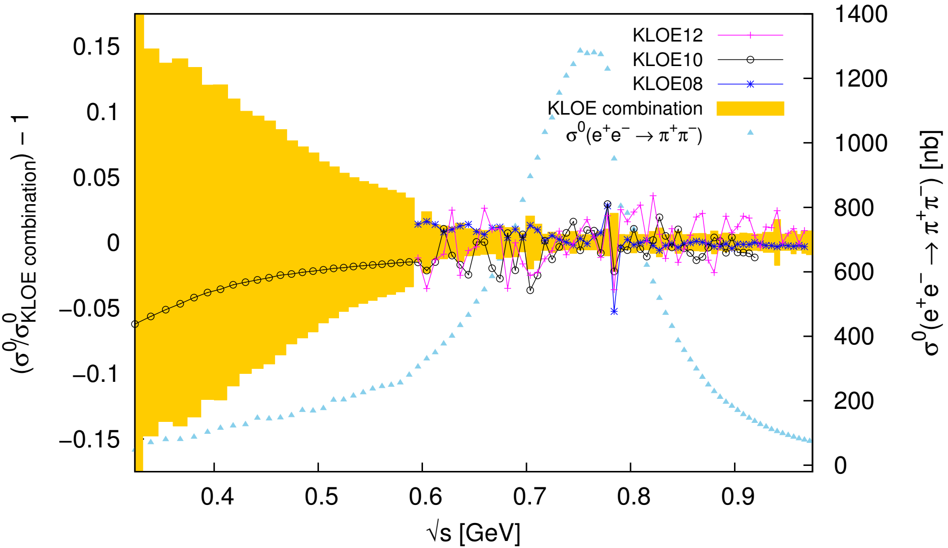
→ Non-trivial influence of correlated uncertainties on resulting mean value

$$a_\mu^{\pi^+\pi^-}(0.1 \leq s' \leq 0.95 \text{ GeV}^2) = (489.9 \pm 2.0_{\text{stat}} \pm 4.3_{\text{sys}}) \times 10^{-10}$$

→ All previous combinations issues now eliminated...

...and consistency between measurements and combination

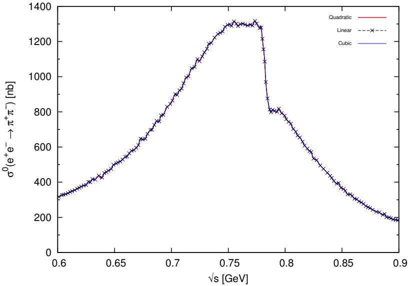
The KLOE $\pi^+\pi^-\gamma(\gamma)$ combination [JHEP 1803 (2018) 173.]

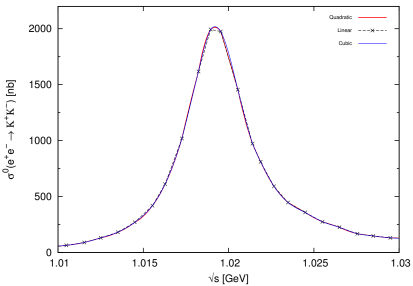


Integration

$\Rightarrow$ Trapezoidal rule integral

- $\rightarrow$ Consistency with linear cluster definition
- $\rightarrow$ High data population $\therefore$ Accurate estimate from linear integral





- $\rightarrow$ Higher order polynomial integrals give (at maximum) differences of $\sim 10\%$ of error

$\Rightarrow$ Estimates of error non-trivial at integral borders

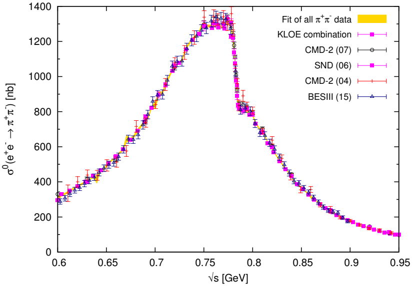
$\longrightarrow$ Extrapolate/interpolate covariance matrices

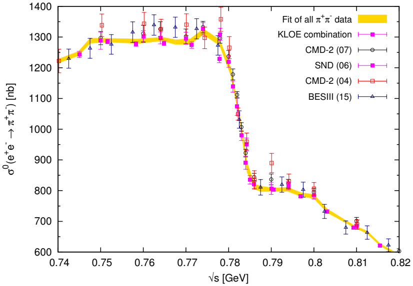
2π without BaBar

$\Rightarrow$ 2π data combination stable without BaBar data

$\rightarrow$ Other data saturate the effect of BaBar

$\rightarrow$ Differences with and without BaBar are now fairly small





In range $0.32 \leq \sqrt{s} \leq 2$ GeV :

$\Rightarrow$ All data: $a_\mu^{\pi^+\pi^-} = 501.45 \pm 1.95$; $\sqrt{\chi^2_{\rm min}/{\rm d.o.f.}} = 1.29$

$\Rightarrow$ No BaBar: $a_\mu^{\pi^+\pi^-} = 500.28 \pm 2.67$; $\sqrt{\chi^2_{\rm min}/{\rm d.o.f.}} = 1.35$

2π CLEO-c data [Phys.Rev. D97 (2018) 032012]

$\Rightarrow$ New 2π data from CLEO-c should be used with caution

- $\rightarrow$ Two measurements taken at different COM energies ($\psi(3770)/\psi(4170)$) have very different cross sections
- $\rightarrow$ Large statistical and systematic errors compared to other radiative return sets
- $\rightarrow$ VP correction has been applied with FJ03VP (needs updated version) and only subtracts real part
- $\rightarrow$ The values for $a_\mu^{\pi^+\pi^-}$ given in the paper only calculated using weighted average
  - $\rightarrow$ Systematics will be highly correlated and should be incorporated
- $\rightarrow$ The authors have fitted the data to Gounaris-Sakurai parametrisation
  - $\rightarrow$ Unreliable representation of cross section at high s
- $\rightarrow$ The authors find (with FJ03VP):

$$a_\mu^{\pi^+\pi^-}(\psi(3770)) = 489.6 \pm 4.5_{\rm stat},\; a_\mu^{\pi^+\pi^-}(\psi(4170)) = 503.6 \pm 5.9_{\rm stat}$$
$$a_\mu^{\pi^+\pi^-}(\text{Weighted average}) = 500.4 \pm 3.6_{\rm stat} \pm 7.5_{\rm sys}$$

- $\rightarrow$ I find (with KNT18VP):

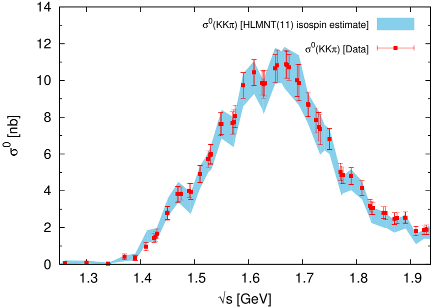
$$a_\mu^{\pi^+\pi^-}(\psi(3770)) = 499.6 \pm 4.5_{\rm stat} \pm 7.5\text{sys},\; a_\mu^{\pi^+\pi^-}(\psi(4170)) = 504.3 \pm 5.9_{\rm stat} \pm 7.6_{\rm sys}$$
$$a_\mu^{\pi^+\pi^-}(\text{Fit} - \text{w/o correlated systematics}) = 500.9 \pm 4.0_{\rm stat} \pm 5.9_{\rm sys}$$
$$a_\mu^{\pi^+\pi^-}(\text{Fit} - \text{with correlated systematics}) = 500.7 \pm 4.0_{\rm stat} \pm 8.3_{\rm sys}$$

$KK\pi$, $KK\pi\pi$ and isospin

$\Rightarrow$ New data for $KK\pi$ and $KK\pi\pi$
removes reliance on isospin (only $K_S^0 \cong K_L^0$)

<u>$KK\pi$</u>

$K_S^0 K_L^0 \pi^0$ [Phys.Rev. D95 (2017), 052001, arXiv:1711.07143]

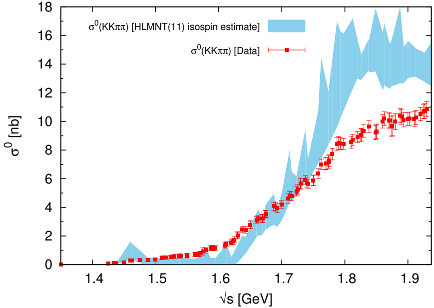


HLMNT11: 2.65 ± 0.14

KNT18: 2.71 ± 0.12

<u>$KK\pi\pi$</u>

$K_S^0 K_L^0 \pi^+ \pi^-$ [Phys.Rev. D80 (2014), 092002]
$K_S^0 K_S^0 \pi^+ \pi^-$ [Phys.Rev. D80 (2014), 092002],
$K_S^0 K_L^0 \pi^0 \pi^0$ [Phys.Rev. D95 (2017), 052001]
$K_S^0 K^\pm \pi^\mp \pi^0$ [Phys.Rev. D95 (2017), 092005]



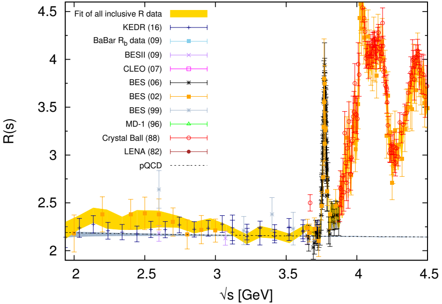
HLMNT11: 2.51 ± 0.35

KNT18: 1.93 ± 0.08

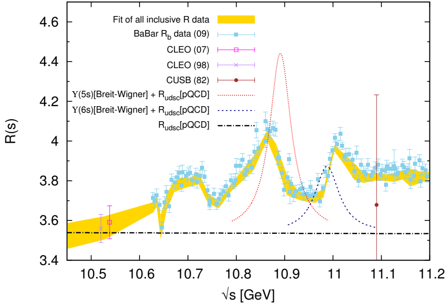
$\Rightarrow$ **But**, still reliant on isospin estimates for $\pi^+\pi^-3\pi^0$, $\pi^+\pi^-4\pi^0$, $KK3\pi$...

Inclusive

⇒ New KEDR inclusive R data [Phys.Lett. B770 (2017) 174-181, Phys.Lett. B753 (2016) 533-541] and BaBar R_b data [Phys. Rev. Lett. 102 (2009) 012001.].



KEDR data improves the inclusive data combination below $c\bar{c}$ threshold



R_b resolves the resonances of the $\Upsilon(5S-6S)$ states.

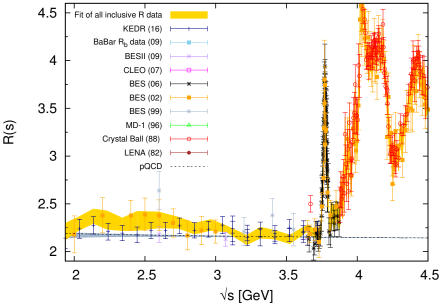
$\Longrightarrow$ **Choose to adopt entirely data driven estimate from threshold to 11.2 GeV**

$$a_\mu^{\text{Inclusive}} = 43.67 \pm 0.17_{\text{stat}} \pm 0.48_{\text{sys}} \pm 0.01_{\text{vp}} \pm 0.44_{\text{fsr}} = 43.67 \pm 0.67_{\text{tot}}$$

KEDR update of $R(s)$ with covariance matrix

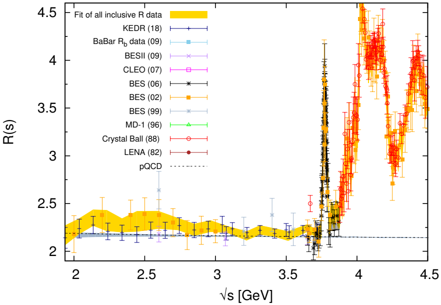
$\Rightarrow$ New precise KEDR update [arXiv:1805.06235] with systematic covariance matrix for all measurements provided by experiment

KNT18



$$a_\mu^{\text{Inclusive}} = 43.67 \pm 0.51$$
$$\sqrt{\chi^2_{\text{min}}/\text{d.o.f.}} = 1.44$$

KNT18 + new KEDR data



$$a_\mu^{\text{Inclusive}} = 43.54 \pm 0.51$$
$$\sqrt{\chi^2_{\text{min}}/\text{d.o.f.}} = 1.47$$

Note: Uncertainties quoted here do not include radiative correction uncertainties

$\Rightarrow$ Observe **very small** changes due to including correlations (slightly closer to pQCD)

Exclusive/inclusive transition point

⇒ New KEDR data allow reconsideration of exclusive/inclusive transition point

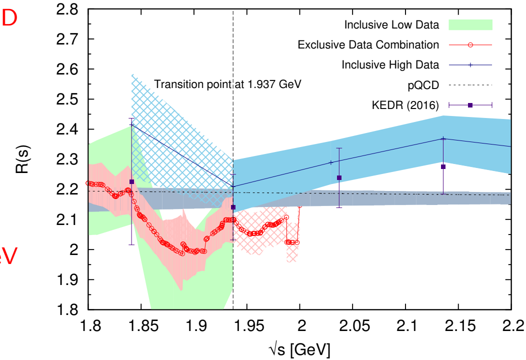
→ KNT18 aim to avoid use of pQCD and keep a data-driven analysis

→ Disagreement between sum of exclusive states and inclusive data/pQCD

→ New $\pi^+\pi^-\pi^0\pi^0$ data result in reduction of the cross section

→ Previous transition point at 2 GeV no longer the preferred choice

→ More natural choice for this transition point at 1.937 GeV



Input	$a_\mu^{\text{had, LO VP}}[1.841 \le \sqrt{s} \le 2.00\ \text{GeV}] \times 10^{10}$
Exclusive sum	6.06 ± 0.17
Inclusive data	6.67 ± 0.26
pQCD	6.38 ± 0.11
Exclusive (< 1.937 GeV) + inclusive (> 1.937 GeV)	6.23 ± 0.13

$\chi^2_{\rm min}/{\rm d.o.f}$ comparison with HLMNT11

Channel	This work (KNT18)	HLMNT11
$\pi^+\pi^-$	1.3	1.4
$\pi^+\pi^-\pi^0$	2.1	3.0
$\pi^+\pi^-\pi^+\pi^-$	1.8	1.7
$\pi^+\pi^-\pi^0\pi^0$	2.0	1.3
$(2\pi^+2\pi^-\pi^0)_{\text{no }\eta}$	1.0	1.2
$(2\pi^+2\pi^-2\pi^0)_{\text{no }\eta\omega}$	3.5	4.0
K^+K^-	2.1	1.9
$K_S^0K_L^0$	0.8	0.8

Table: Comparison of the global $\sqrt{\chi^2_{\rm min}/{\rm d.o.f}}$ for the leading and major sub-leading channels determined in the HLMNT11 analysis and in this work (KNT18). The first column indicates the final state or individual contribution, the second column gives the KNT18 value, the third column states the HLMNT11 value and the last column gives the difference between the two numbers.

KNT18 $\alpha(M_Z^2)$ update

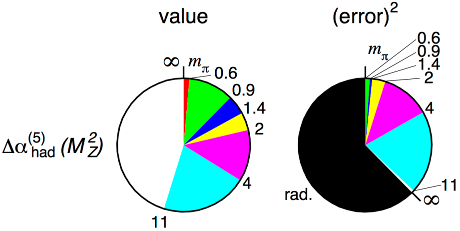
$\Delta\alpha^{(5)}(M_Z^2)$ HLMNT11: $(276.26 \pm 1.38_{\rm tot}) \times 10^{-4}$

$\downarrow$

This work: $\Delta\alpha^{(5)}_{\rm had}(M_Z^2) = (276.11 \pm 0.26{\rm stat} \pm 0.68_{\rm sys} \pm 0.14_{\rm vp} \pm 0.82_{\rm fsr}) \times 10^{-4}$

$$= (276.11 \pm 0.73_{\rm exp} \pm 0.84_{\rm rad}) \times 10^{-4}$$

$$= (276.11 \pm 1.11_{\rm tot}) \times 10^{-4}$$



$$\Rightarrow \alpha^{-1}(M_Z^2) = 128.946 \pm 0.015$$

Comparison with HLMNT11

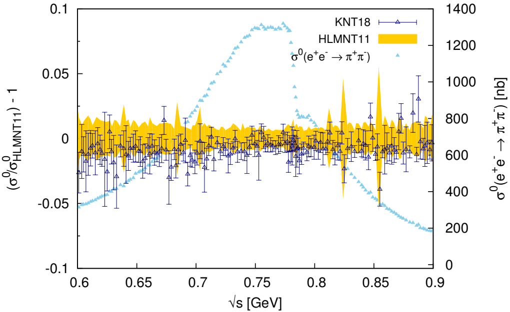
Channel	This work (KNT18)	HLMNT11	Difference
$\pi^+\pi^-$	502.99 ± 1.97	505.77 ± 3.09	-2.78
$\pi^+\pi^-\pi^0$	47.82 ± 0.89	47.51 ± 0.99	0.31
$\pi^+\pi^-\pi^+\pi^-$	15.17 ± 0.21	14.65 ± 0.47	0.52
$\pi^+\pi^-\pi^0\pi^0$	19.80 ± 0.79	20.37 ± 1.26	-0.57
K^+K^-	23.05 ± 0.22	22.15 ± 0.46	0.90
$K_S^0 K_L^0$	13.05 ± 0.19	13.33 ± 0.16	-0.28
Inclusive channel	41.27 ± 0.62	41.40 ± 0.87	-0.13
Total	693.27 ± 2.46	694.91 ± 4.27	-1.64

$\Rightarrow$ Biggest difference in 2π channel
$\rightarrow$ large reduction in mean and uncertainty

$\Rightarrow$ Tensions with HLMNT11 analysis for both two-kaon channels

$\Rightarrow$ Overall agreement with HLMNT11

$\Rightarrow$ Notable improvement of about one third in uncertainty



Comparison with other similar works

Channel	This work (KNT18)	DHMZ17	Difference
$\pi^+\pi^-$	503.74 ± 1.96	507.14 ± 2.58	-3.40
$\pi^+\pi^-\pi^0$	47.70 ± 0.89	46.20 ± 1.45	1.50
$\pi^+\pi^-\pi^+\pi^-$	13.99 ± 0.19	13.68 ± 0.31	0.31
$\pi^+\pi^-\pi^0\pi^0$	18.15 ± 0.74	18.03 ± 0.54	0.12
K^+K^-	23.00 ± 0.22	22.81 ± 0.41	0.19
$K^0_S K^0_L$	13.04 ± 0.19	12.82 ± 0.24	0.22
$1.8 \le \sqrt{s} \le 3.7$ GeV	34.54 ± 0.56 (data)	33.45 ± 0.65 (pQCD)	1.09
Total	693.3 ± 2.5	693.1 ± 3.4	0.2

$\Rightarrow$ Total estimates from two analyses in very good agreement

$\Rightarrow$ Masks much larger differences in the estimates from individual channels

$\Rightarrow$ Unexpected tension for 2π considering the data input likely to be similar

- $\rightarrow$ Points to marked differences in way data are combined
- $\rightarrow$ From 2π discussion: $a_\mu^{\pi^+\pi^-}$ (Weighted average) $= 509.1 \pm 2.9$

$\Rightarrow$ Compensated by lower estimates in other channels

- $\rightarrow$ For example, the choice to use pQCD instead of data above 1.8 GeV

$\Rightarrow$ FJ17: $a_{\mu,\,\mathrm{FJ17}}^{\mathrm{had,\,LO\,VP}} = 688.07 \pm 41.4$

- $\rightarrow$ Much lower mean value, but in agreement within errors

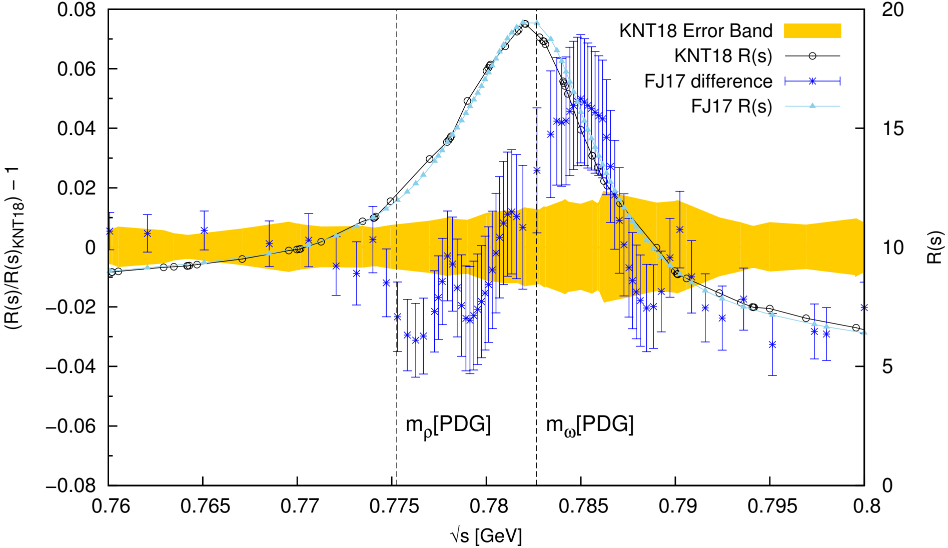
Comparison tables

Channel	KNT18	DHMZ17	Difference
Data based channels ($\sqrt{s} \leq 1.8$ GeV)			
$\pi^+\pi^-$	503.74 ± 1.96	506.70 ± 2.58	-2.96
$\pi^+\pi^-\pi^0$	47.70 ± 0.89	46.20 ± 1.45	1.50
$\pi^+\pi^-\pi^+\pi^-$	13.99 ± 0.19	13.68 ± 0.31	0.31
$\pi^+\pi^-\pi^0\pi^0$	18.15 ± 0.74	18.03 ± 0.54	0.12
K^+K^-	23.00 ± 0.22	23.06 ± 0.41	-0.06
$K_S^0K_L^0$	13.04 ± 0.19	12.82 ± 0.24	0.22
Total	693.3 ± 2.5	693.1 ± 3.4	0.2

Channel	KNT18	FJ17	Difference
Data based channels ($0.318 \leq \sqrt{s} \leq 2$ GeV)			
$\pi^+\pi^-$	501.68 ± 1.71	502.16 ± 2.44	-0.48
$\pi^+\pi^-\pi^0$	47.83 ± 0.89	44.32 ± 1.48	3.51
$\pi^+\pi^-\pi^+\pi^-$	15.17 ± 0.21	14.80 ± 0.36	0.37
$\pi^+\pi^-\pi^0\pi^0$	19.80 ± 0.79	19.69 ± 2.32	0.11
K^+K^-	23.05 ± 0.22	21.99 ± 0.61	1.06
$K_S^0K_L^0$	13.05 ± 0.19	13.10 ± 0.41	-0.05
Total	693.27 ± 2.46	688.07 ± 4.14	5.20

Channel	KNT18	Benayoun et. al	Difference
Data based channels ($\sqrt{s} \leq 1.05$ GeV)			
$\pi^+\pi^-$	495.86 ± 1.94	489.83 ± 1.22	6.03
$\pi^+\pi^-\pi^0$	44.49 ± 0.80	42.94 ± 0.52	1.55
K^+K^-	18.12 ± 0.18	17.18 ± 0.25	0.94
$K_S^0K_L^0$	11.97 ± 0.17	11.87 ± 0.25	0.10

Comparison of $\rho - \omega$ with FJ17



Comparison of ϕ with FJ17