Hadronic contribution to $(g-2)_\mu$ from e^+e^- annihilations

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2nd plenary workshop of the g-2 theory initiative

- June 2018 -

Content of the talk

- Data on $e^+e^- \rightarrow$ hadrons

- Updated combination of all e^+e^- data:
  focus on the combination procedure (HVPTools)

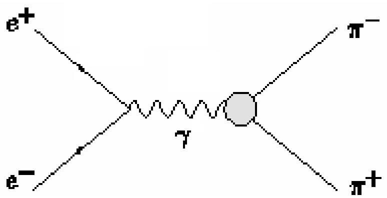
→ Updated KLOE data with correlations ($\pi\pi$)

→ New data from CLEO ($\pi\pi$)

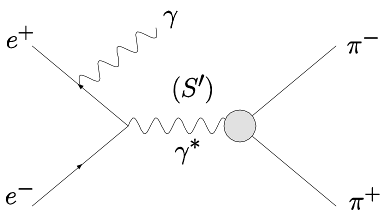
- Results on a_μ

- Discussion and conclusions

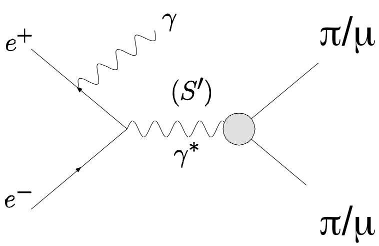
HVP: Low-energy data on $e^+e^- \to$ hadrons

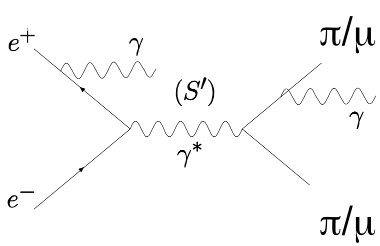


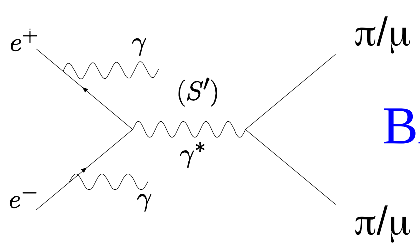
$\sqrt{s}$ scan + radiative corrections: CMD-2&3, SND, BES etc.



KLOE (08&10) + μμ (12) (ISR)







BABAR (09) (ISR + Add. rad.)

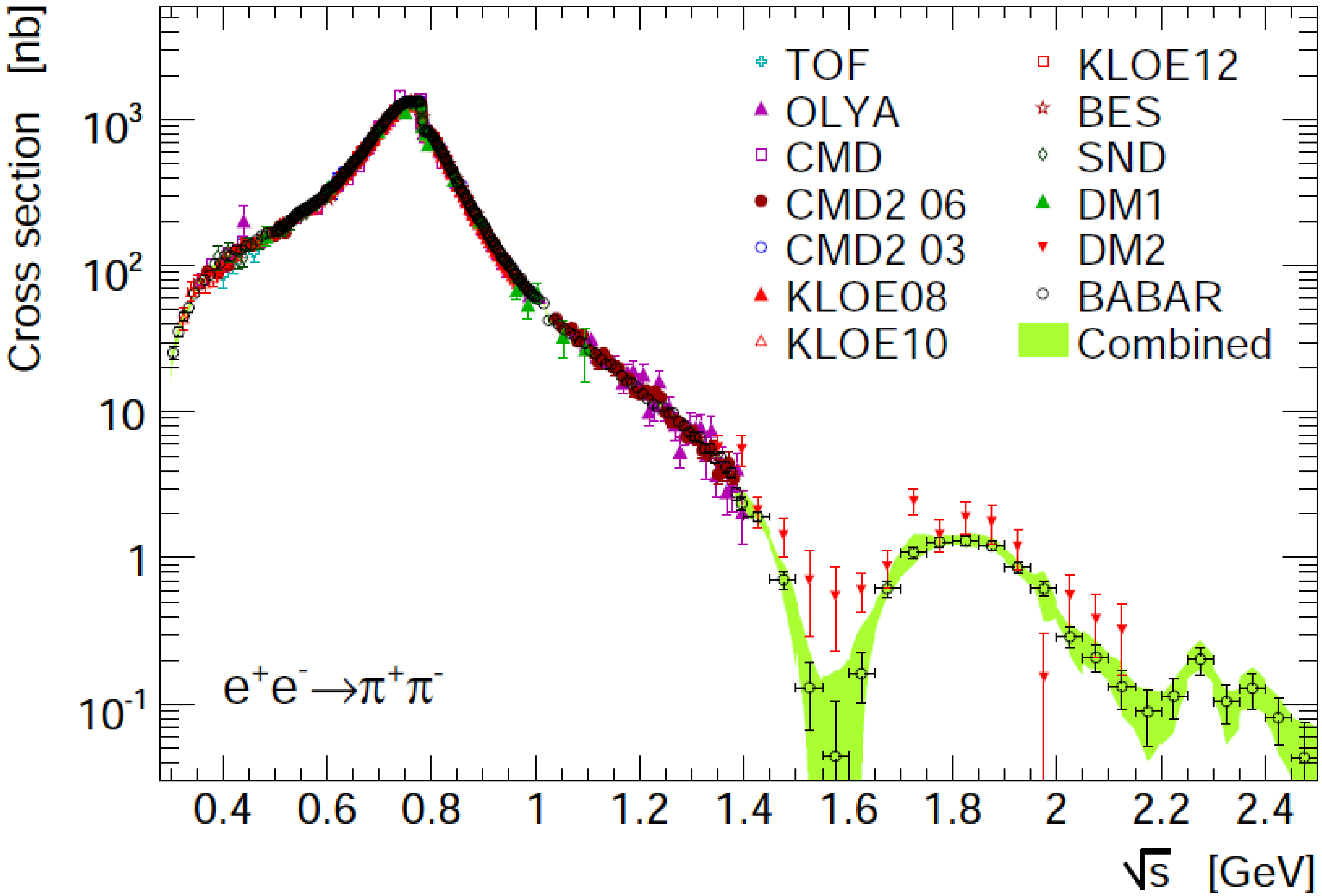
Need: $e^+e^- \to$ hadrons bare (no VP) cross section

→ in addition to the dominant ππ channel, need to account for KK, $\pi^0\gamma$, $\eta\gamma$ + channels with higher multiplicities

→ need to combine measurements in each channel & sum channels

→ Do not use hadronic τ decays data (less precise + theory uncertainties)

Combination for the $e^+e^- \rightarrow \pi^+\pi^-$ channel (2017)



arXiv: 1706.09436 (EPJ C) Davier-Hoecker-BM-Zhang

Improved procedure and software (HVPTools) for combining cross section data with arbitrary point spacing/binning

Combine Cross Section Data: goal and requirements

→ Goal: combine experimental spectra with arbitrary point spacing / binning

→ Requirements:

- Properly propagate uncertainties and correlations

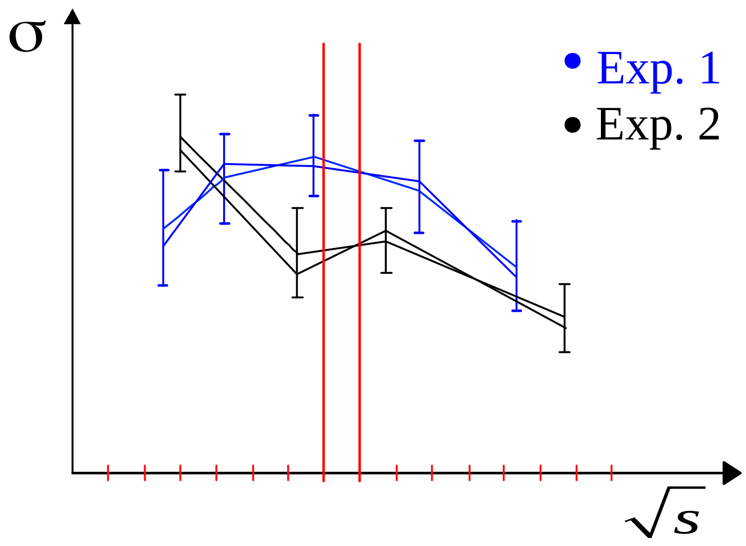
- *Between measurements (data points/bins) of a given experiment* (covariance matrices and/or detailed split of uncertainties in sub-components)
- *Between experiments* (common systematic uncertainties, e.g. VP) – based on detailed information provided in publications
- *Between different channels* – motivated by understanding of the meaning of systematic uncertainties and identifying the common ones:

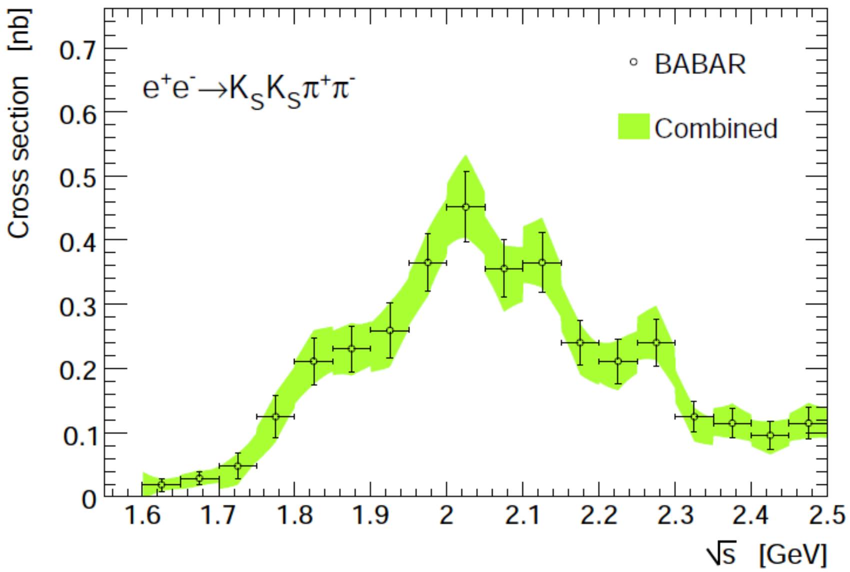
BABAR luminosity (ISR or BhaBha), efficiencies (photon, Ks, Kl, modeling);
BABAR radiative corrections; $4\pi 2\pi^0$-$\eta\omega$
CMD2 $\eta\gamma$ – $\pi^0\gamma$; CMD2/3 luminosity; SND luminosity;
FSR; hadronic VP (old experiments)

- Minimize biases

- Optimize g-2 integral uncertainty (without overestimating the precision with which the uncertainties of the measurements are known)

Combination procedure implemented in HVPTools software





→ Define a (fine) final binning (to be filled and used for integrals etc.)

→ Linear/quadratic splines to interpolate between the points/bins of each experiment
- for binned measurements: preserve integral inside each bin

→ Fluctuate data points taking into account correlations and re-do the splines for each (pseudo-)experiment
- each uncertainty fluctuated coherently for all the points/bins that it impacts
- eigenvector decomposition for (statistical & systematic) covariance matrices

Combination procedure implemented in HVPTools software

For each final bin:

→ Compute an average value for each measurement and its uncertainty

→ Compute correlation matrix between experiments

→ Minimize χ^2 and get average coefficients (weights)

→ Compute average between experiments and its uncertainty

Evaluation of integrals and propagation of uncertainties:

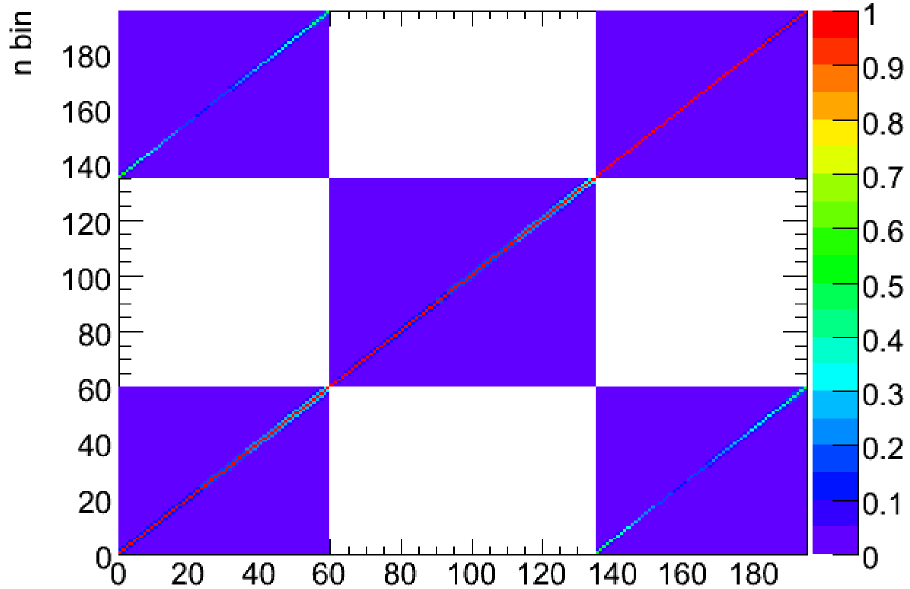
→ Integral(s) evaluated for nominal result and for each set of toy pseudo-experiments; uncertainty of integrals from RMS of results for all toys

→ The pseudo-experiments also used to derive (statistical & systematic) covariance matrices of combined cross sections → Integral evaluation

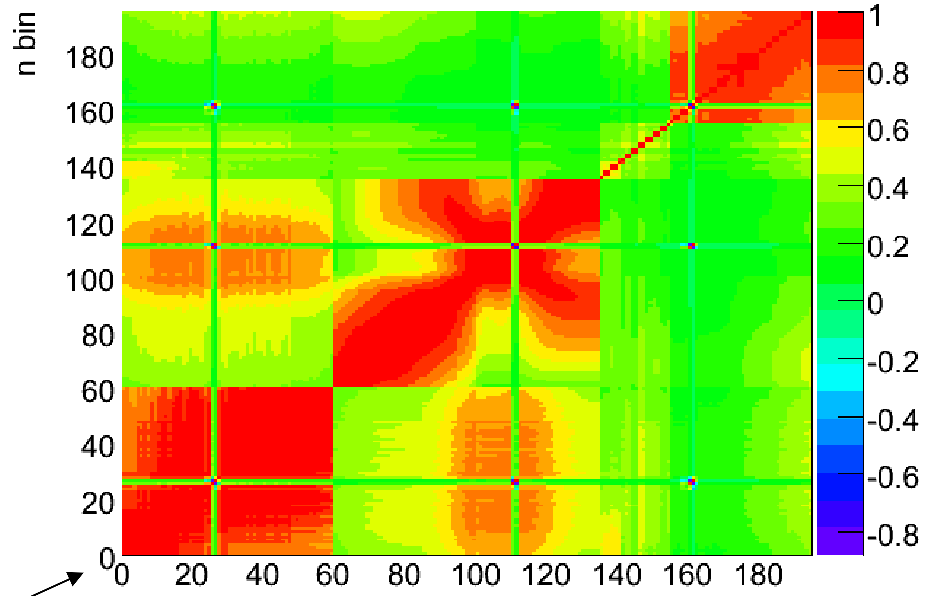
→ Uncertainties also propagated through $\pm 1\sigma$ shifts of each uncertainty:
- allows to account for correlations between different channels (for integrals and spectra)

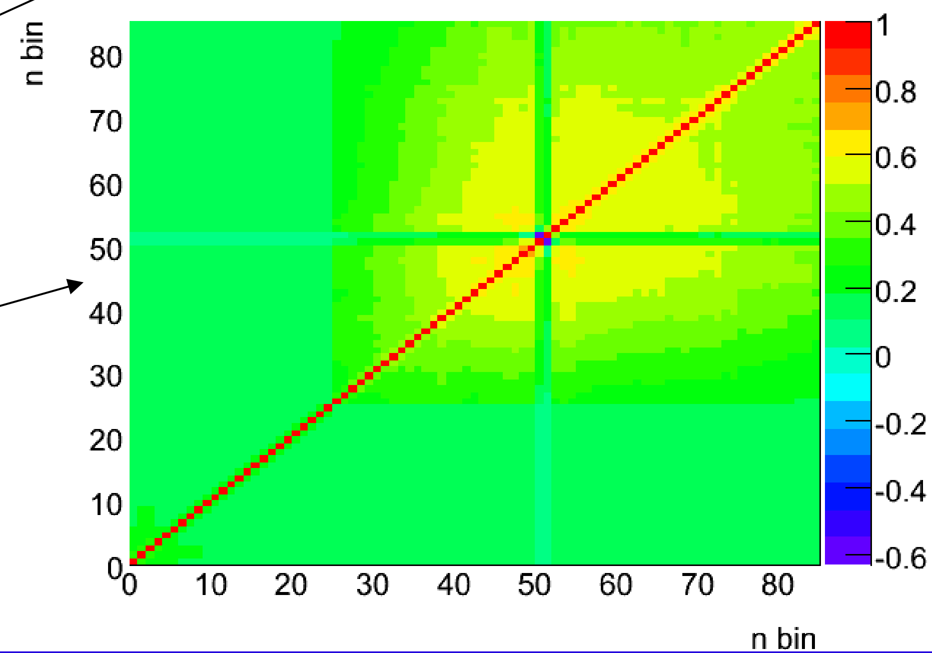
→ *Checked consistency between the different approaches*

Treatment of the KLOE data – correlation matrices



KLOE: 08 10 12 n bin



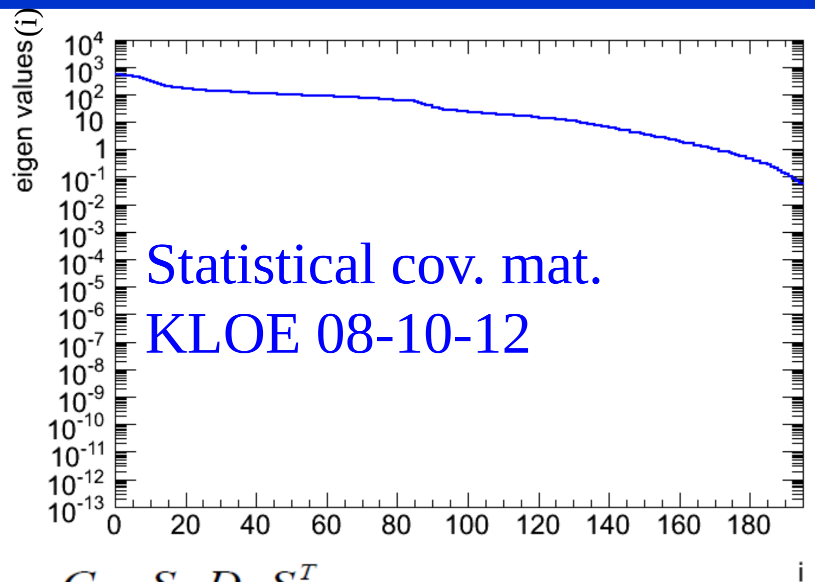


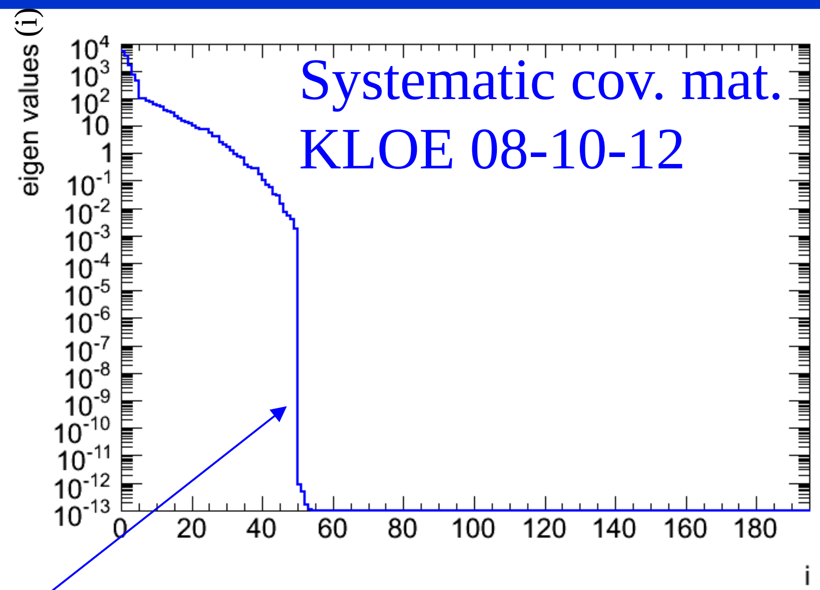
→ Statistical and systematic covariance matrices among the 3 measurements

→ Total covariance matrix for the combination of the 3 measurements

→ Lacking information on correlations with BES (VP, FSR, rad. func.) : need individual uncertainties

Treatment of the KLOE data – eigenvector decomposition

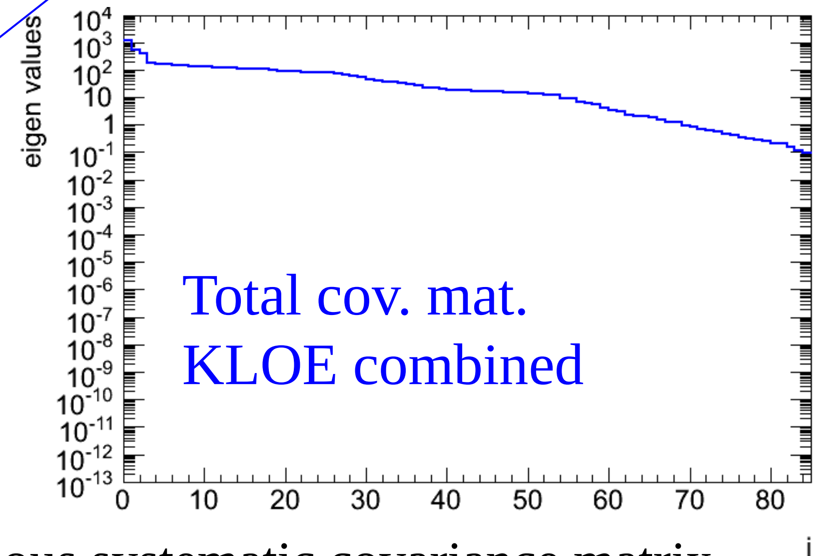




$$C = S \cdot D \cdot S^T$$

$$D = \begin{pmatrix} \ddots & 0 & 0 \\ 0 & \sigma_i^2 & 0 \\ 0 & 0 & \ddots \end{pmatrix}$$

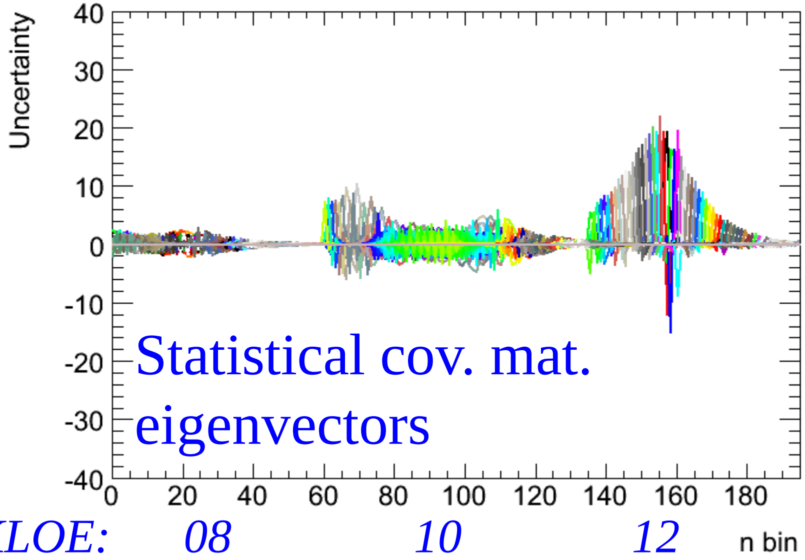
$$S = \begin{pmatrix} V_1 & \dots & V_n \end{pmatrix}$$

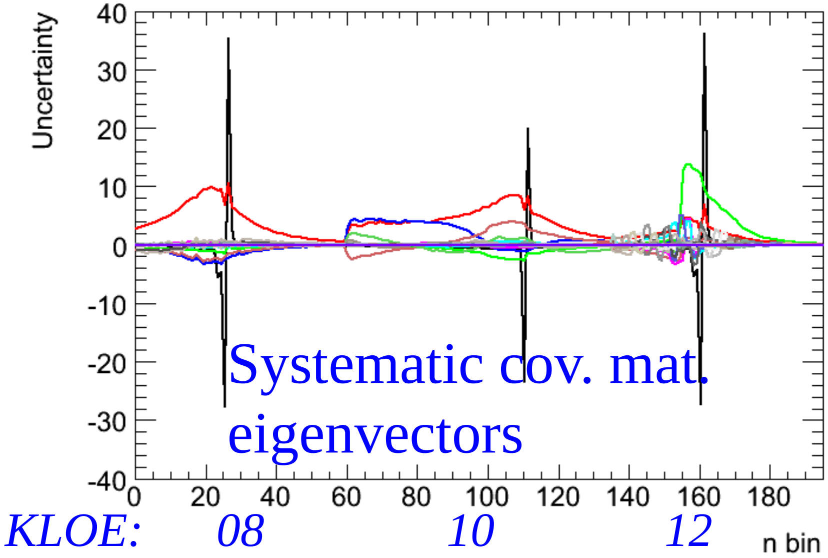


→ "counting" the number of independent components (50) used to build the covariance matrix

→ Problem of negative eigenvalues for previous systematic covariance matrix solved (informed KLOE collaboration about the problem in summer 2016)

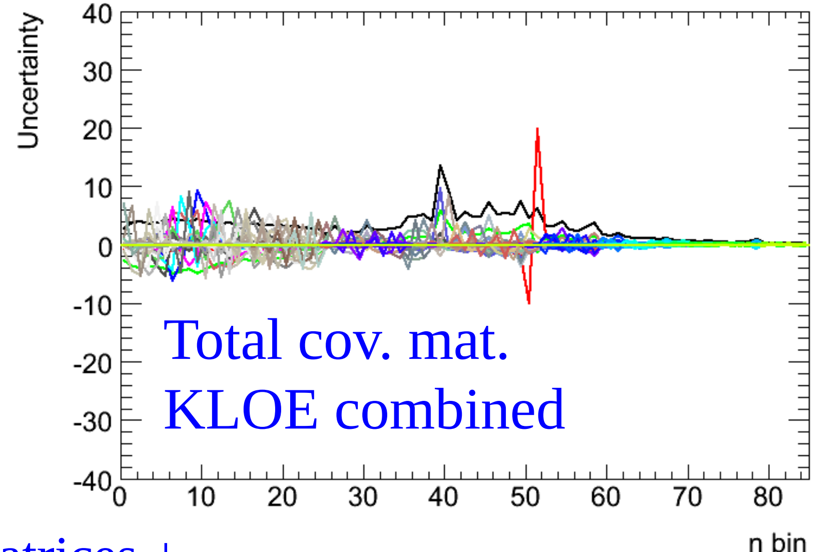
Treatment of the KLOE data – eigenvector decomposition





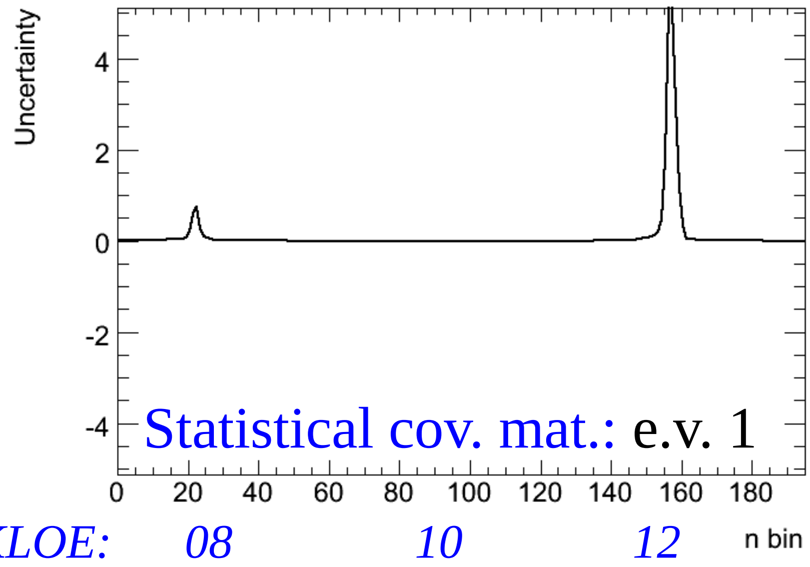
- → Each normalized eigenvector ($\sigma_i * V_i$) treated as an uncertainty fully correlated between the bins
- → All these uncertainties are independent between each-other

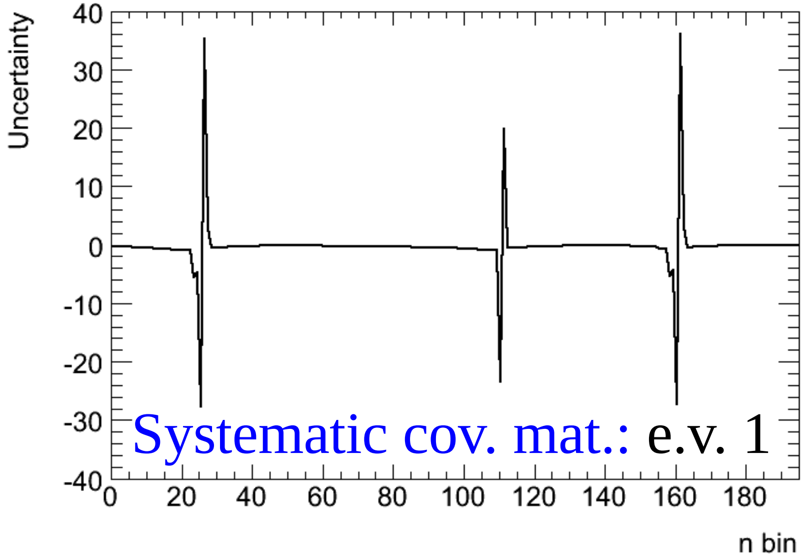
$$C = \sum_{i=1}^{N_{bins}} \sigma_i^2 \cdot C\left(V_i\right)$$

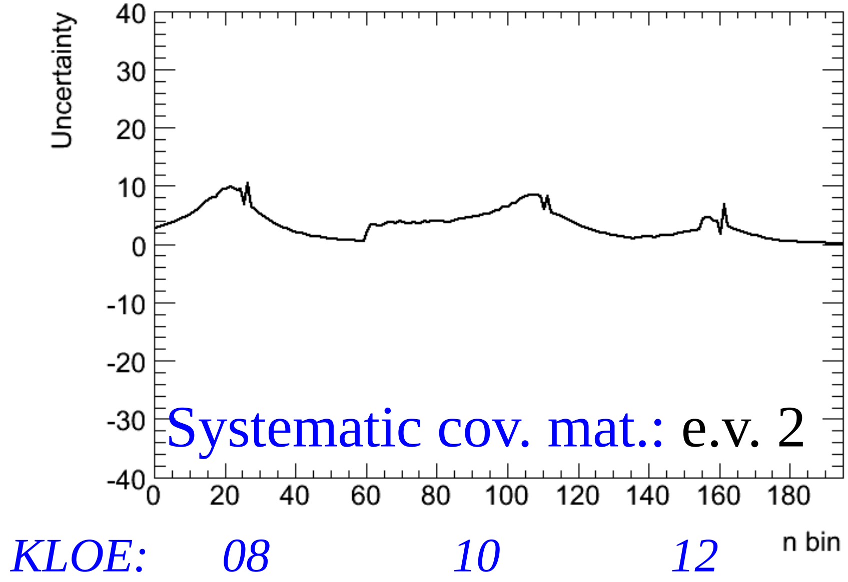


- → Checked exact matching with the original matrices + with all a_μ integrals and uncertainties published by KLOE

Treatment of the KLOE data – eigenvector decomposition







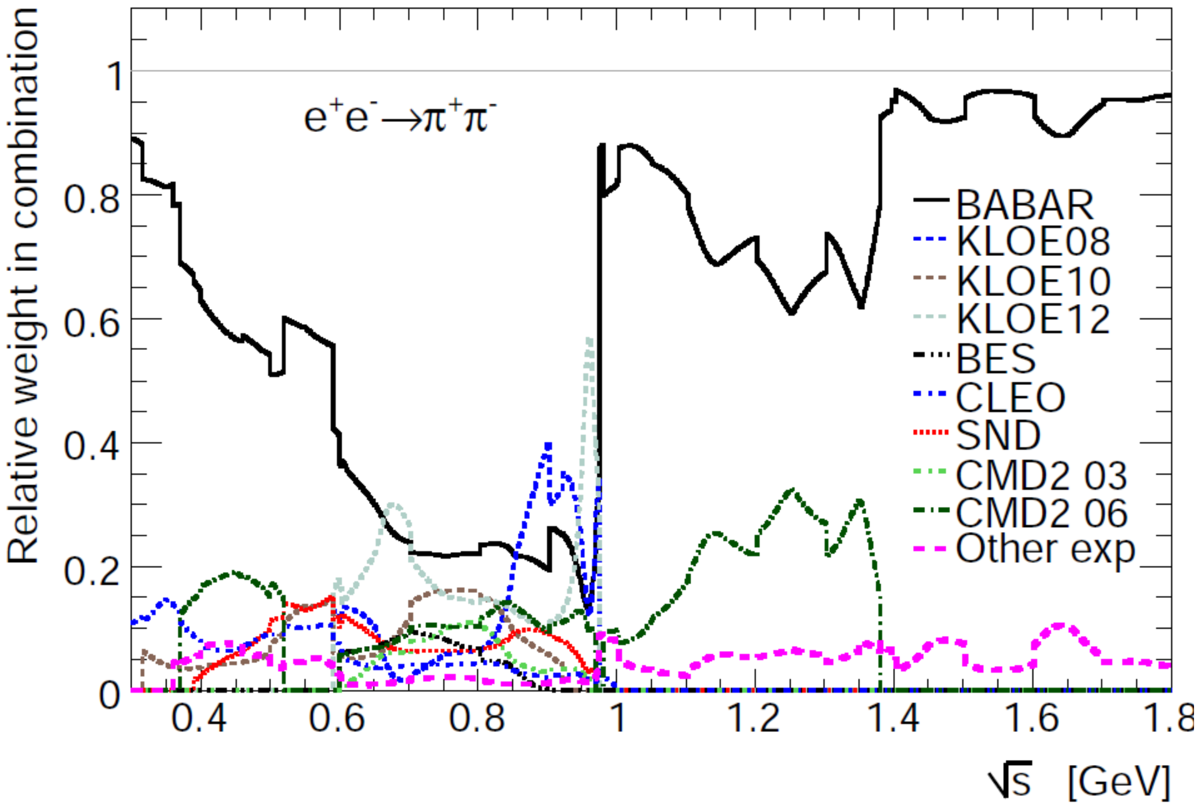
→ Eigenvectors carry the general features of the correlations:

- long-range for systematics
- ~short-range for statistical uncertainties + correlations between KLOE 08 & 12

Combination procedure: weights of various measurements

For each final bin:
→ Minimize χ^2 and get average coefficients

Note: average weights must account for bin sizes / point spacing of measurements (do not over-estimate the weight of experiments with large bins)
→ weights in fine bins evaluated using a common (large) binning for measurements + interpolation → compare the precisions on the same footing



→ Bins used by KLOE larger than the ones by BABAR in ρ-ω interference region (factor ~3)

→ Average dominated by BaBar and KLOE, BaBar covering full range

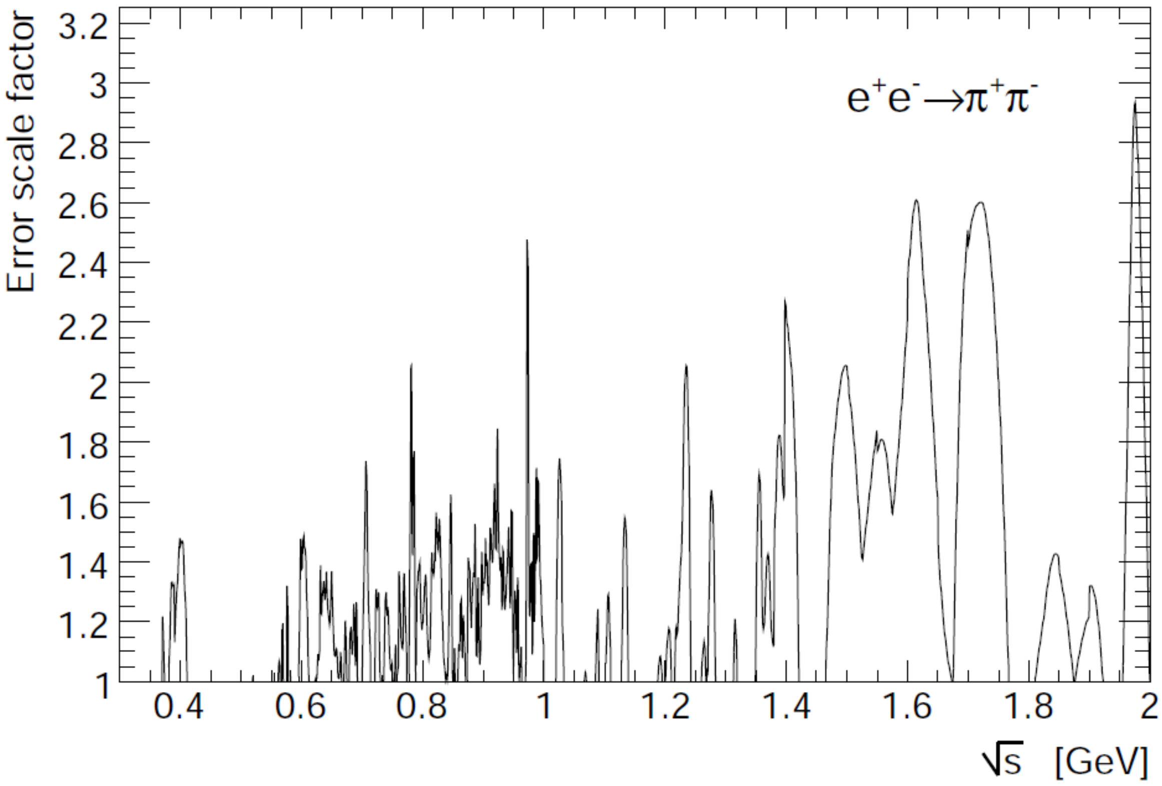
Combination procedure: compatibility between measurements

For each final bin:

→ χ^2/ndof: test locally the level of agreement between input measurements, *taking into account the correlations*

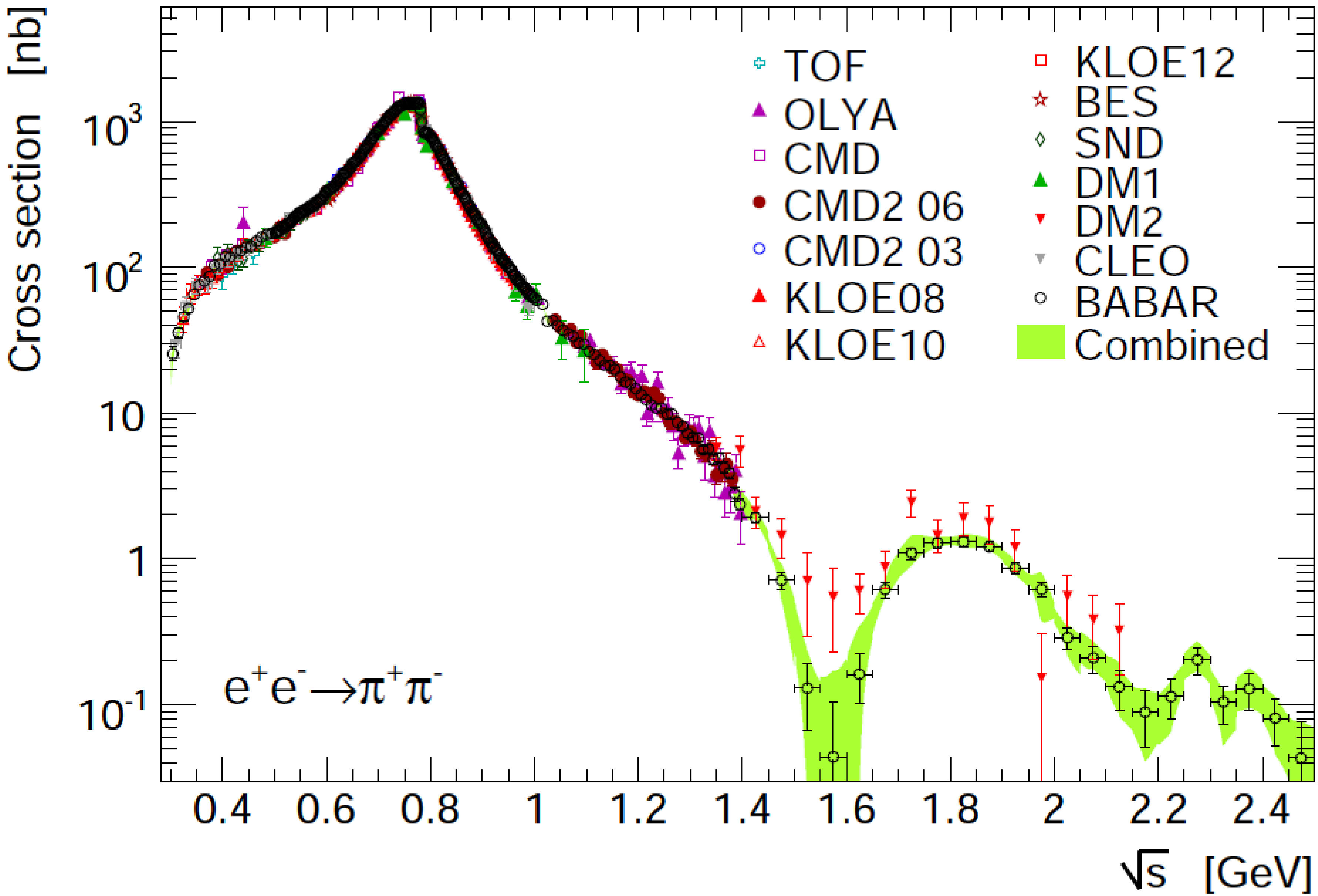
→ Conservatively scale uncertainties in bins where χ^2/ndof > 1 (PDG)

→ Observed tension between BABAR and KLOE measurements

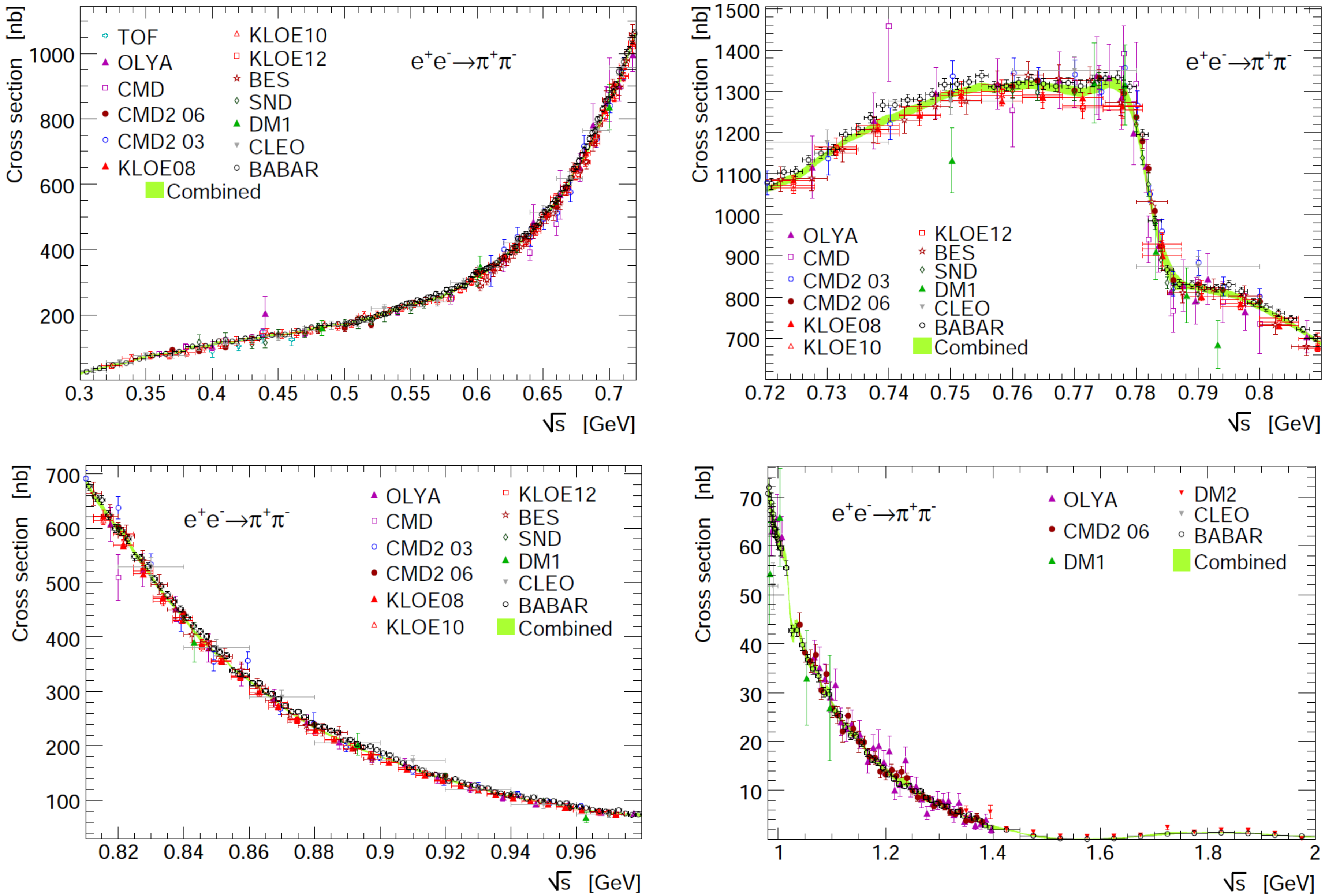


→ Also motivates conservative uncertainty treatment in evaluation of weights

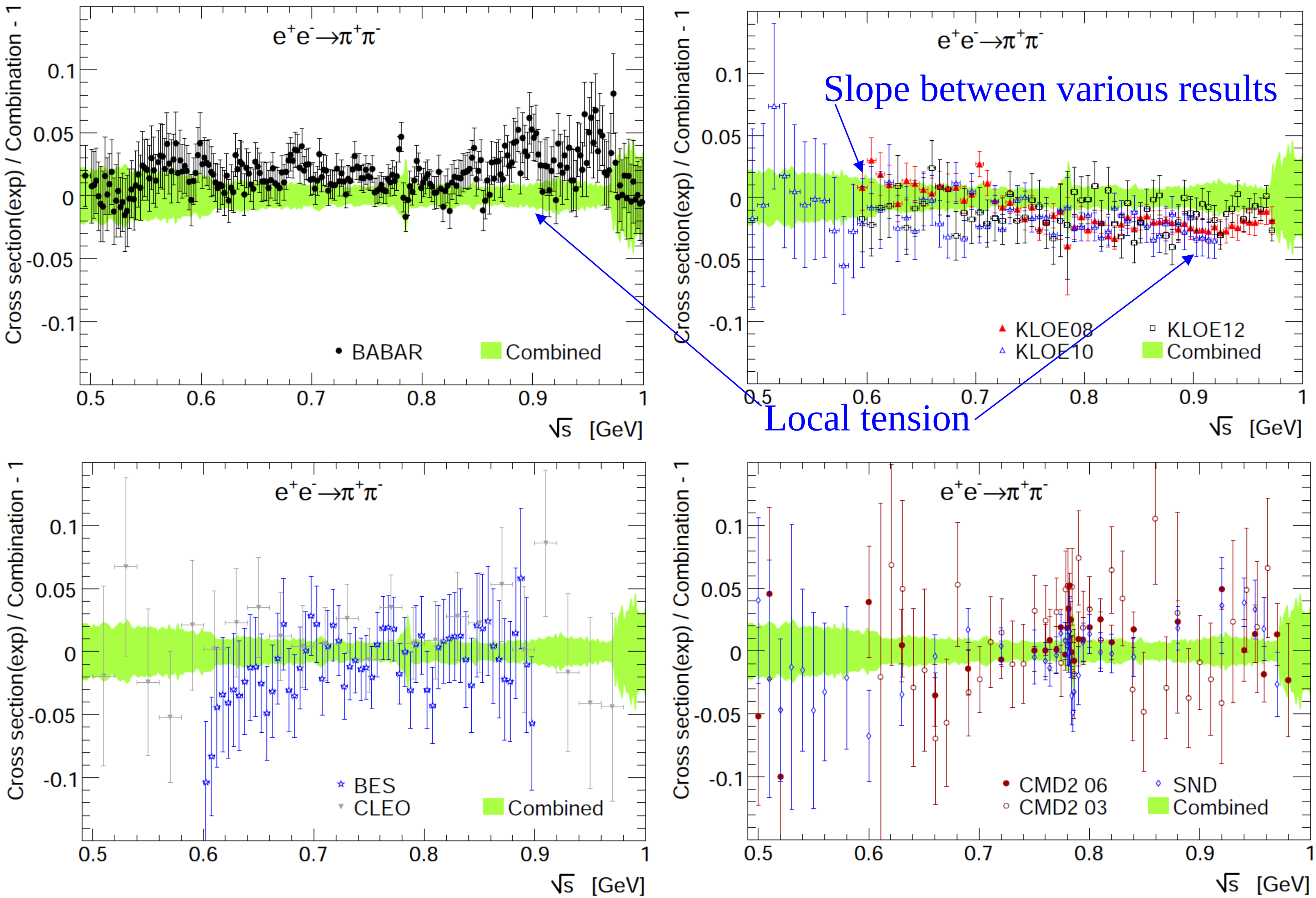
Combination for the $e^+e^- \rightarrow \pi^+\pi^-$ channel



Combination for the $e^+e^- \to \pi^+\pi^-$ channel



Combination for the $e^+e^- \to \pi^+\pi^-$ channel



$a_\mu^{\pi\pi}$ contribution [0.28; 1.8] GeV

→ Closure test of the combination method:
- replace all central values of the measured cross sections by predictions from of a Gounaris-Sakurai model (keeping uncertainties unchanged)
- perform combination and integration procedure
- compare integration result with expectation from integral of the model

→ Bias ~ $0.1 \cdot 10^{-10}$ when using linear interpolation

→ *Negligible bias for quadratic interpolation*

→ Updated result:

506.70 ± 2.32 ($\pm$ 1.01 (stat.) $\pm$ 2.08 (syst.)) [10^{-10}]

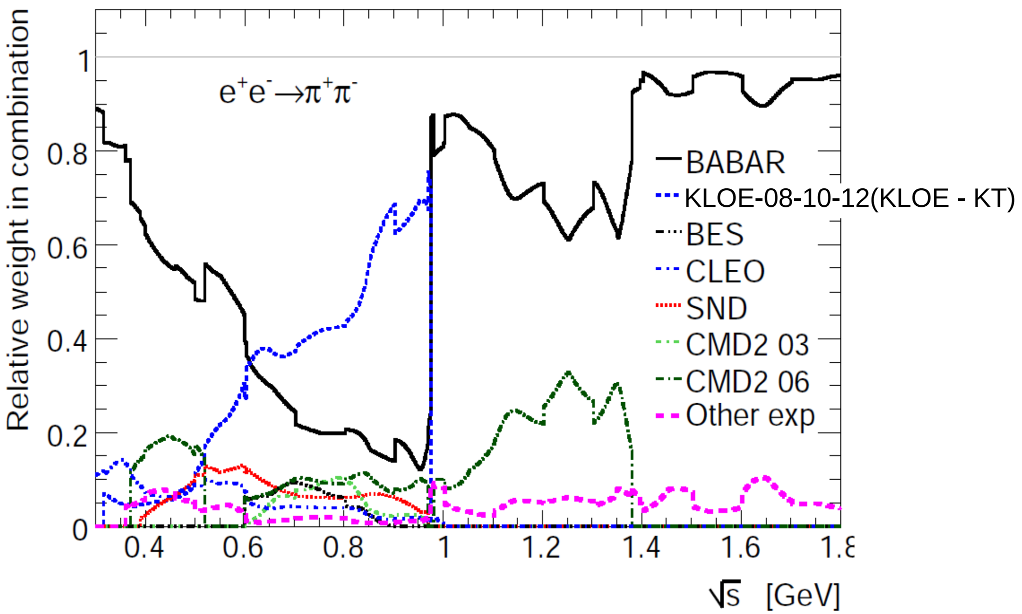
(after uncertainty enhancement by 14% caused by the tension between inputs)

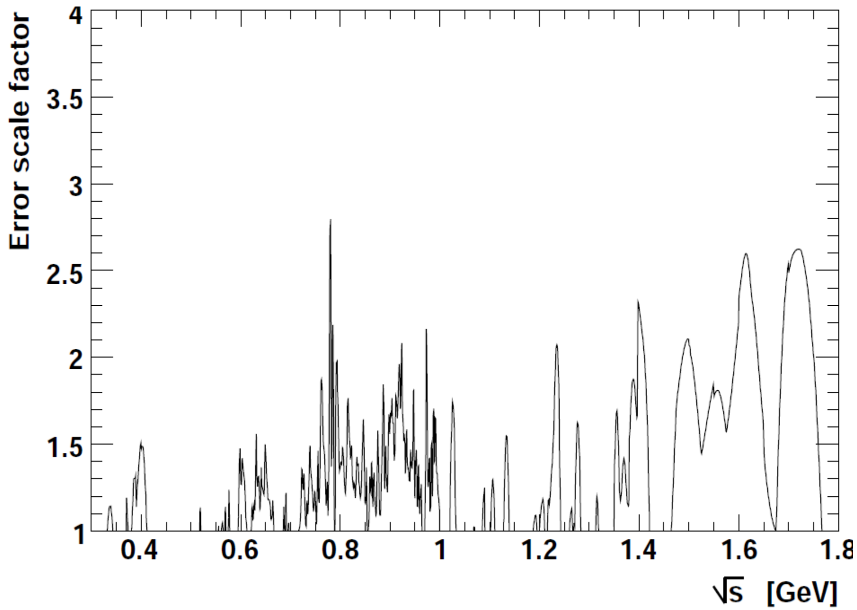
Total uncertainty: 5.9 (2003) → 2.8 (2011) → 2.6 (2017) → 2.3 (2018)

$a_\mu^{\pi\pi}$ contribution [0.28; 1.8] GeV

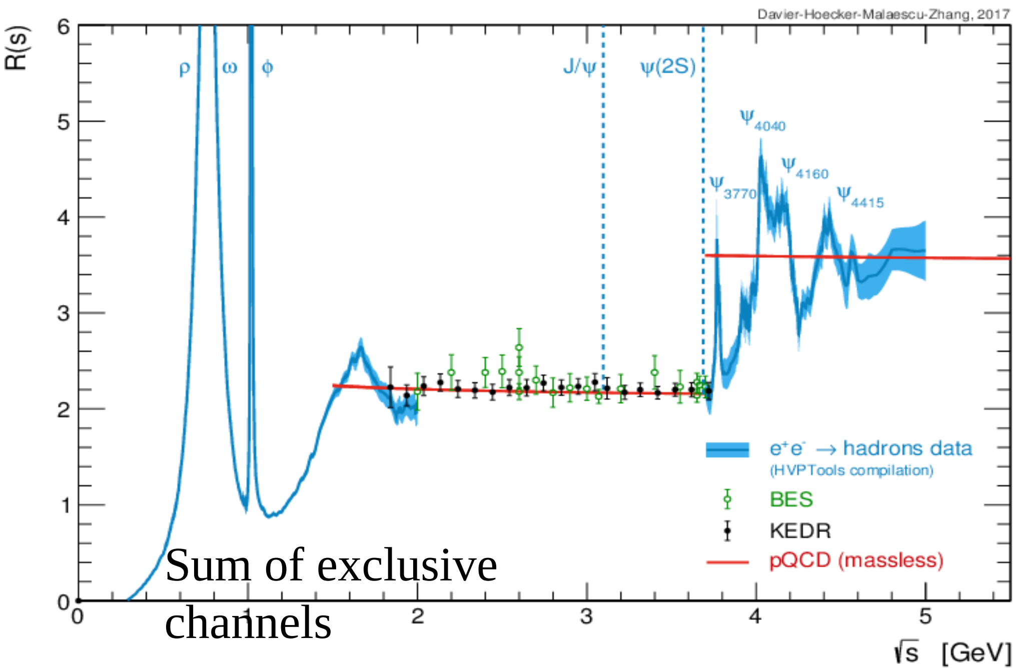
→ with KLOE-08-10-12 (KLOE-KT) used as input: 506.55 ± 2.38 [10^{-10}]
(after uncertainty enhancement by 18% caused by the tension between inputs)

→ Compensation between uncertainty reduction for KLOE-08-10-12 (KLOE-KT), inducing a change of weights in DHMZ combination,
and tension enhancement

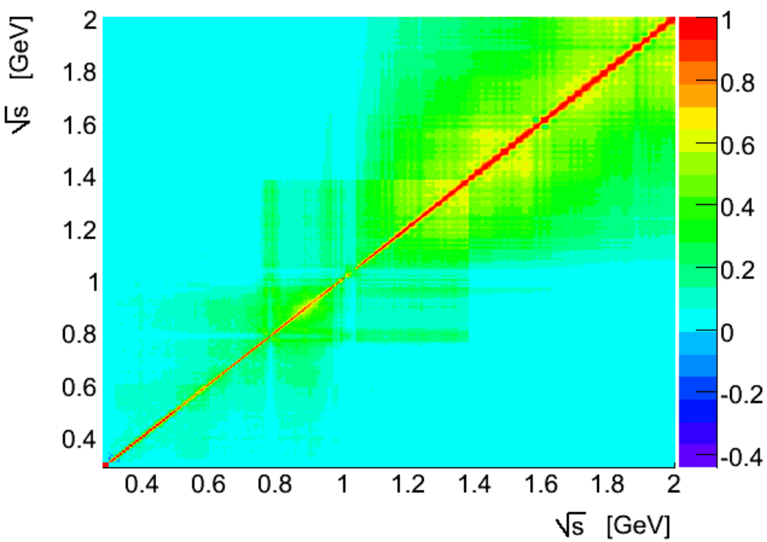




$R_{e+e-} \rightarrow$ Hadrons



Sum of exclusive channels



→ Full propagation of uncertainties and correlations

→ Performed non-trivial check: a_μ from sum of individual channels and from Ree integral < 1.8 GeV

Conclusion

→ Long standing discrepancy between data and SM on a_μ : *3.6σ in this update*

→ The evaluation of the HVP contribution to a_μ^{SM} is a continuous effort, following the release of new experimental data: *692.9 ± 3.2 [10^{-10}]*

→ Precision on $a_\mu^{Had,LO}$ improved by more than a factor 2 in the last 14 years

→ Need split of KLOE systematic uncertainties (as in the original publications)

→ Looking forward to the improved experimental result

Backup Slides

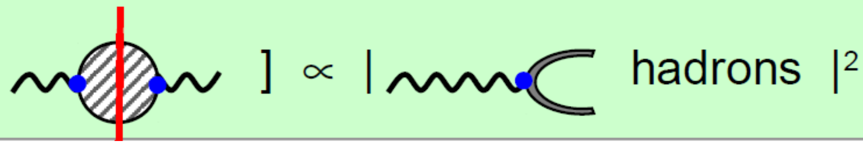
Hadronic Vacuum Polarization and Muon $(g-2)_\mu$

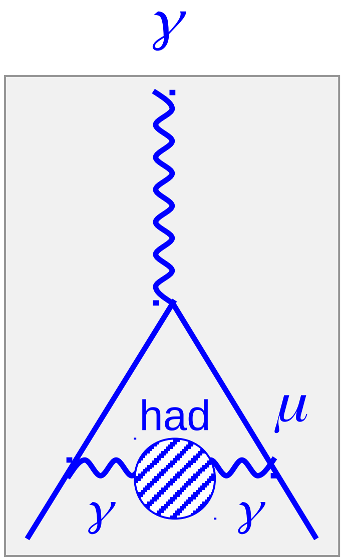
Dominant uncertainty for the theoretical prediction: from lowest-order HVP piece

Cannot be calculated from QCD (low mass scale), but one can use experimental data on $e^+e^- \rightarrow$hadrons cross section

Born: $\sigma^{(0)}(s) = \sigma(s)\left(\alpha/\alpha(s)\right)^2$

$$12\pi\ \mathrm{Im}\Pi_\gamma(s) = \frac{\sigma^0\ [e^+e^- \rightarrow \text{hadrons}\ (\gamma_{FSR})]}{\sigma_{pt}} \equiv R(s)$$

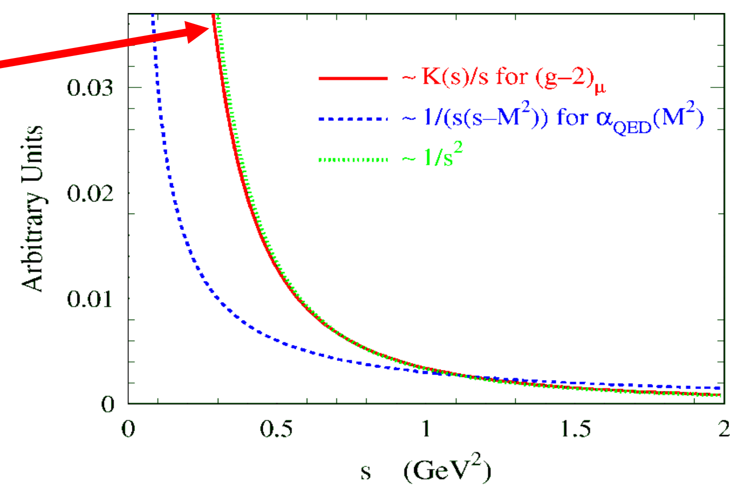




$$a_\mu^{\text{had}} = \frac{\alpha^2}{3\pi^2}\int_{4m_\pi^2}^{\infty} ds\ \frac{K(s)}{s} R(s)$$

Dispersion relation

Bouchiat and Michel, 1961



→ Precise $\sigma(e^+e^- \rightarrow \text{hadrons})$ measurements at low energy are very important

a_μ contributions and sum (1706.09436, EPJC)

Channel	$a_\mu^{\mathrm{had,LO}}$ $[10^{-10}]$	
$\pi^0\gamma$	$4.29 \pm 0.06 \pm 0.04 \pm 0.07$	
$\eta\gamma$	$0.65 \pm 0.02 \pm 0.01 \pm 0.01$	
$\pi^+\pi^-$	$507.14 \pm 1.13 \pm 2.20 \pm 0.75$	Updated
$\pi^+\pi^-\pi^0$	$46.20 \pm 0.40 \pm 1.10 \pm 0.86$	
$2\pi^+2\pi^-$	$13.68 \pm 0.03 \pm 0.27 \pm 0.14$	
$\pi^+\pi^-2\pi^0$	$18.03 \pm 0.06 \pm 0.48 \pm 0.26$	
$2\pi^+2\pi^-\pi^0$ (η excl.)	$0.69 \pm 0.04 \pm 0.06 \pm 0.03$	
$\pi^+\pi^-3\pi^0$ (η excl., isospin)	$0.35 \pm 0.02 \pm 0.03 \pm 0.01$	
$3\pi^+3\pi^-$	$0.11 \pm 0.00 \pm 0.01 \pm 0.00$	
$2\pi^+2\pi^-2\pi^0$ (η excl.)	$0.72 \pm 0.06 \pm 0.07 \pm 0.14$	
$\pi^+\pi^-4\pi^0$ (η excl., isospin)	$0.11 \pm 0.01 \pm 0.11 \pm 0.00$	
$\eta\pi^+\pi^-$	$1.18 \pm 0.03 \pm 0.06 \pm 0.02$	
$\eta\omega$	$0.32 \pm 0.02 \pm 0.02 \pm 0.01$	
$\eta\pi^+\pi^-\pi^0$ (non-ω,ϕ)	$0.39 \pm 0.03 \pm 0.11 \pm 0.03$	
$\eta 2\pi^+2\pi^-$	$0.03 \pm 0.01 \pm 0.00 \pm 0.00$	
$\eta\pi^+\pi^-2\pi^0$	$0.03 \pm 0.01 \pm 0.01 \pm 0.00$	
$\omega\pi^0$ ($\omega \to \pi^0\gamma$)	$0.94 \pm 0.01 \pm 0.02 \pm 0.02$	
$\omega(\pi\pi)^0$ ($\omega \to \pi^0\gamma$)	$0.08 \pm 0.00 \pm 0.01 \pm 0.00$	
ω (non-$3\pi,\pi\gamma,\eta\gamma$)	$0.36 \pm 0.00 \pm 0.01 \pm 0.00$	
K^+K^-	$22.81 \pm 0.24 \pm 0.28 \pm 0.17$	Updated
K_SK_L	$12.82 \pm 0.06 \pm 0.18 \pm 0.15$	
ϕ (non-$K\overline{K},3\pi,\pi\gamma,\eta\gamma$)	$0.05 \pm 0.00 \pm 0.00 \pm 0.00$	
$K\overline{K}\pi$	$2.45 \pm 0.06 \pm 0.12 \pm 0.07$	
$K\overline{K}2\pi$	$0.85 \pm 0.02 \pm 0.05 \pm 0.01$	
$K\overline{K}3\pi$ (estimate)	$-0.03 \pm 0.01 \pm 0.02 \pm 0.00$	
$\eta\phi$	$0.36 \pm 0.02 \pm 0.02 \pm 0.01$	
$\eta K\overline{K}$ (non-ϕ)	$0.01 \pm 0.01 \pm 0.01 \pm 0.00$	
$\omega K\overline{K}$ ($\omega \to \pi^0\gamma$)	$0.01 \pm 0.00 \pm 0.00 \pm 0.00$	
$\omega\eta\pi^0$	$0.06 \pm 0.04 \pm 0.00 \pm 0.00$	

J/ψ (BW integral)	6.28 ± 0.07
$\psi(2S)$ (BW integral)	1.57 ± 0.03
R_{data} [3.7–5.0 GeV]	$7.29 \pm 0.05 \pm 0.30 \pm 0.00$
R_{QCD} $[1.8\text{–}3.7\ \mathrm{GeV}]_{uds}$	$33.45 \pm 0.28 \pm 0.59_{\mathrm{dual}}$
R_{QCD} $[5.0\text{–}9.3\ \mathrm{GeV}]_{udsc}$	6.86 ± 0.04
R_{QCD} $[9.3\text{–}12.0\ \mathrm{GeV}]_{udscb}$	1.21 ± 0.01
R_{QCD} $[12.0\text{–}40.0\ \mathrm{GeV}]_{udscb}$	1.64 ± 0.00
R_{QCD} $[> 40.0\ \mathrm{GeV}]_{udscb}$	0.16 ± 0.00
R_{QCD} $[> 40.0\ \mathrm{GeV}]_{t}$	0.00 ± 0.00
Sum	$693.1 \pm 1.2 \pm 2.6 \pm 1.7 \pm 0.1_\psi \pm 0.7_{\mathrm{QCD}}$

→ Included 39 channels (22 in 2010 update)

→ Precision improved by 21%

→ Only 0.10 ± 0.03% in missing (estimated) channels

Situation in arXiv:1010.4180 (EPJC)

Channel	$a_\mu^{\mathrm{had,LO}}$ $[10^{-10}]$	$\Delta\alpha_{\mathrm{had}}(M_Z^2)$ $[10^{-4}]$
$\pi^0\gamma$	$4.42 \pm 0.08 \pm 0.13 \pm 0.12$	$0.36 \pm 0.01 \pm 0.01 \pm 0.01$
$\eta\gamma$	$0.64 \pm 0.02 \pm 0.01 \pm 0.01$	$0.08 \pm 0.00 \pm 0.00 \pm 0.00$
$\pi^+\pi^-$	$507.80 \pm 1.22 \pm 2.50 \pm 0.56$	$34.43 \pm 0.07 \pm 0.17 \pm 0.04$
$\pi^+\pi^-\pi^0$	$46.00 \pm 0.42 \pm 1.03 \pm 0.98$	$4.58 \pm 0.04 \pm 0.11 \pm 0.09$
$2\pi^+2\pi^-$	$13.35 \pm 0.10 \pm 0.43 \pm 0.29$	$3.49 \pm 0.03 \pm 0.12 \pm 0.08$
$\pi^+\pi^-2\pi^0$	$18.01 \pm 0.14 \pm 1.17 \pm 0.40$	$4.43 \pm 0.03 \pm 0.29 \pm 0.10$
$2\pi^+2\pi^-\pi^0$ (η excl.)	$0.72 \pm 0.04 \pm 0.07 \pm 0.03$	$0.22 \pm 0.01 \pm 0.02 \pm 0.01$
$\pi^+\pi^-3\pi^0$ (η excl., from isospin)	$0.36 \pm 0.02 \pm 0.03 \pm 0.01$	$0.11 \pm 0.01 \pm 0.01 \pm 0.00$
$3\pi^+3\pi^-$	$0.12 \pm 0.01 \pm 0.01 \pm 0.00$	$0.04 \pm 0.00 \pm 0.00 \pm 0.00$
$2\pi^+2\pi^-2\pi^0$ (η excl.)	$0.70 \pm 0.05 \pm 0.04 \pm 0.09$	$0.25 \pm 0.02 \pm 0.02 \pm 0.03$
$\pi^+\pi^-4\pi^0$ (η excl., from isospin)	$0.11 \pm 0.01 \pm 0.11 \pm 0.00$	$0.04 \pm 0.00 \pm 0.04 \pm 0.00$
$\eta\pi^+\pi^-$	$1.15 \pm 0.06 \pm 0.08 \pm 0.03$	$0.33 \pm 0.02 \pm 0.02 \pm 0.01$
$\eta\omega$	$0.47 \pm 0.04 \pm 0.00 \pm 0.05$	$0.15 \pm 0.01 \pm 0.00 \pm 0.02$
$\eta 2\pi^+2\pi^-$	$0.02 \pm 0.01 \pm 0.00 \pm 0.00$	$0.01 \pm 0.00 \pm 0.00 \pm 0.00$
$\eta\pi^+\pi^-2\pi^0$ (estimated)	$0.02 \pm 0.01 \pm 0.01 \pm 0.00$	$0.01 \pm 0.00 \pm 0.00 \pm 0.00$
$\omega\pi^0$ ($\omega \to \pi^0\gamma$)	$0.89 \pm 0.02 \pm 0.06 \pm 0.02$	$0.18 \pm 0.00 \pm 0.02 \pm 0.00$
$\omega\pi^+\pi^-, \omega 2\pi^0$ ($\omega \to \pi^0\gamma$)	$0.08 \pm 0.00 \pm 0.01 \pm 0.00$	$0.03 \pm 0.00 \pm 0.00 \pm 0.00$
ω (non-$3\pi, \pi\gamma, \eta\gamma$)	$0.36 \pm 0.00 \pm 0.01 \pm 0.00$	$0.03 \pm 0.00 \pm 0.00 \pm 0.00$
K^+K^-	$21.63 \pm 0.27 \pm 0.58 \pm 0.36$	$3.13 \pm 0.04 \pm 0.08 \pm 0.05$
$K_S^0K_L^0$	$12.96 \pm 0.18 \pm 0.25 \pm 0.24$	$1.75 \pm 0.02 \pm 0.03 \pm 0.03$
ϕ (non-$K\overline{K}, 3\pi, \pi\gamma, \eta\gamma$)	$0.05 \pm 0.00 \pm 0.00 \pm 0.00$	$0.01 \pm 0.00 \pm 0.00 \pm 0.00$
$K\overline{K}\pi$ (partly from isospin)	$2.39 \pm 0.07 \pm 0.12 \pm 0.08$	$0.76 \pm 0.02 \pm 0.04 \pm 0.02$
$K\overline{K}2\pi$ (partly from isospin)	$1.35 \pm 0.09 \pm 0.38 \pm 0.03$	$0.48 \pm 0.03 \pm 0.14 \pm 0.01$
$K\overline{K}3\pi$ (partly from isospin)	$-0.03 \pm 0.01 \pm 0.02 \pm 0.00$	$-0.01 \pm 0.00 \pm 0.01 \pm 0.00$
$\phi\eta$	$0.36 \pm 0.02 \pm 0.02 \pm 0.01$	$0.13 \pm 0.01 \pm 0.01 \pm 0.00$
$\omega K\overline{K}$ ($\omega \to \pi^0\gamma$)	$0.00 \pm 0.00 \pm 0.00 \pm 0.00$	$0.00 \pm 0.00 \pm 0.00 \pm 0.00$
J/ψ (Breit-Wigner integral)	6.22 ± 0.16	7.03 ± 0.18
$\psi(2S)$ (Breit-Wigner integral)	1.57 ± 0.03	2.50 ± 0.04
R_{data} $[3.7 - 5.0$ GeV$]$	$7.29 \pm 0.05 \pm 0.30 \pm 0.00$	$15.79 \pm 0.12 \pm 0.66 \pm 0.00$
R_{QCD} $[1.8 - 3.7\ \mathrm{GeV}]_{uds}$	33.45 ± 0.28	24.27 ± 0.19
R_{QCD} $[5.0 - 9.3\ \mathrm{GeV}]_{udsc}$	6.86 ± 0.04	34.89 ± 0.18
R_{QCD} $[9.3 - 12.0\ \mathrm{GeV}]_{udscb}$	1.21 ± 0.01	15.56 ± 0.04
R_{QCD} $[12.0 - 40.0\ \mathrm{GeV}]_{udscb}$	1.64 ± 0.01	77.94 ± 0.12
R_{QCD} $[> 40.0\ \mathrm{GeV}]_{udscb}$	0.16 ± 0.00	42.70 ± 0.06
R_{QCD} $[> 40.0\ \mathrm{GeV}]_{t}$	0.00 ± 0.00	-0.72 ± 0.01
Sum	$692.3 \pm 1.4 \pm 3.1 \pm 2.4 \pm 0.2_\psi \pm 0.3_{\mathrm{QCD}}$	$274.97 \pm 0.17 \pm 0.78 \pm 0.37 \pm 0.18_\psi \pm 0.52_{\mathrm{QCD}}$

Lepton Magnetic Anomaly: from Dirac to QED

$$\vec{\mu} = g\frac{e}{2m}\vec{s}, \qquad\qquad a = (g-2)/2$$

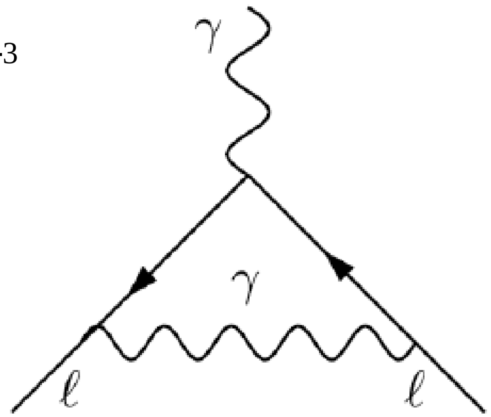
Dirac (1928) $g_e=2$ $a_e=0$

anomaly discovered:

Kusch-Foley (1948) $a_e = (1.19 \pm 0.05)\ 10^{-3}$

and explained by $O(\alpha)$ QED contribution:

Schwinger (1948) $a_e = \alpha/2\pi = 1.16\ 10^{-3}$



first triumph of QED

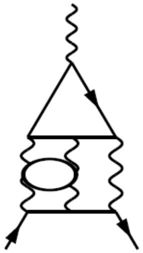
$\Rightarrow$ a_e sensitive to quantum fluctuations of fields

More Quantum Fluctuations

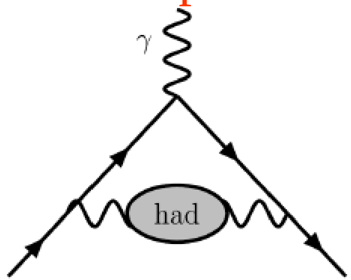
$$a = a^{\mathrm{QED}} + a^{\mathrm{had}} + a^{\mathrm{weak}} \quad + ?\ a^{\text{new physics}}\ ?$$

typical contributions:

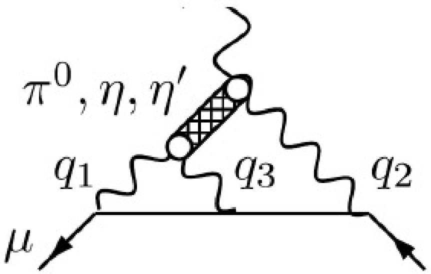
QED up to $O(\alpha^5)$ (Kinoshita et al.)



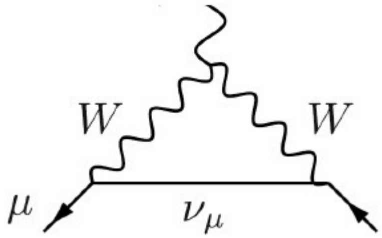
Hadrons vacuum polarization



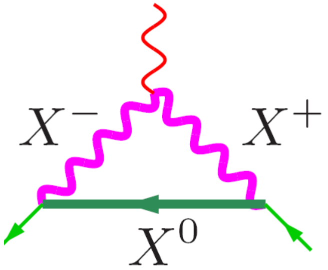
light-by-light (models)



Electroweak

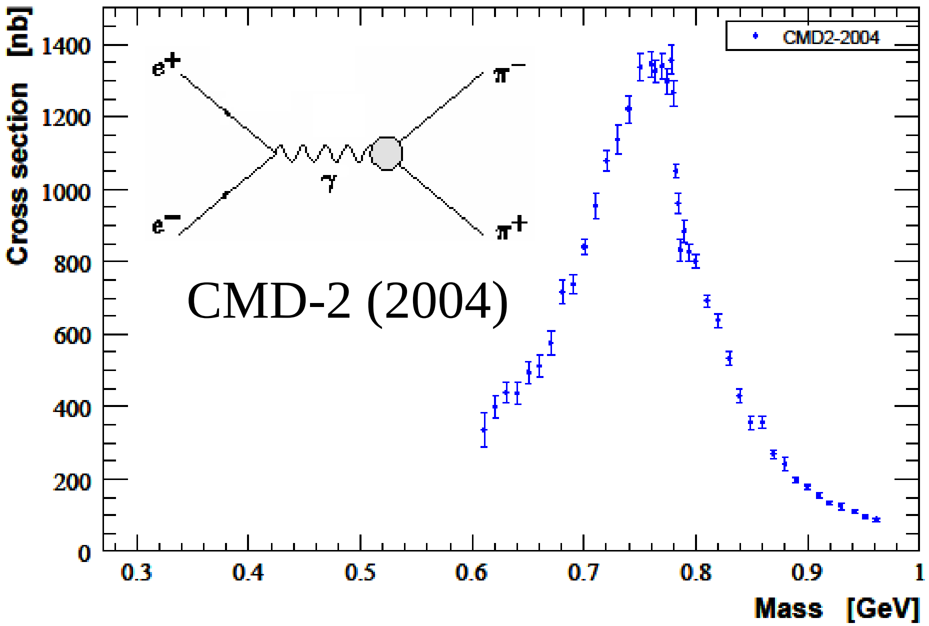


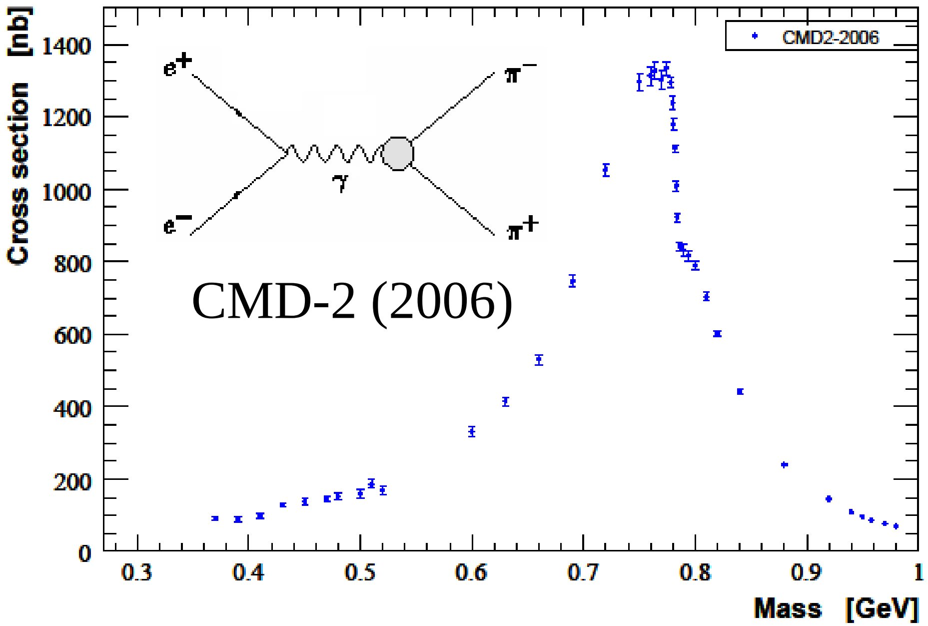
new physics at high mass scale

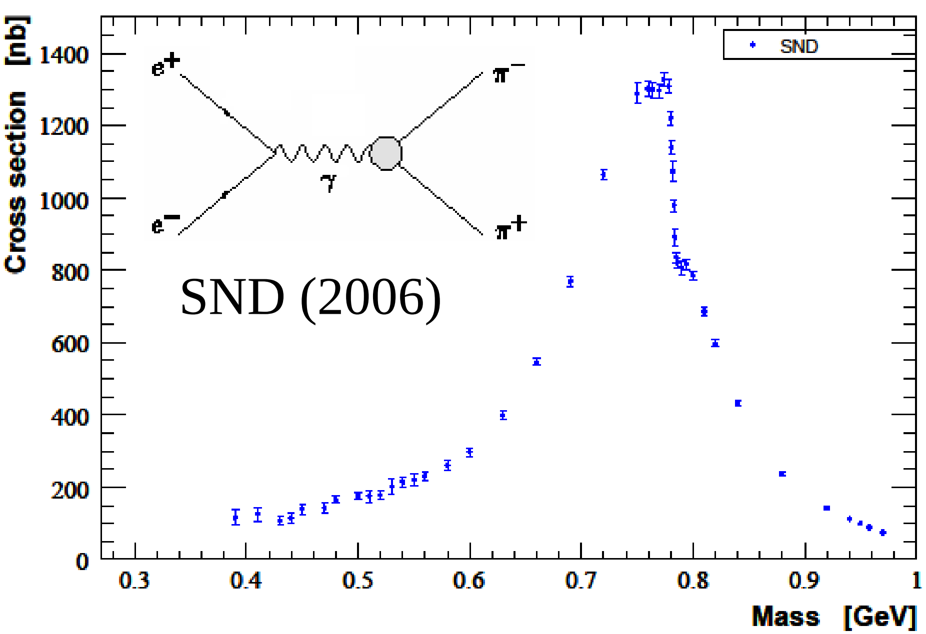


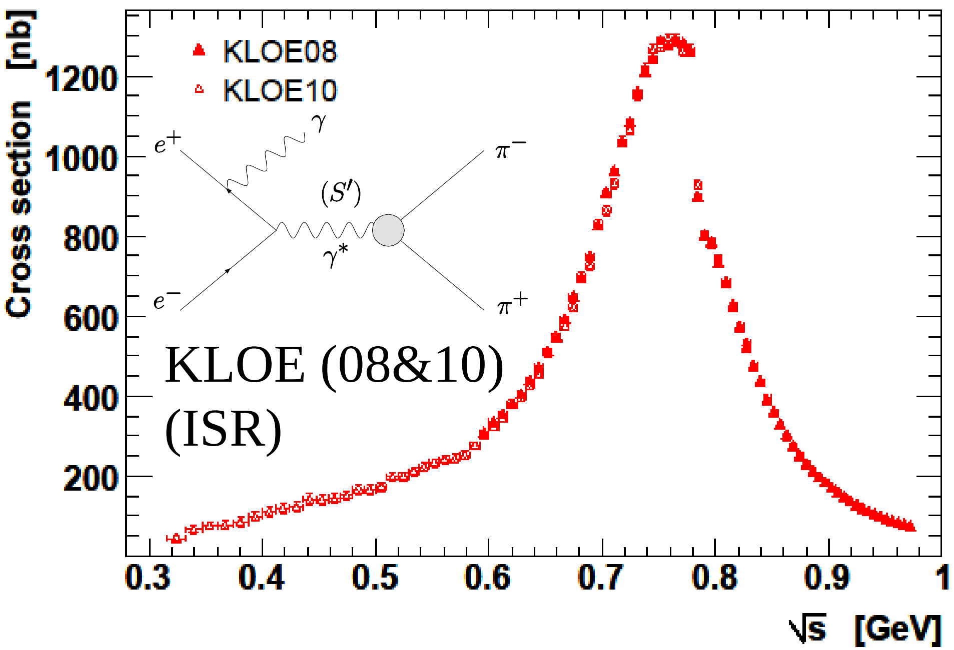
$$\delta a_l \propto \frac{m_{l^2}}{M^2}$$ $\Rightarrow$ a_μ much more sensitive to high scales

HVP: Data on $e^+e^- \to$ hadrons

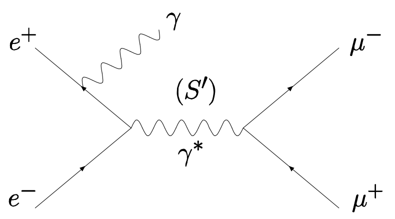


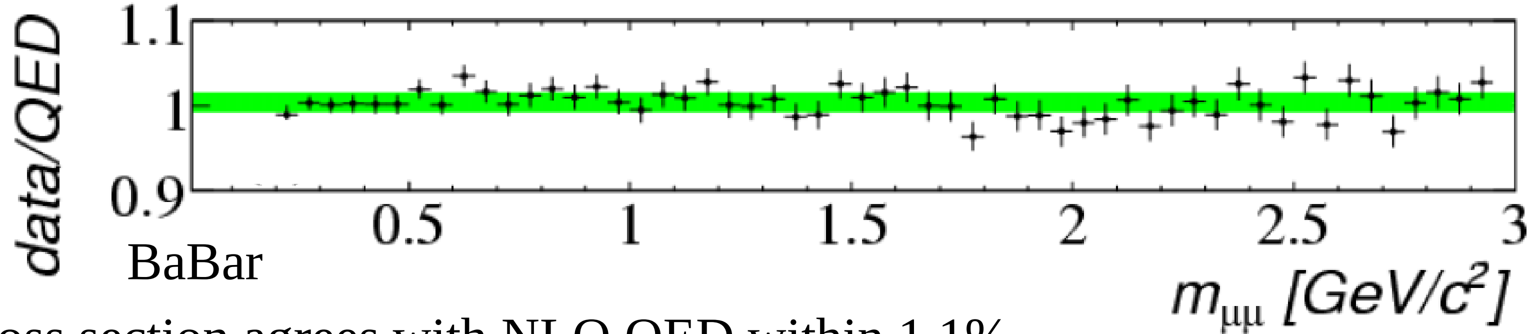




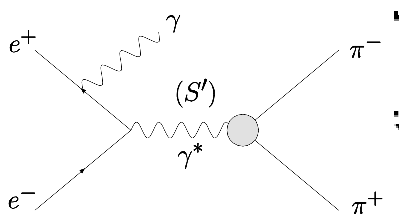


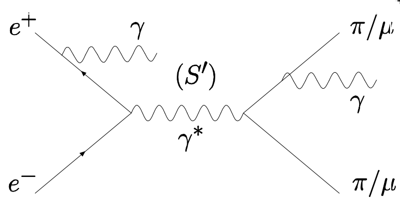
BaBar results (arXiv:0908.3589, PRL 103, 231801 (2009); arXiv:1205.2228)

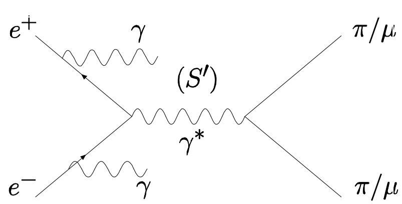


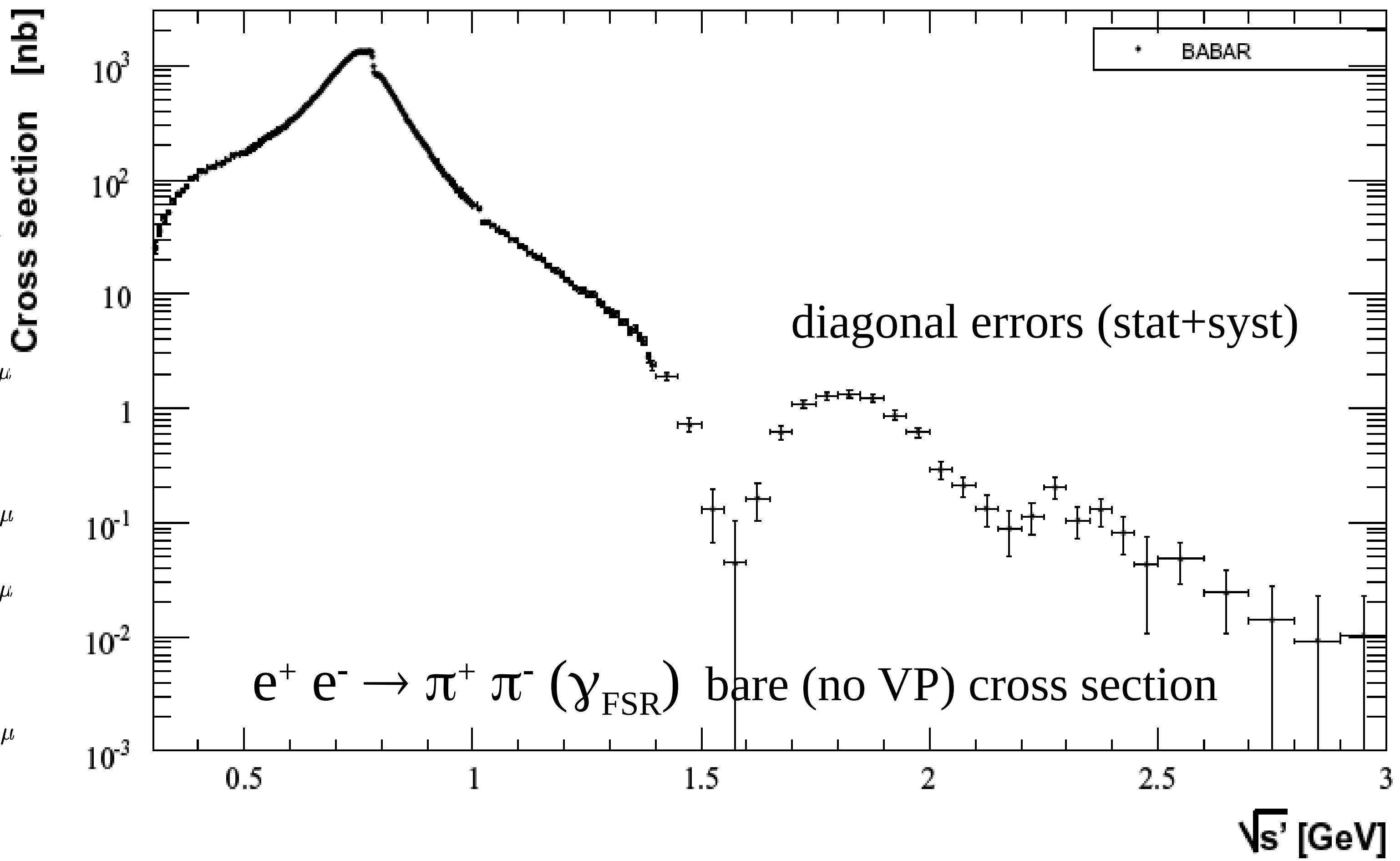


Absolute $\mu^+\mu^-$ cross section agrees with NLO QED within 1.1%

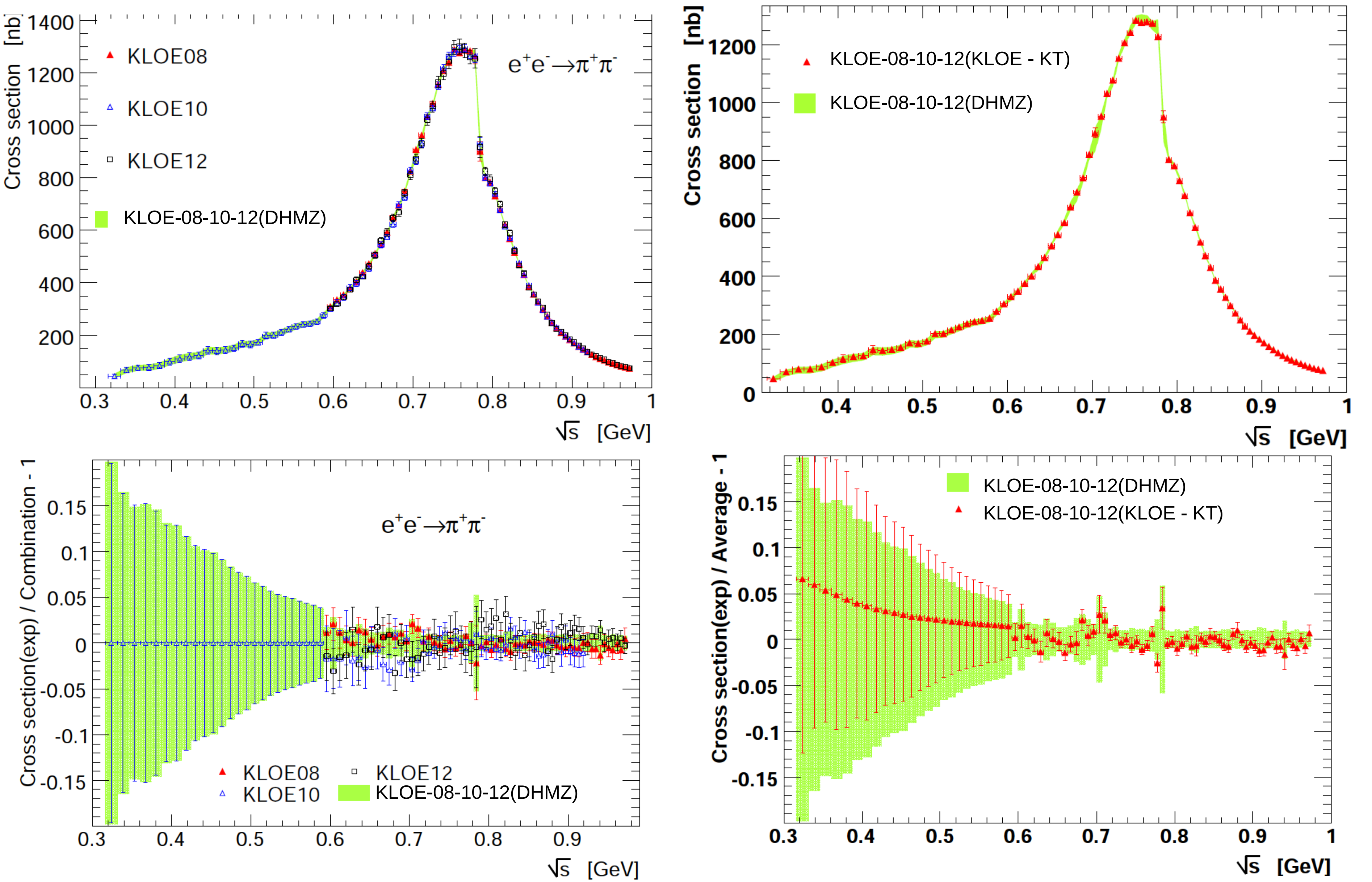








Combining the 3 KLOE measurements



Combining the 3 KLOE measurements - $a_\mu^{\pi\pi}$ contribution

KLOE08 a_μ[0.6 ; 0.9] : 368.3 ± 3.2 [10^{-10}]

KLOE10 a_μ[0.6 ; 0.9] : 365.6 ± 3.3

KLOE12 a_μ[0.6 ; 0.9] : 366.8 ± 2.5

→ Correlation matrix:

	08	10	12
08	1	0.70	0.35
10	0.70	1	0.19
12	0.35	0.19	1

→ Amount of independent information provided by each measurement

→ KLOE-08-10-12(DHMZ) - a_μ[0.6 ; 0.9] : 366.5 ± 2.8 (Without χ^2 rescaling: ± 2.2)

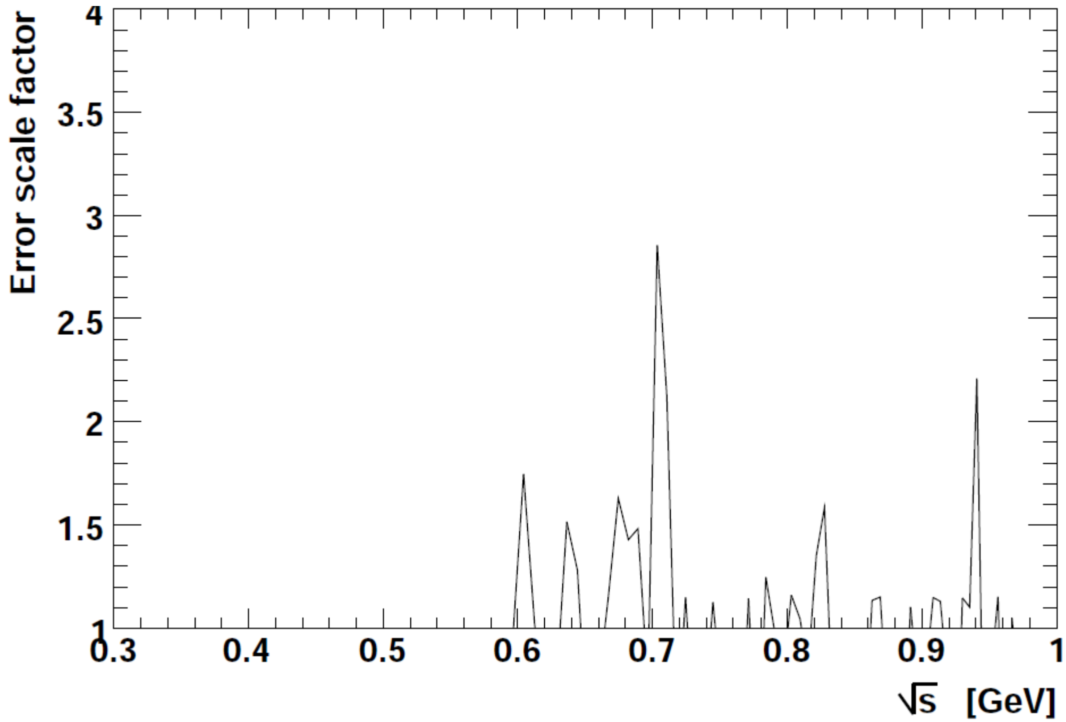
→ Conservative treatment of uncertainties and correlations (not perfectly known) in weight determination

→ KLOE-08-10-12(KLOE-KT) - a_μ[0.6 ; 0.9]GeV : 366.9 ± 2.2

→ Assuming perfect knowledge of the correlations to minimize average uncertainty

→ Impact of the scaling factor?

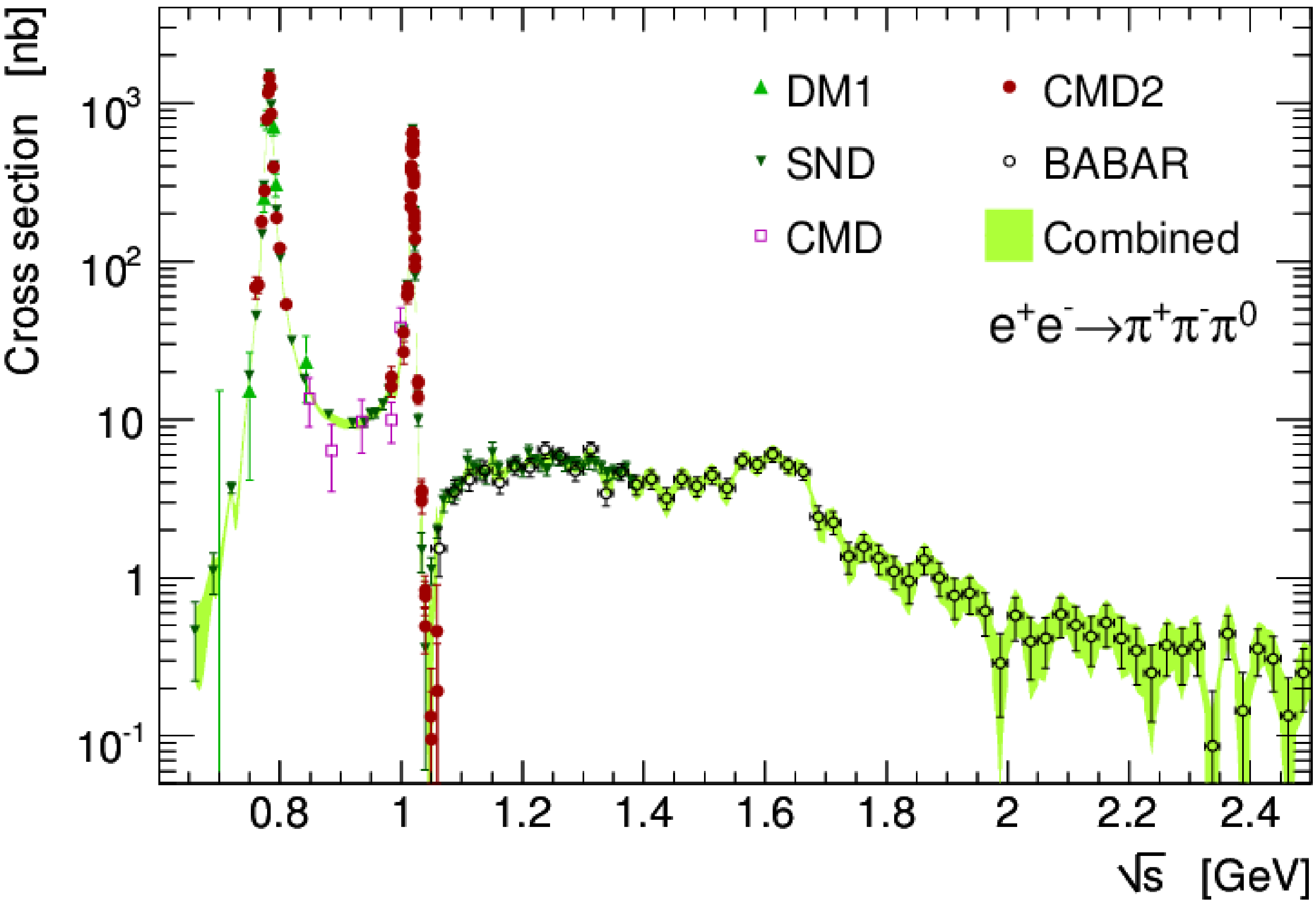
Direct comparison of the 3 KLOE measurements



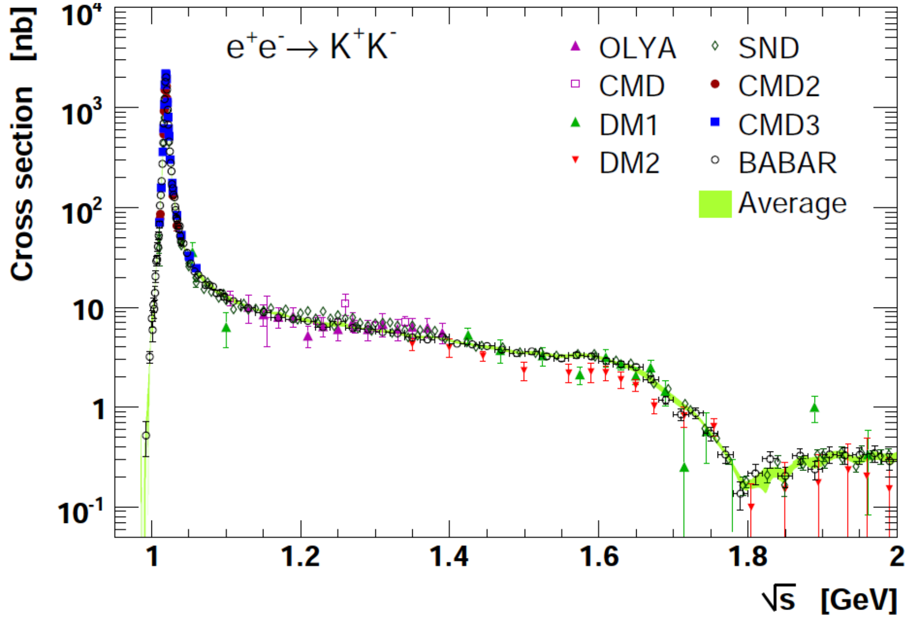
→ Local χ^2/ndof test of the local compatibility between KLOE 08 & 10 & 12, taking into account the correlations: some tensions observed

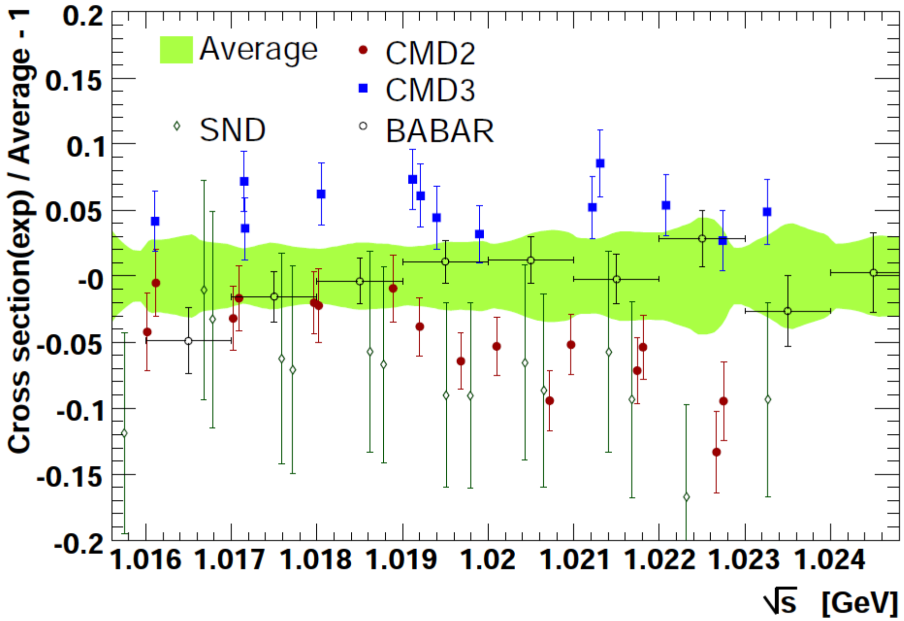
→ Does not probe general trends of the difference between the measurements (e.g. slopes in the ratio)

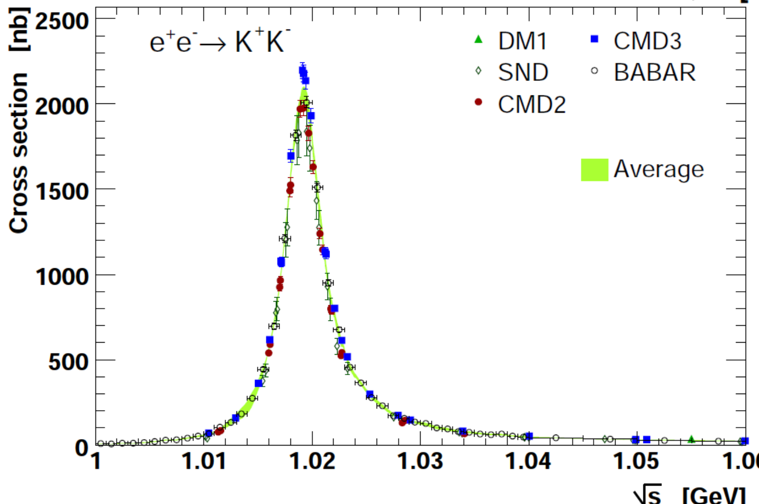
Combination for the $e^+e^- \rightarrow \pi^+\pi^-\pi^0$ channel

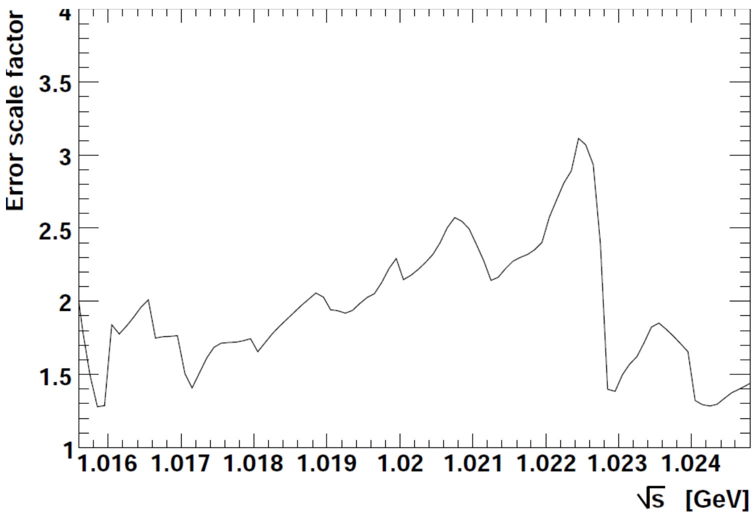


Combination for the $e^+e^- \to K^+K^-$ channel





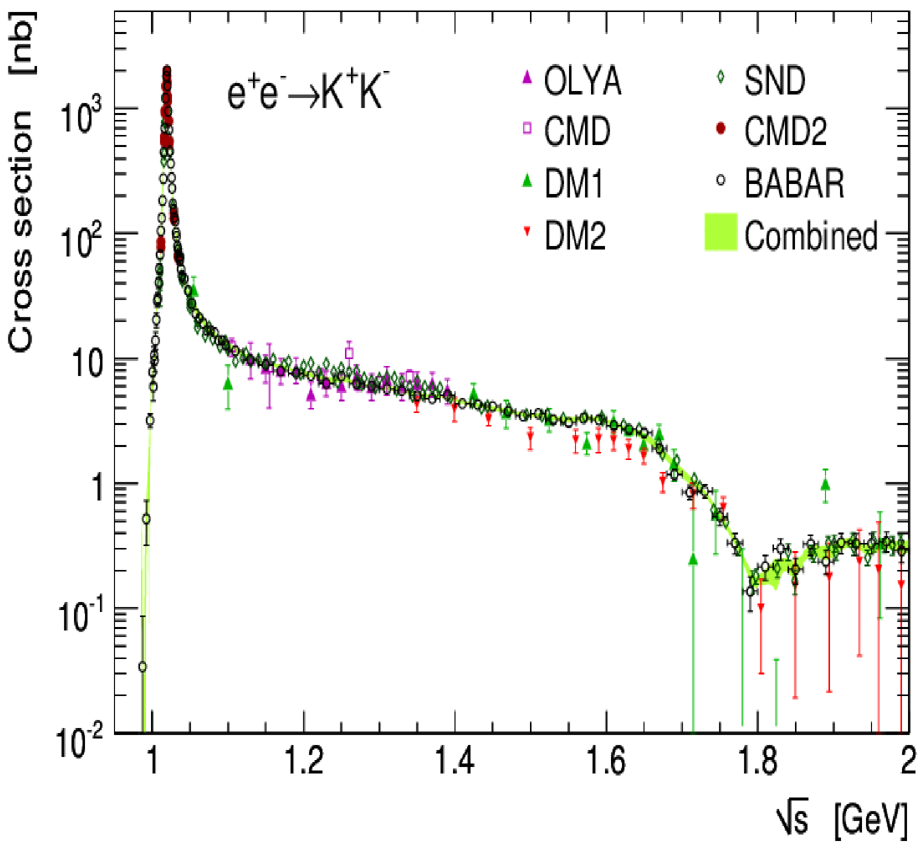


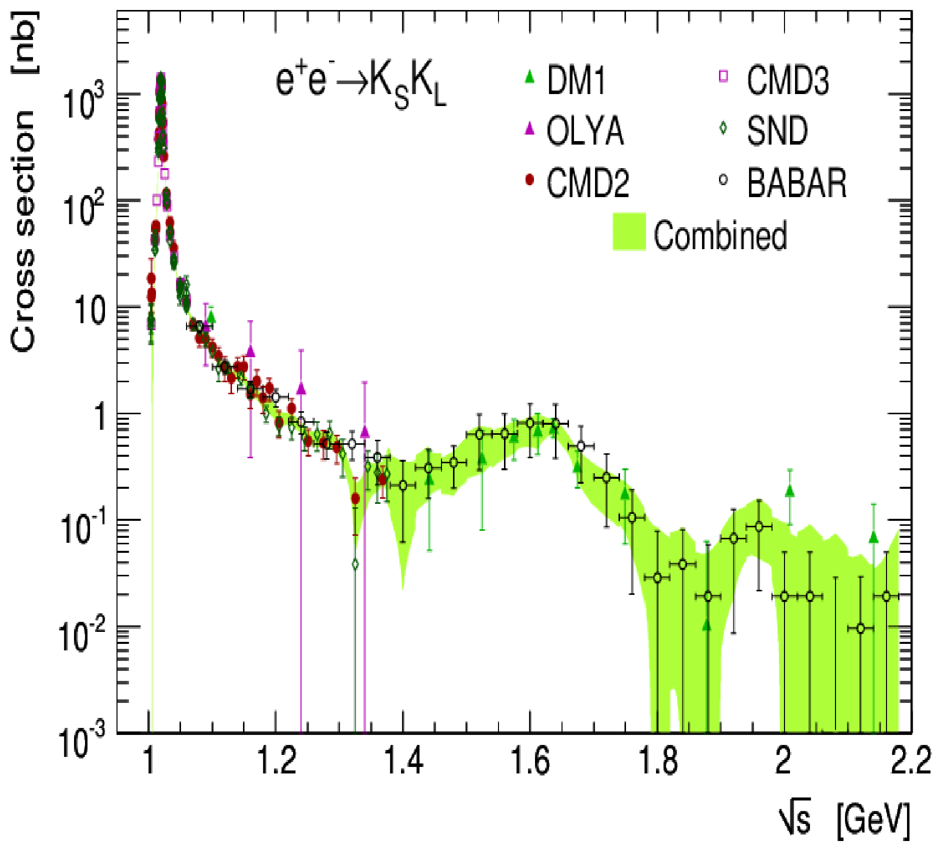


→ Tension between measurements

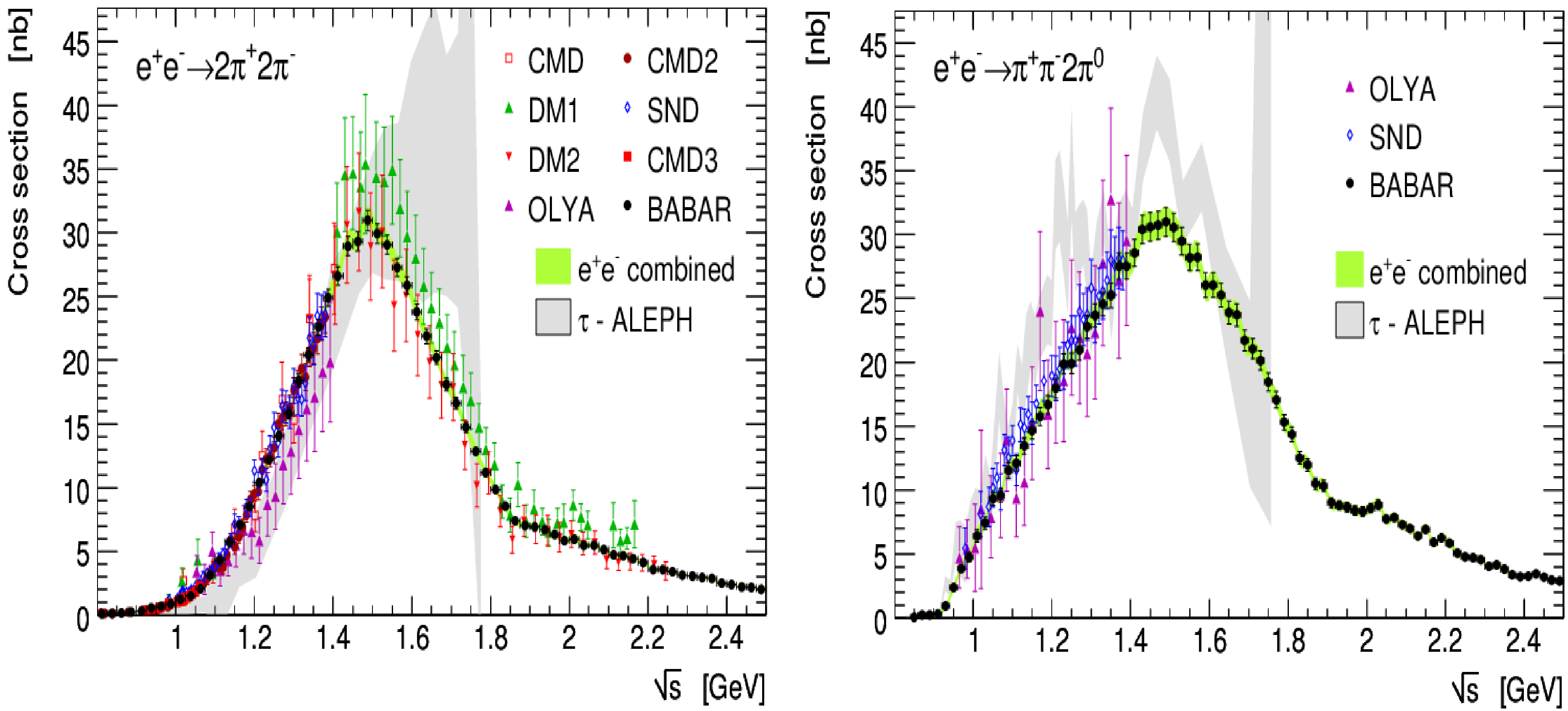
→ $a_\mu[\to 1.8\text{GeV}]$: 23.06 ± 0.22 (stat.) ± 0.40 (syst.) [10^{-10}] (enhancement x 2.2)

Combination for the $e^+e^- \to K^+K^-$, K_sK_l channels



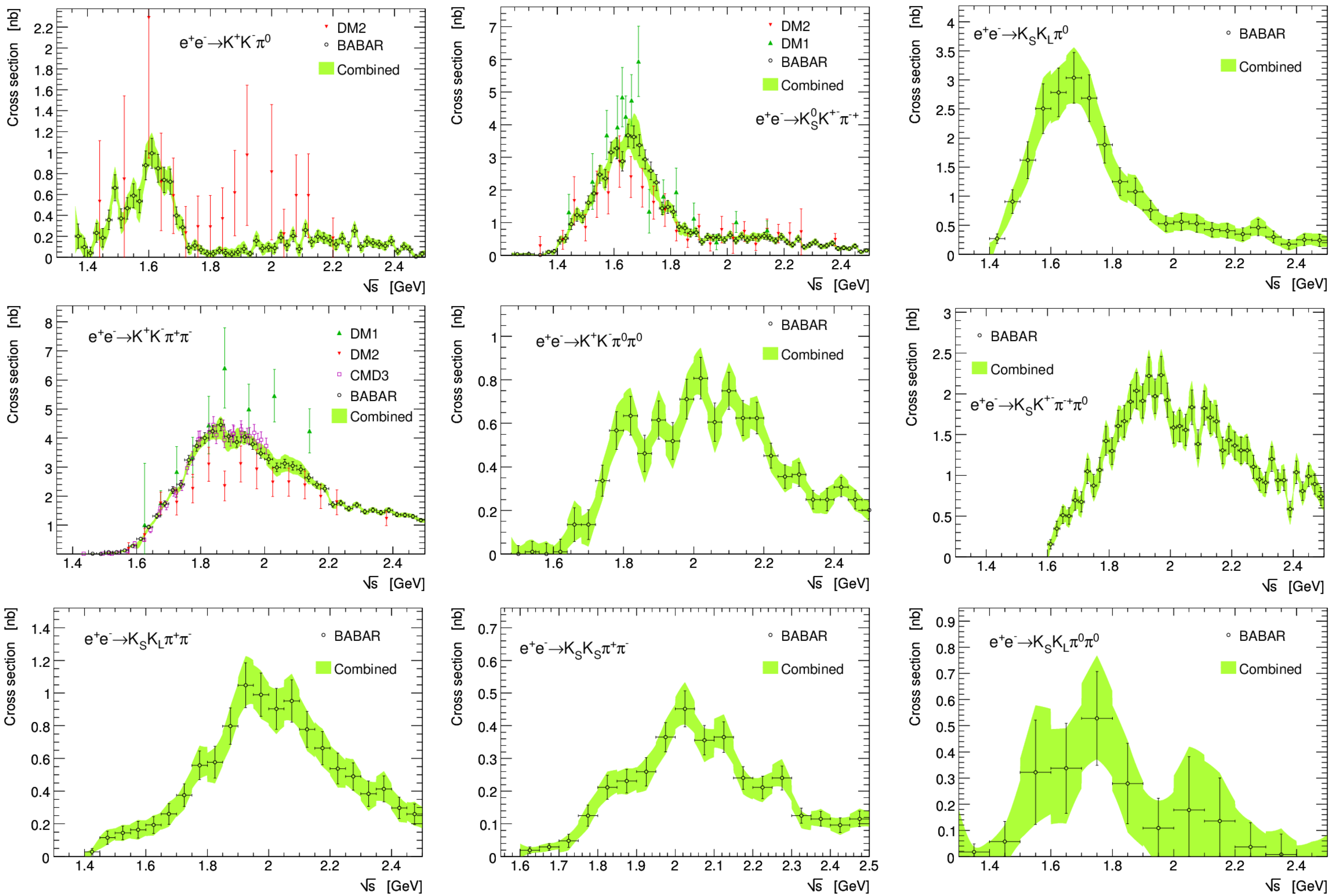


$e^+e^- \rightarrow \pi^+\pi^-\pi^+\pi^-$, $e^+e^- \rightarrow \pi^+\pi^-\pi^0\pi^0$

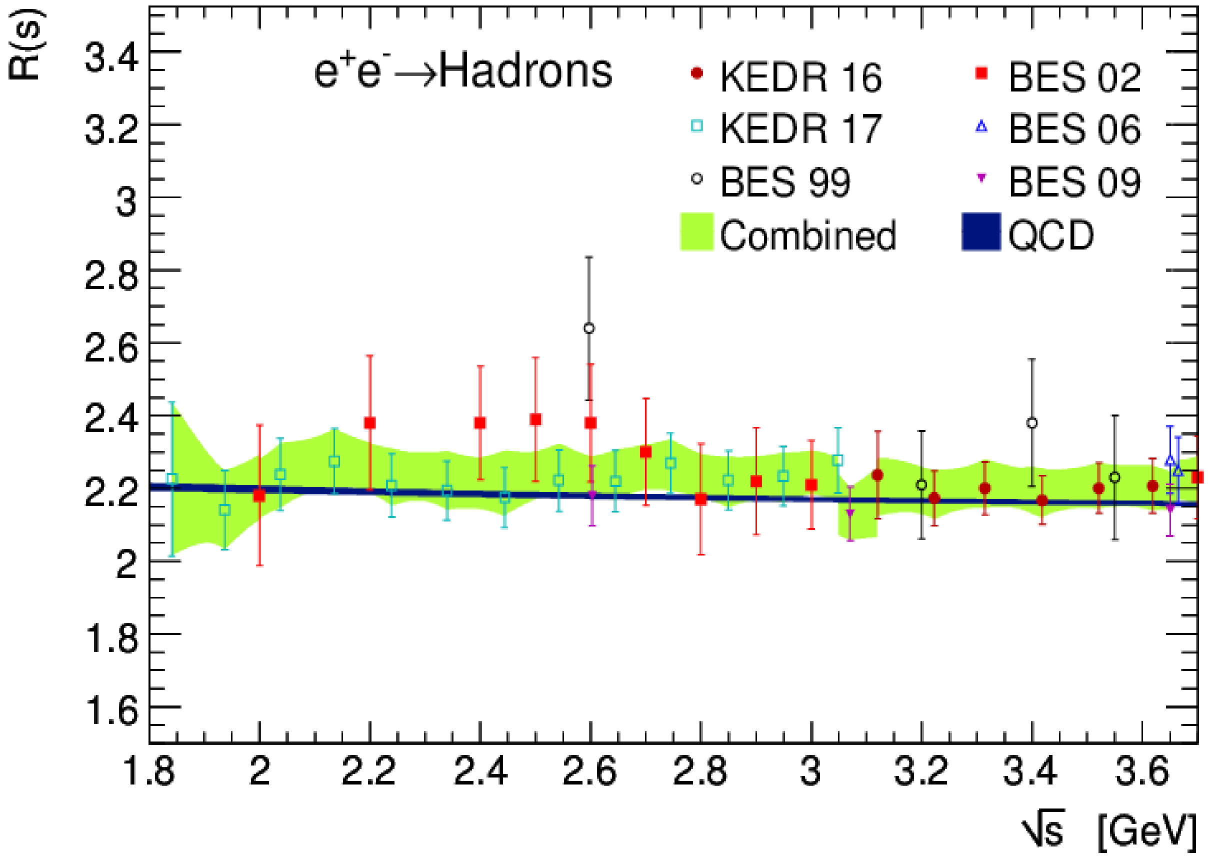


→ Essentially normalization differences w.r.t. τ data: cross-checks very desirable

Combination for the $e^+e^- \to KK\pi$ and $KK2\pi$ channels



Contributions from the 1.8 – 3.7 GeV region



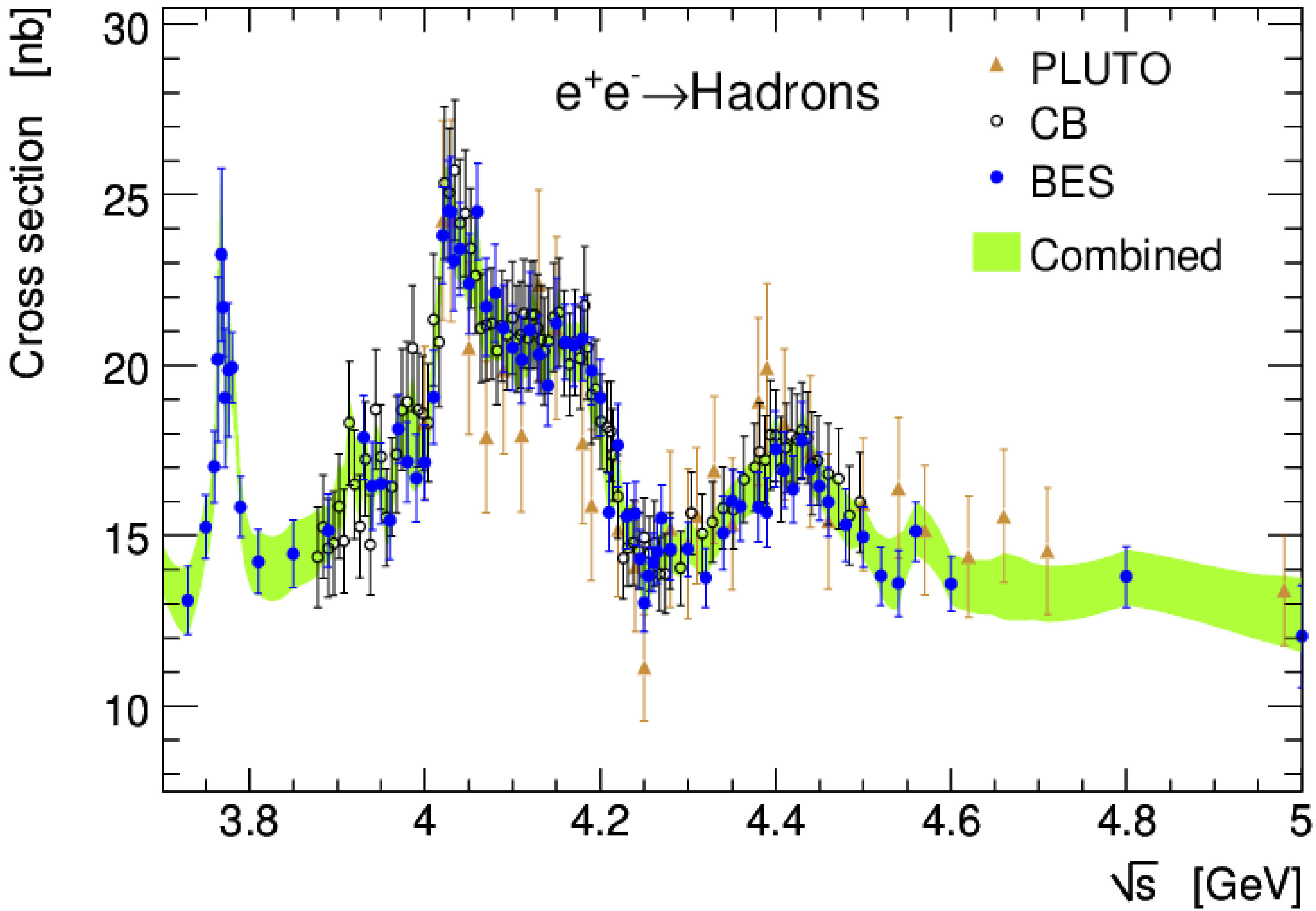
→ Contribution evaluated from pQCD (4 loops) + O(α_s^2) quark mass corrections

→ Uncertainties: α_s, truncation of perturbative series, CIPT/FOPT, m_q

→ 1.8-2.0 GeV: 7.71±0.37(data); 8.30±0.09(QCD); added syst. 0.59 [10^{-10}]

→ 2.0-3.7 GeV: 25.82±0.61(data); 25.15 ± 0.19(QCD); agreement within 1σ

Contributions from the charm resonance region



Status of a_μ - 2017 update

- Including latest results on e+e- → hadrons in the combination

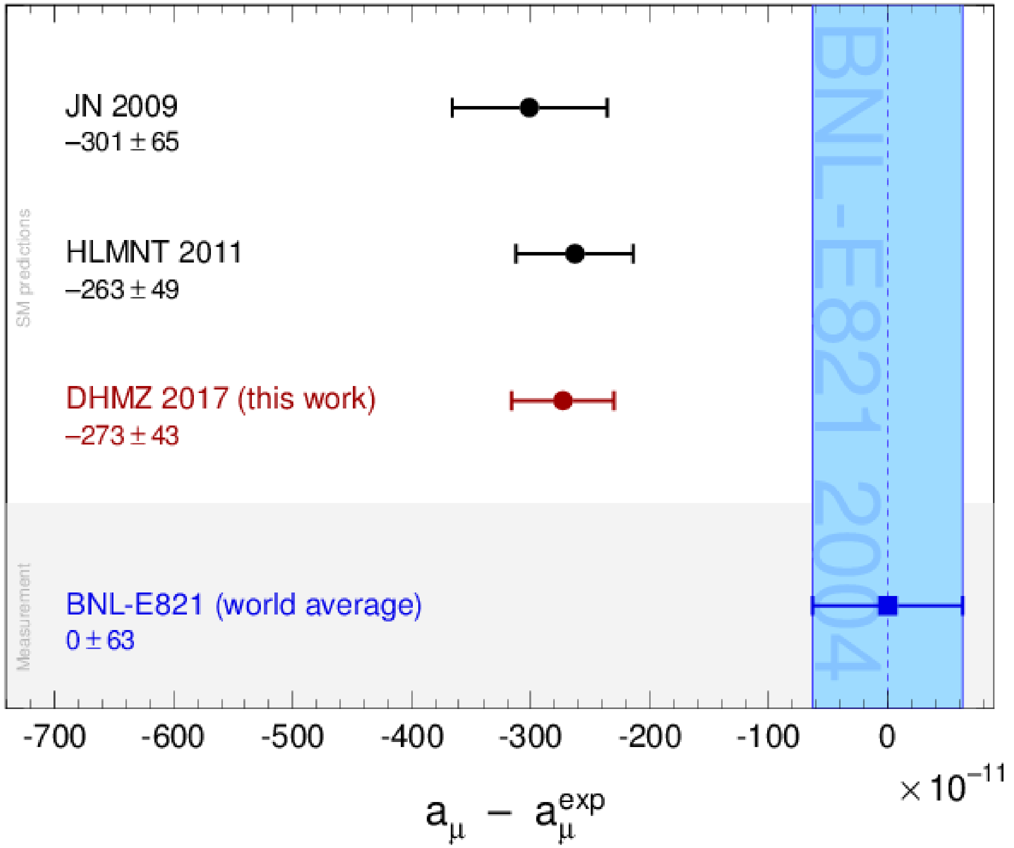
+ latest QED calculation (Kinoshita et al.) yields

$$a_\mu^{SM}[e+e-] = (11\ 659\ 182.3 \pm 3.4 \pm 2.6 \pm 0.2)\ 10^{-10}$$

HVP LBL EW (±4.3)

- E-821 updated result $(11\ 659\ 209.1 \pm 6.3)\ 10^{-10}$

- Deviation $(26.8 \pm 7.6)\ 10^{-10}$ $(3.5\ \sigma)$



Improving a_μ through fits for the $e^+e^- \to \pi^+\pi^-$ channel

→ Fit using form-factor model based on *analyticity* and *unitarity*

F(s)=R(s) * J(s)

$$R(s) = 1 + \alpha_V\, s + \frac{\kappa_1\, s}{m_\omega^2 - s - i m_\omega \Gamma_\omega^{\rm tot}}$$ (1611.09359, C. Hanhart et al.)

$$J(s) = \mathrm{e}^{1-\delta_1^1(t_0)/\pi} \left(1 - \frac{s}{t_0}\right)^{[1-\delta_1^1(t_0)/\pi]t_0/s} \left(1 - \frac{s}{t_0}\right)^{-1} \exp\left\{\frac{s}{\pi}\int_{4m_\pi^2}^{t_0} \mathrm{d}t\, \frac{\delta_1^1(t)}{t(t-s)}\right\}$$

(hep-ph/0402285, F.J. Yndurain et al.) Omnès integral

$$\cot\delta_1(s) = \frac{s^{1/2}}{2k^3}(M_\rho^2 - s)\left\{\frac{2M_\pi^3}{M_\rho^2\sqrt{s}} + B_0 + B_1 w(s)\right\}$$ (1102.2183, F.J. Yndurain et al.)

$$w(s) = \frac{\sqrt{s} - \sqrt{s_0 - s}}{\sqrt{s} + \sqrt{s_0 - s}}, \qquad s_0^{1/2} = 1.05\,\mathrm{GeV}\,,$$

→ Fit with 8 parameters on BABAR data, with full uncertainty propagation

	BABAR data	Fit	
a_μ[0.3-0.6] GeV:	111.00 ± 1.35	109.78 ± 0.78	[10^{-10}]
a_μ[0.6-0.9] GeV:	376.71 ± 2.72	376.68 ± 2.71	
a_μ[0.3-1] GeV:	503.56 ± 3.76	502.31 ± 3.41	

a_μ constraint through fits for the $e^+e^- \to \pi^+\pi^-$ channel