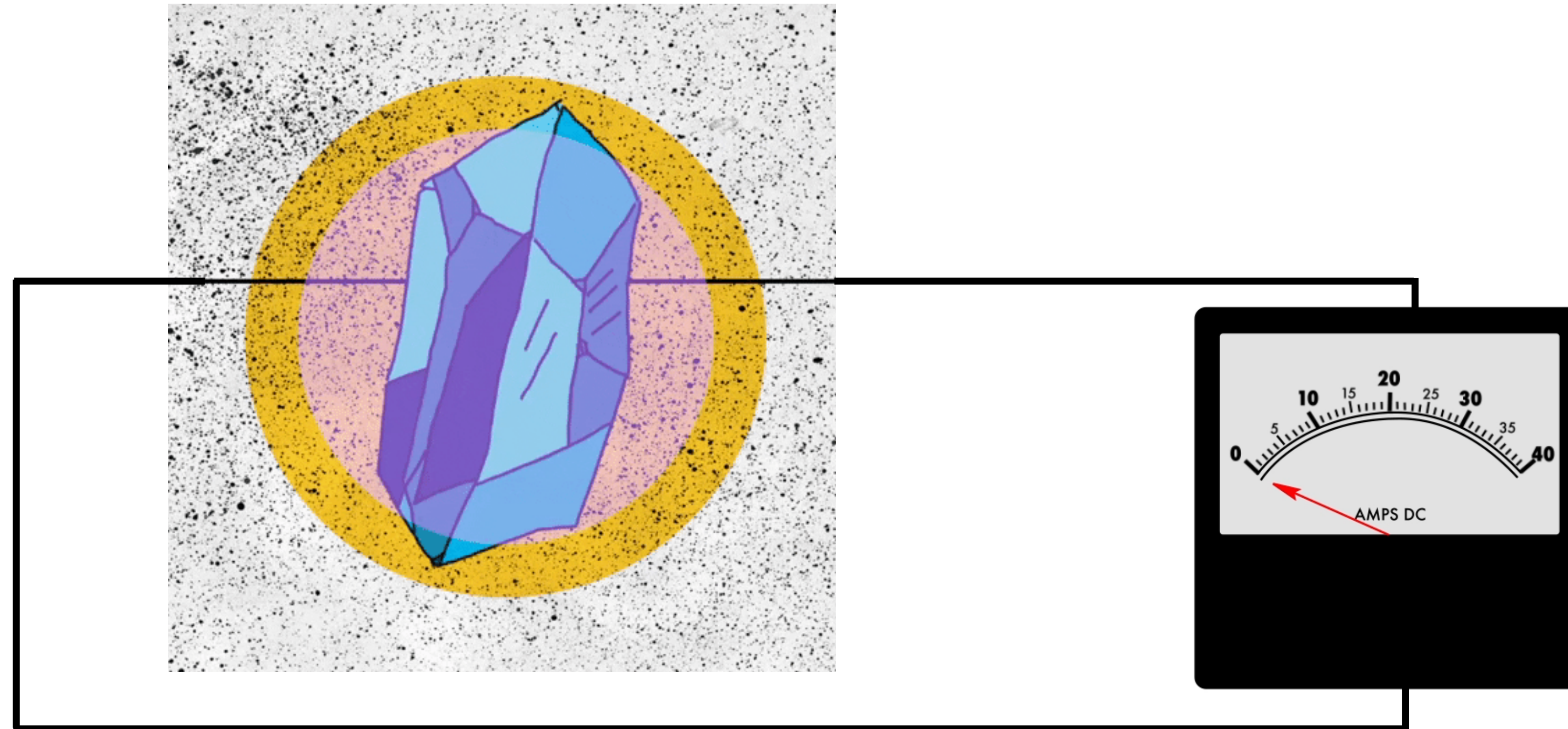


The Piezoaxionic Effect



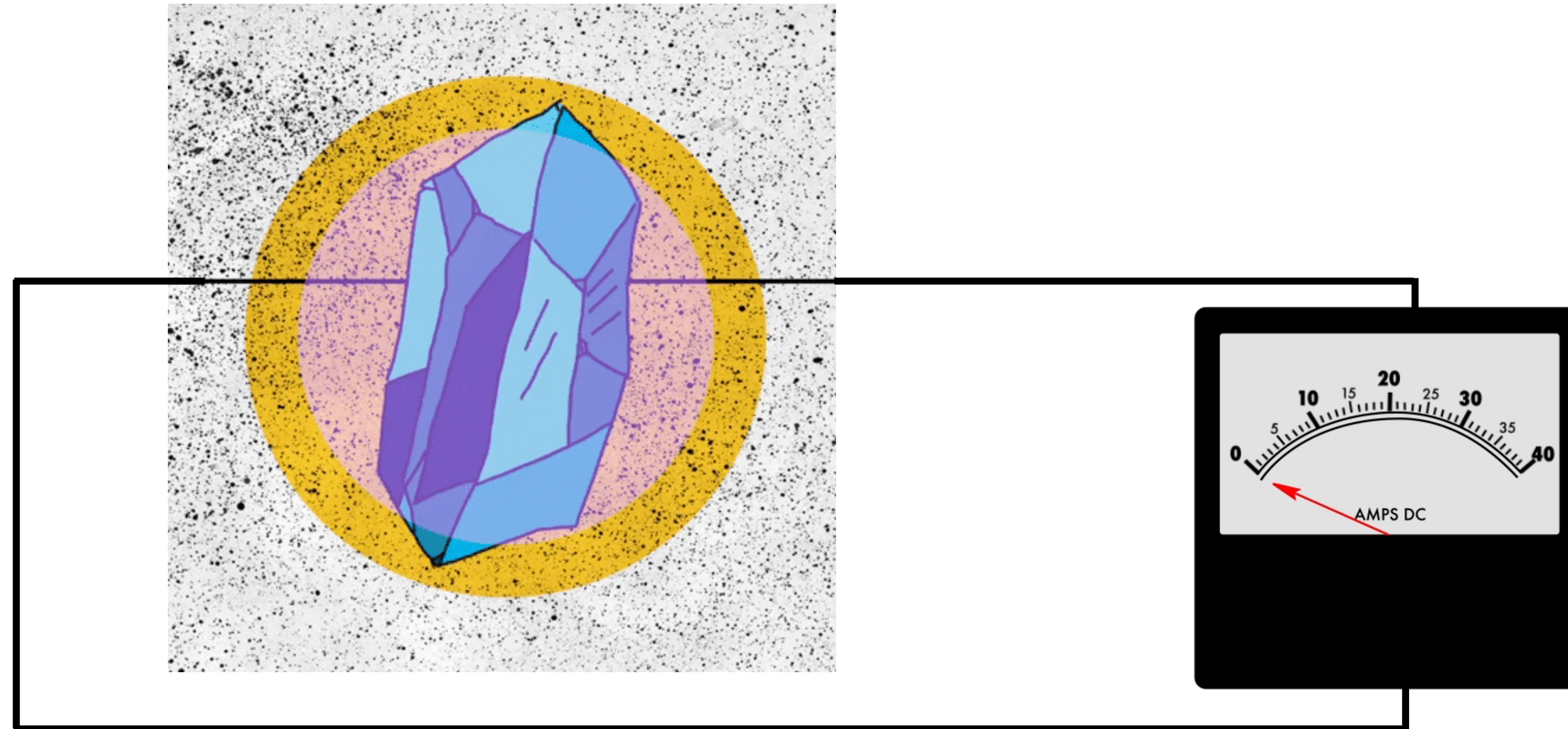
Amalia Madden

Perimeter Institute for Theoretical Physics

Based on 2112.11466

with Asimina Arvanitaki (Perimeter Institute) and Ken Van Tilburg (NYU & Flatiron Institute)

The Piezoaxionic Effect



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Properties of light axion dark matter

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- Small frequency spread (coherence)

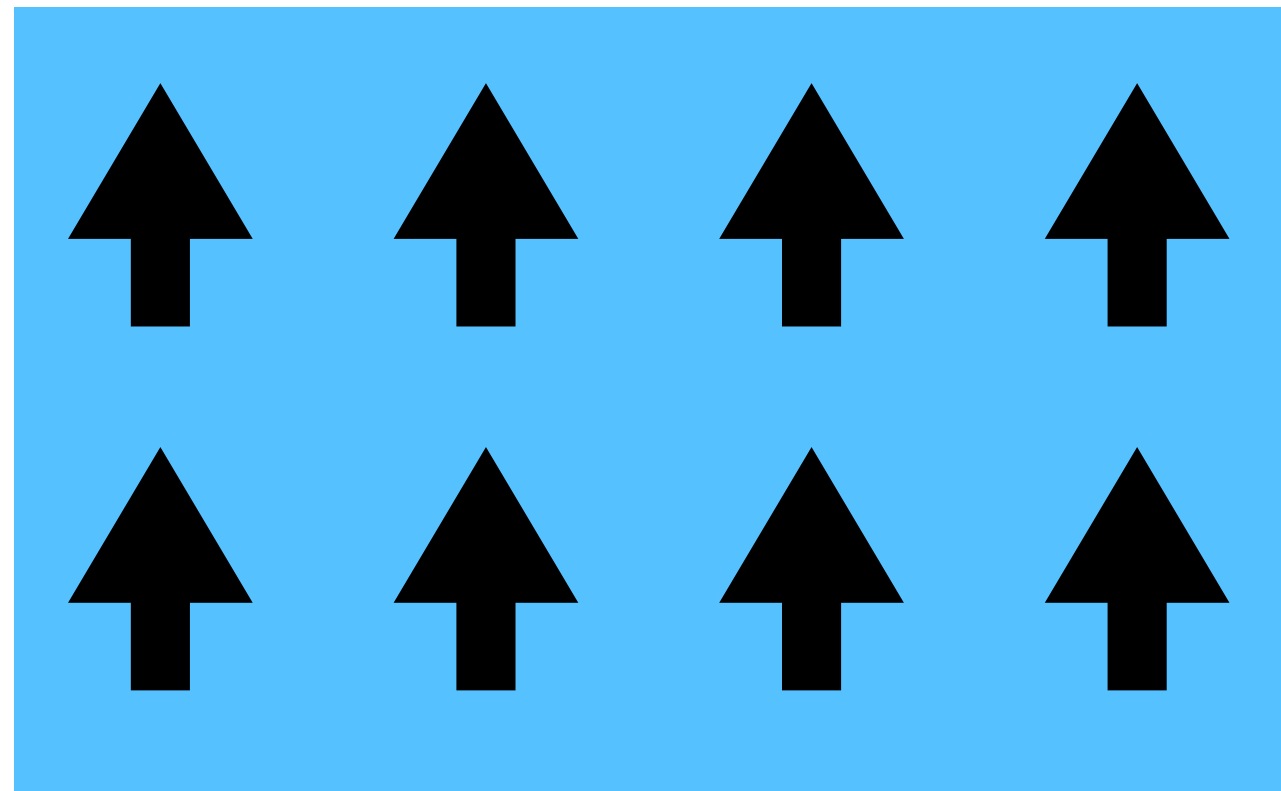
$$\delta\omega_a \approx \frac{v^2}{2c^2} \omega_a \approx 10^{-6} \omega_a$$



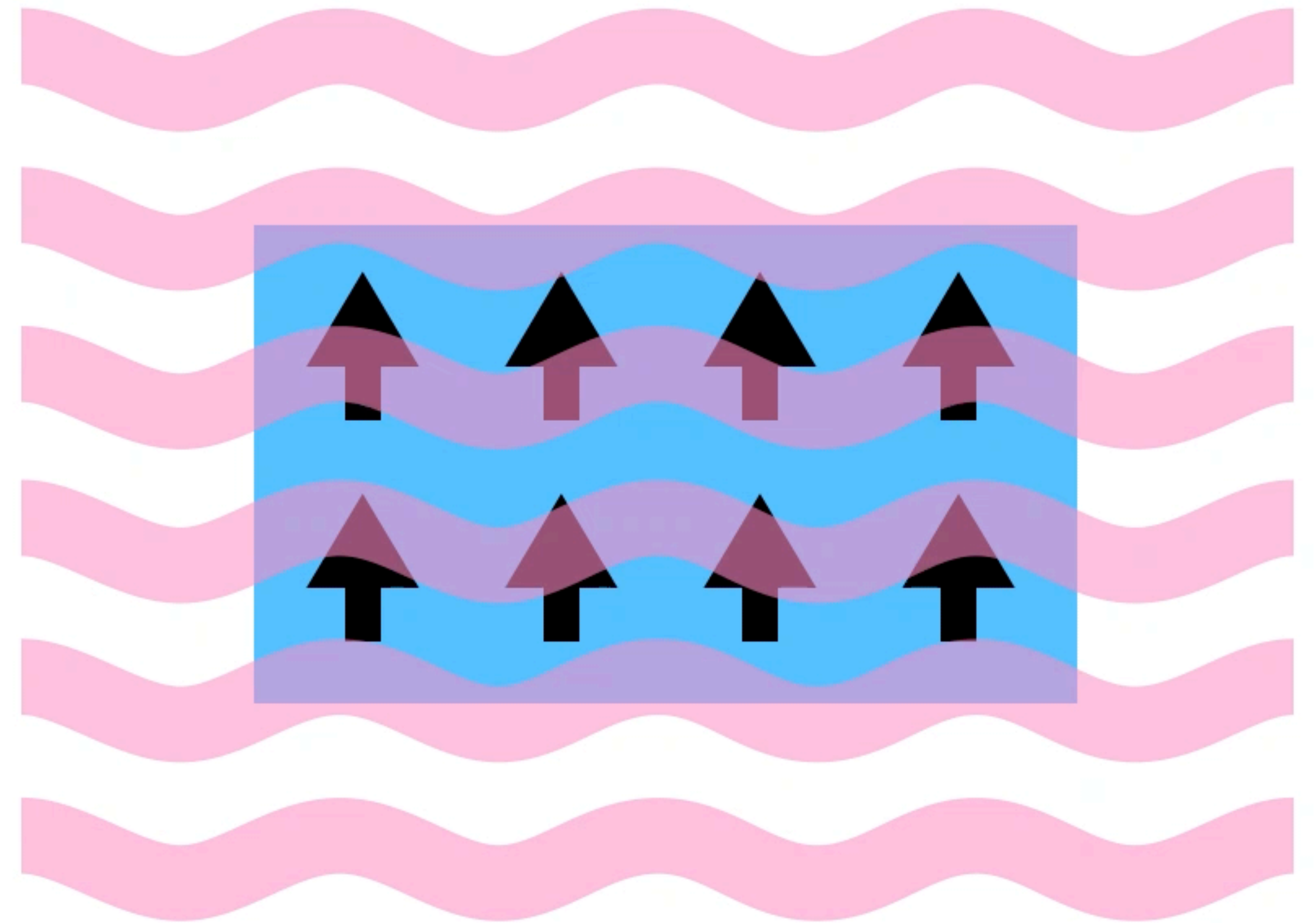
Piezoaxionic effect - a preview

$$\mathcal{L} \supset \frac{\alpha_s a}{8\pi f_a} G \tilde{G}$$

\uparrow
 \vec{B}



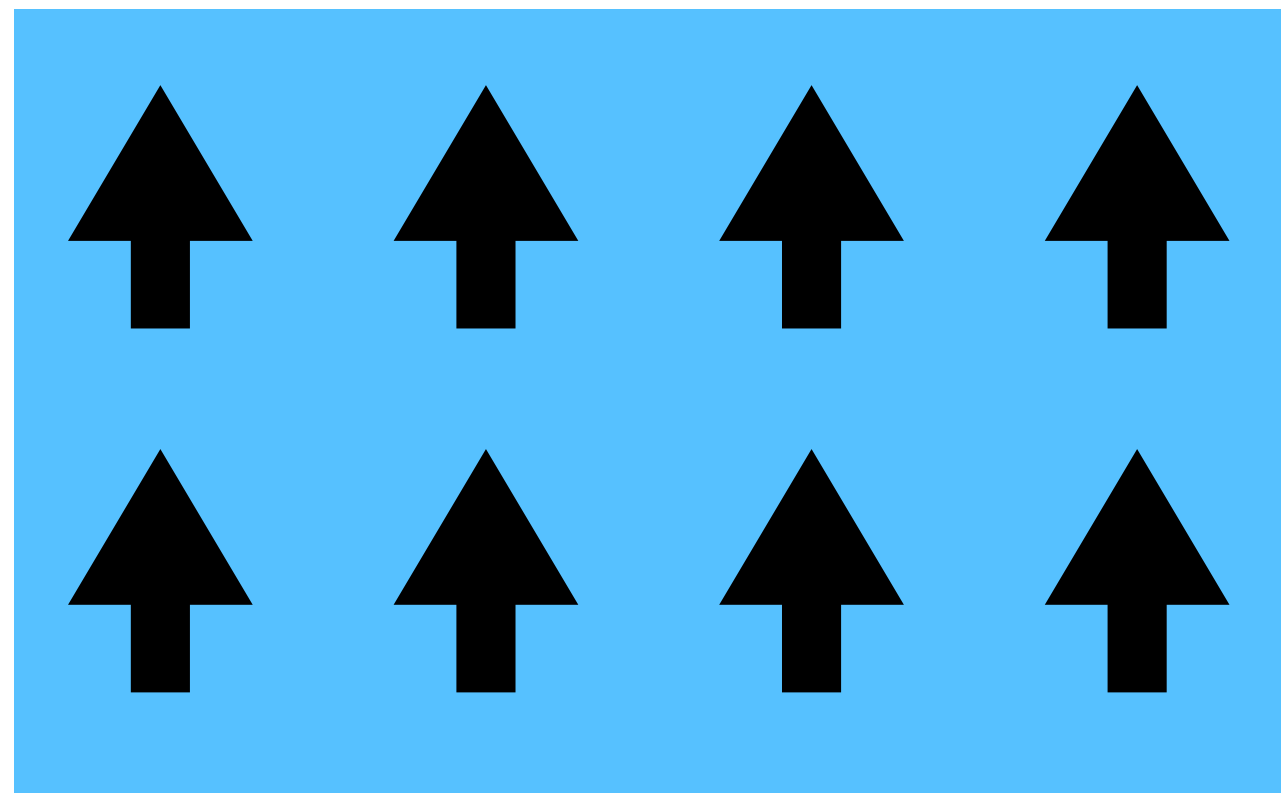
Axion DM
background



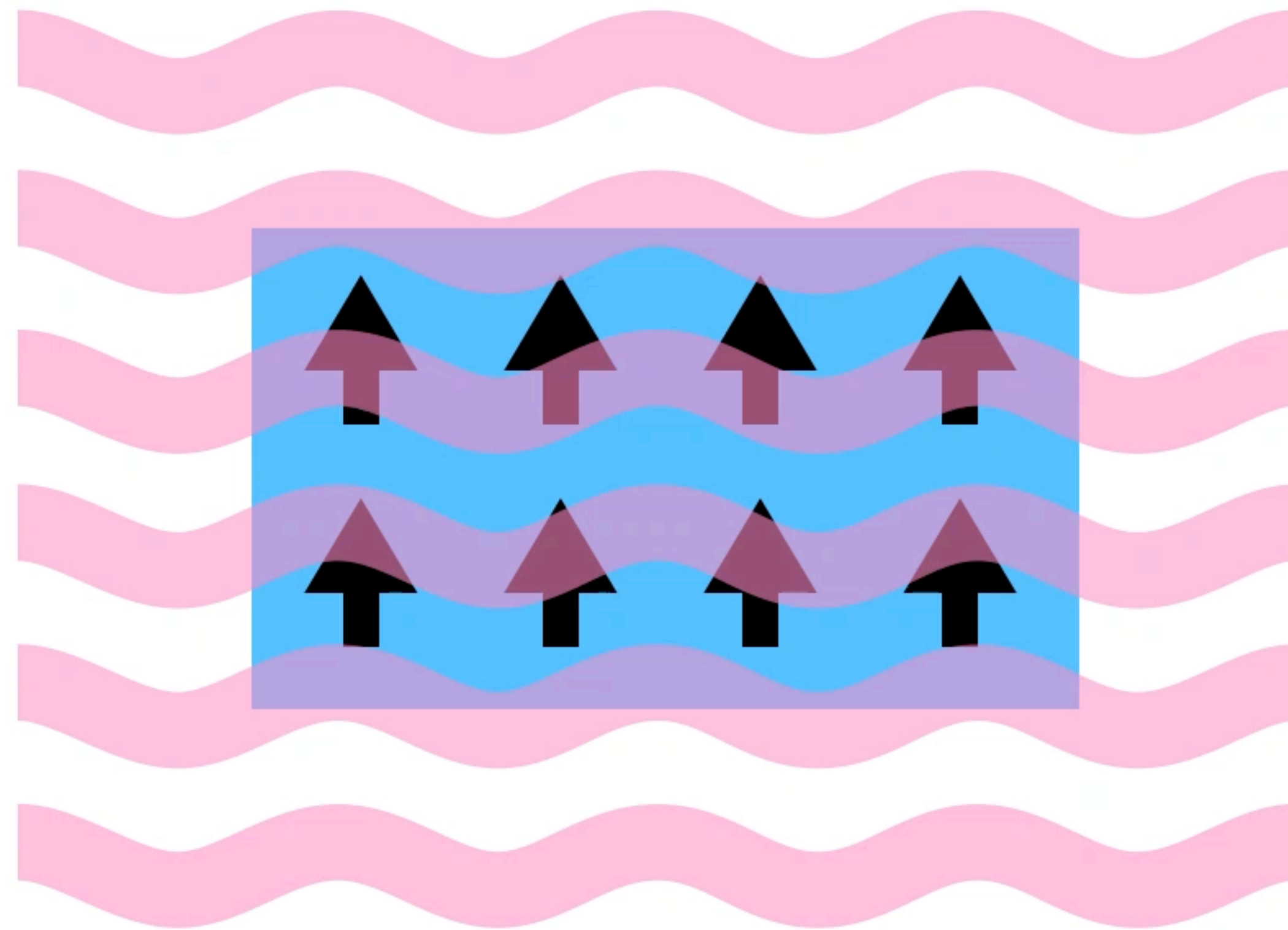
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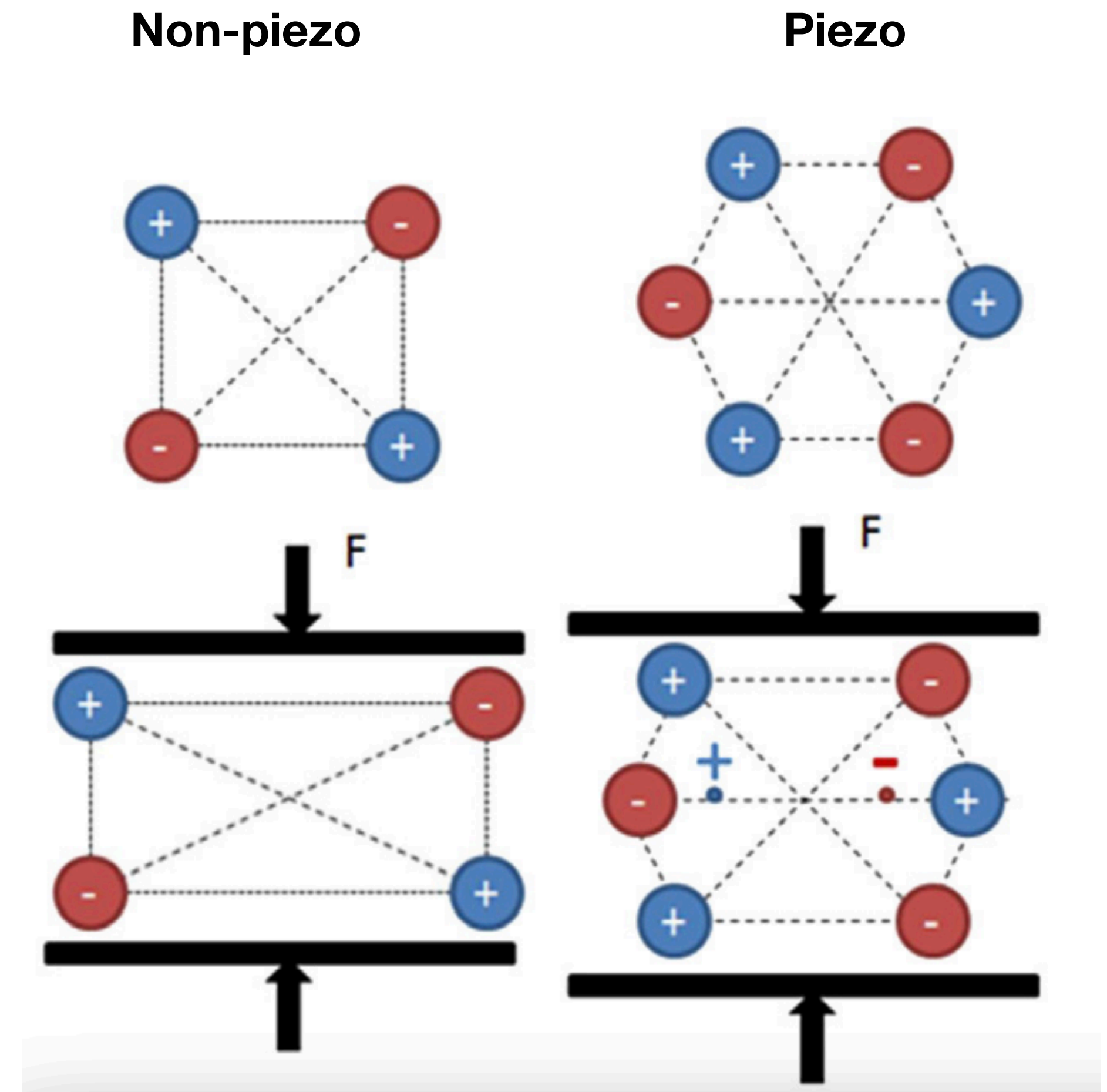


Outline

- Parity violation: piezoelectric crystals and axion couplings
- Schiff's theorem: how to detect parity violating nuclear multipole moments
- Resonant mass detectors
- Proposed experimental setup and sensitivity

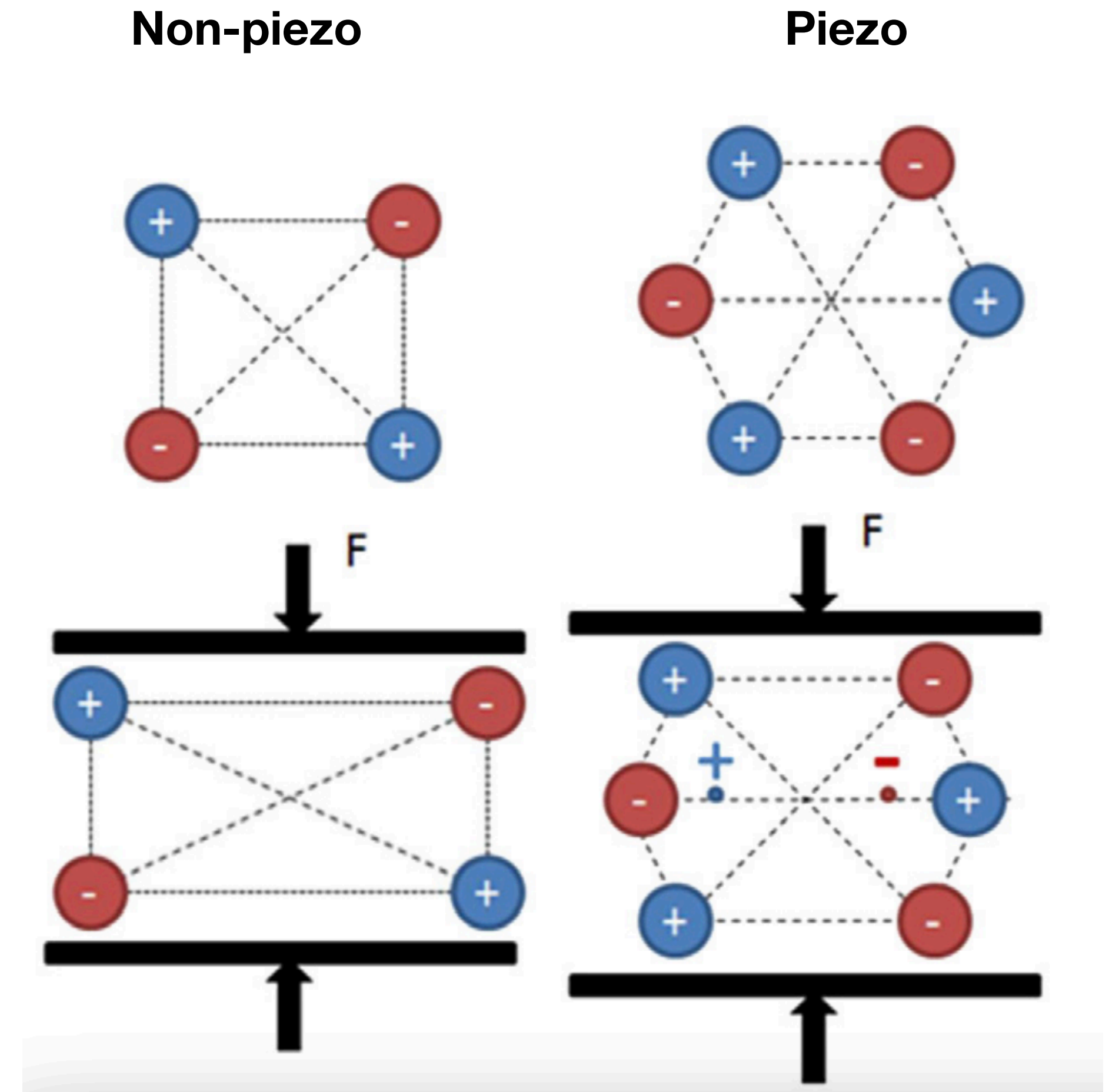
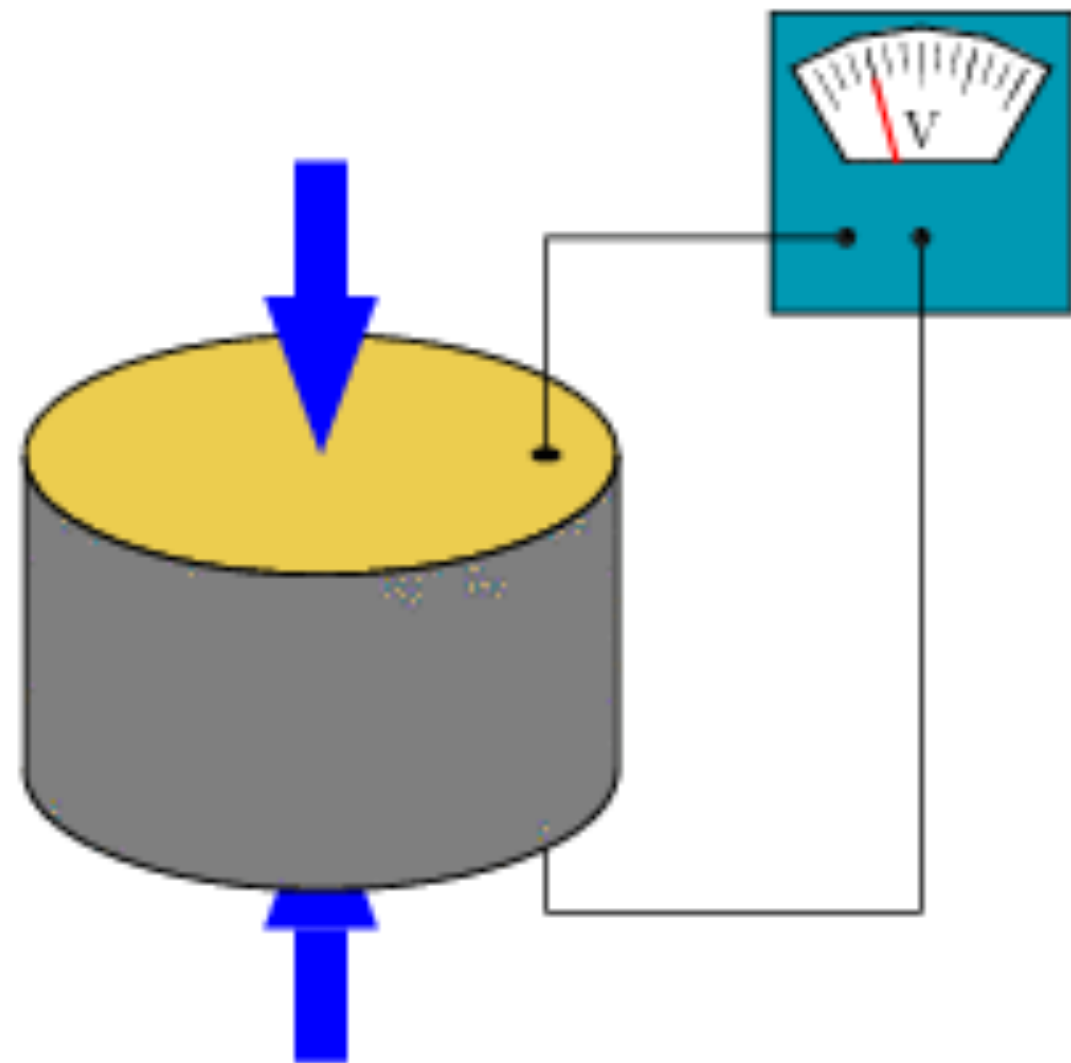
Piezoelectric Crystals

- Piezoelectric are a large class of materials: 20 out of 32 symmetry groups
- Crystal structure breaks parity symmetry
 $(x, y, z) \neq (-x, -y, -z)$
- Deformation causes net charge across unit cell (and vice versa).



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Constitutive Equations for Piezoelectricity

$$\text{Stress} = + \overset{\text{Stiffness}}{\downarrow} c \cdot \text{Strain} - \overset{\text{Piezoelectric}}{\downarrow} h \cdot \text{Electric Displacement}$$

$$\text{Electric Field} = - h \cdot \text{Strain} + \frac{1}{\epsilon} \cdot \text{Electric Displacement}$$

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parity even
parity odd

Constitutive Equations for Piezoelectricity

$$\theta_a(t) \equiv \frac{a(t)}{f_a}$$

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 \text{Electric Field} &= - \underline{h} \cdot \text{Strain} + \frac{1}{\overset{\text{Permittivity}}{\uparrow} \underline{\epsilon}} \cdot \text{Electric Displacement} - \overset{\text{Electroaxionic}}{\uparrow} \underline{\zeta} \theta_a(t) \cdot \text{Nuclear Spin Direction}
 \end{aligned}$$

parity even
 parity odd
 time-reversal odd

The piezoaxionic tensor ξ is **ODD** under parity, and can only be present in piezoelectric materials.

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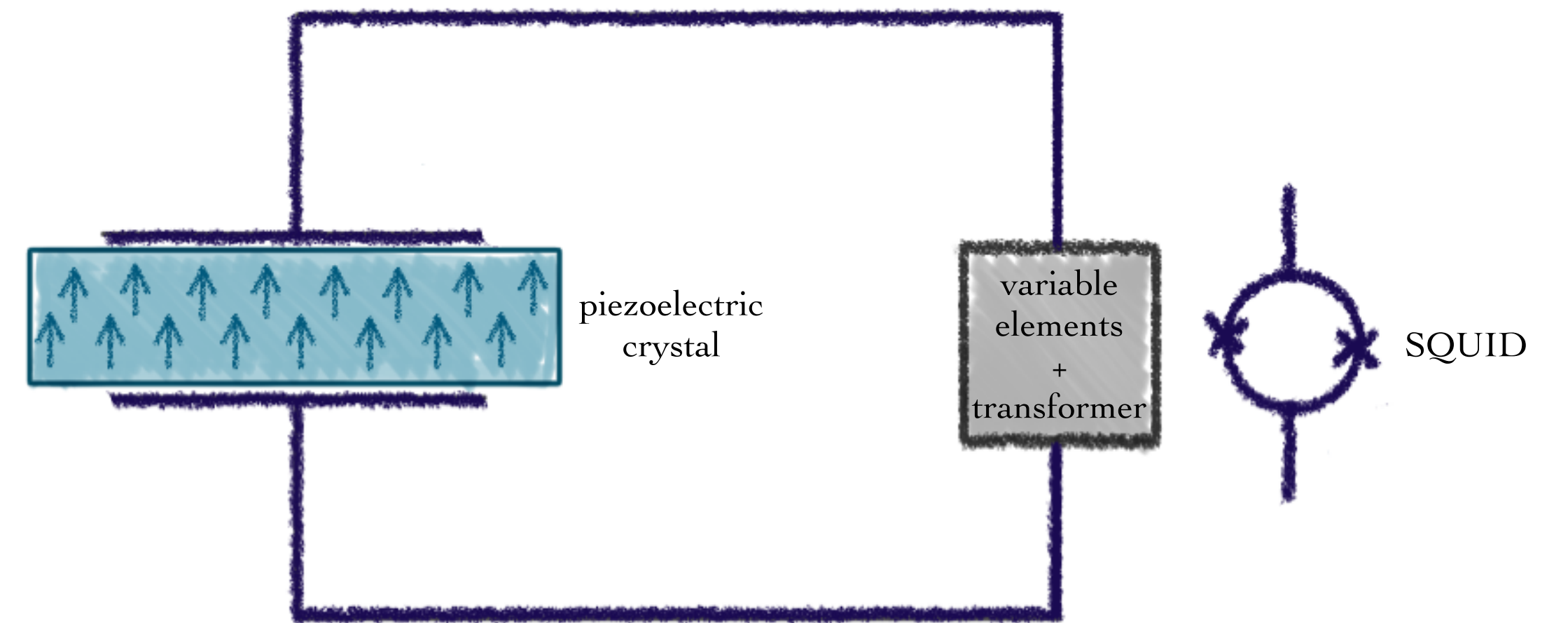
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We will focus on ξ in this talk!

Journey from microscopic to macroscopic

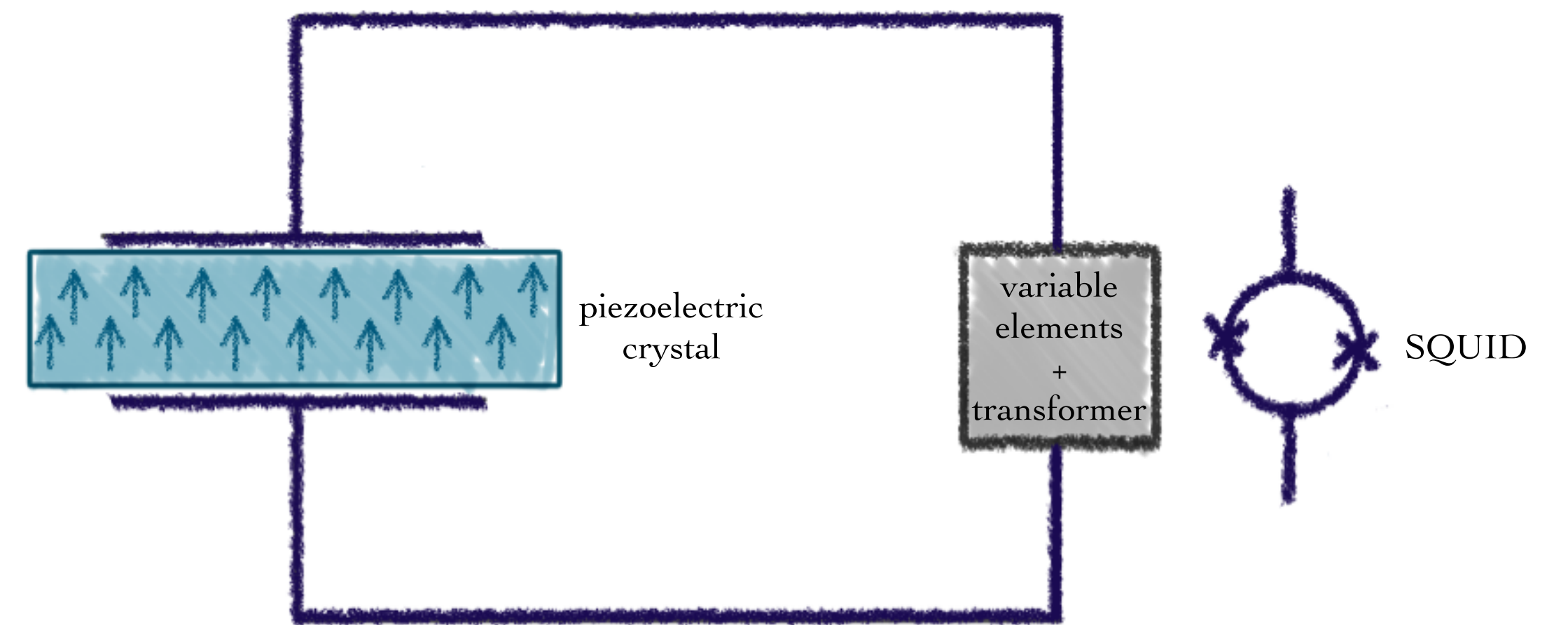
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AXION DM



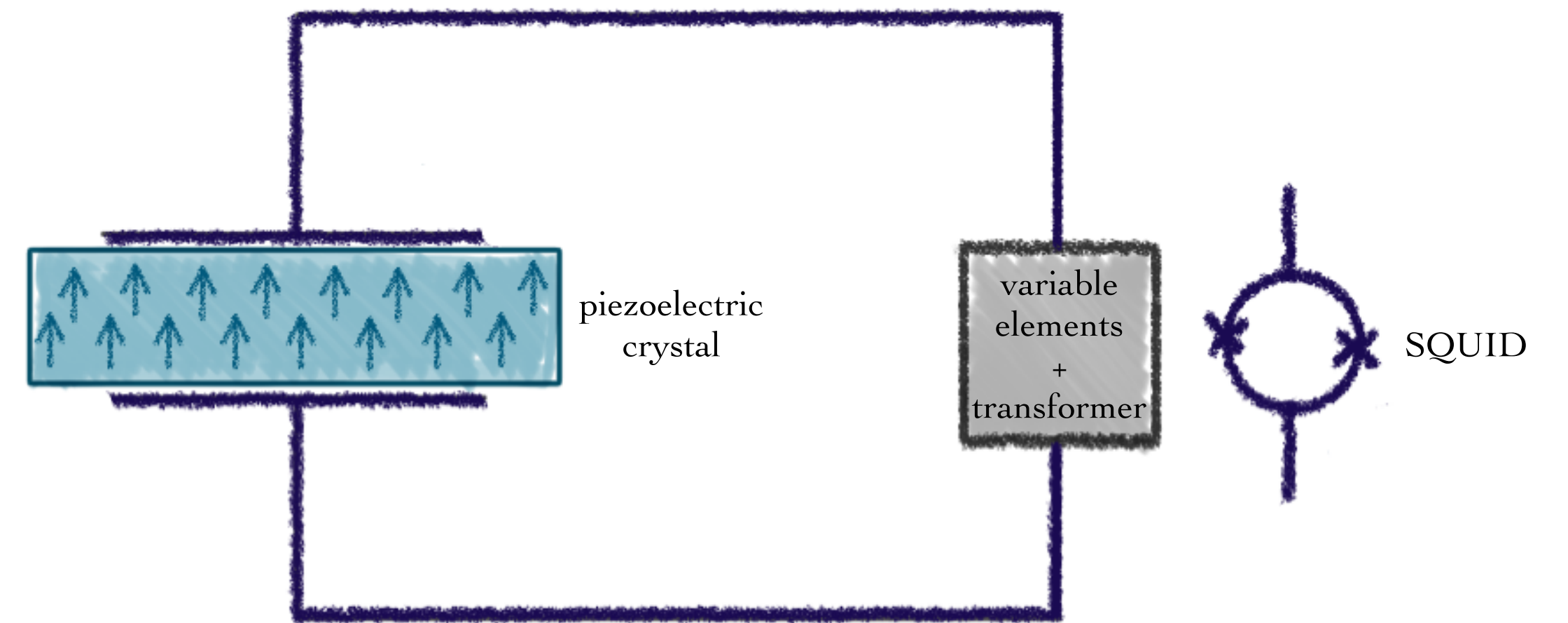
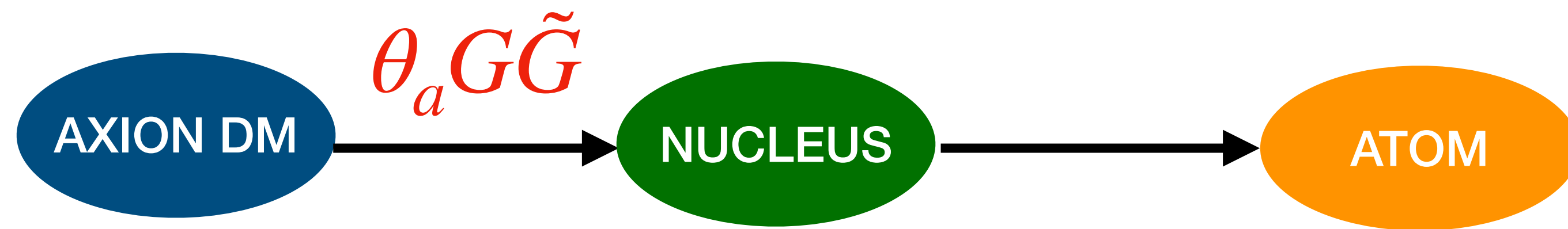
Journey from microscopic to macroscopic

START HERE



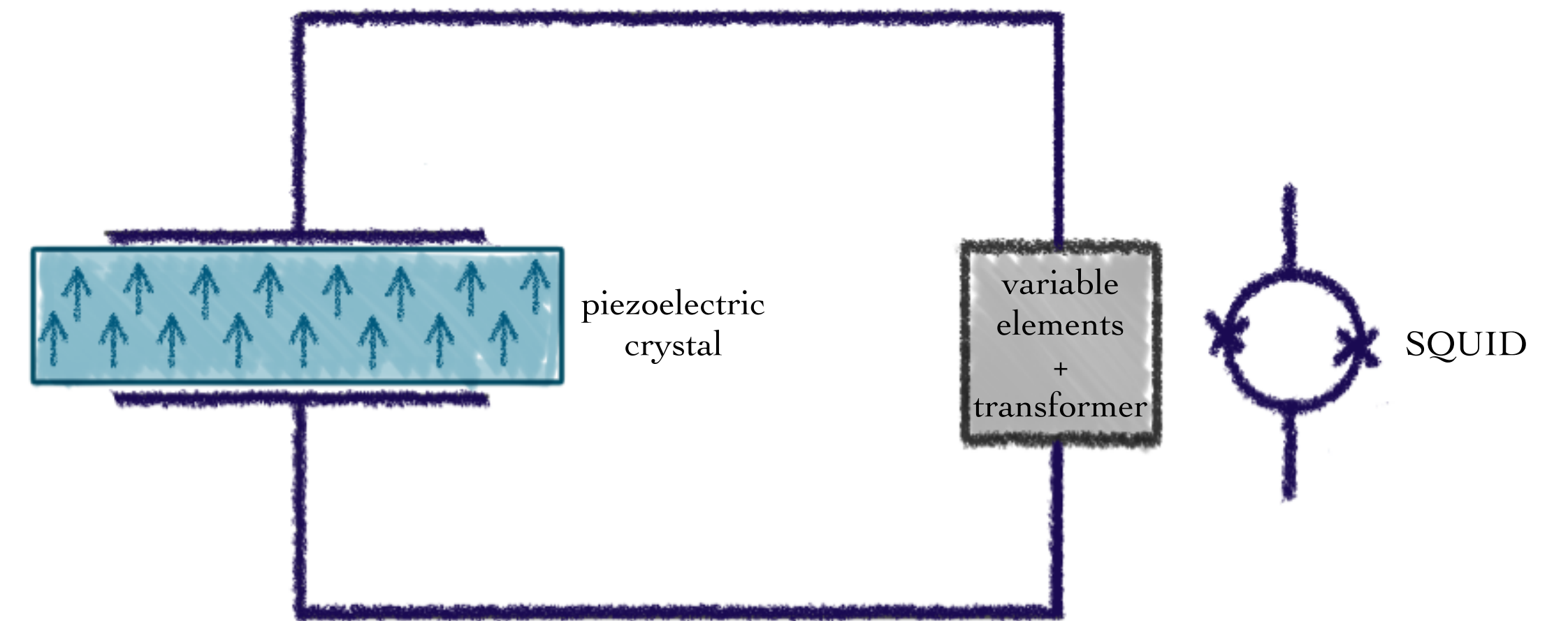
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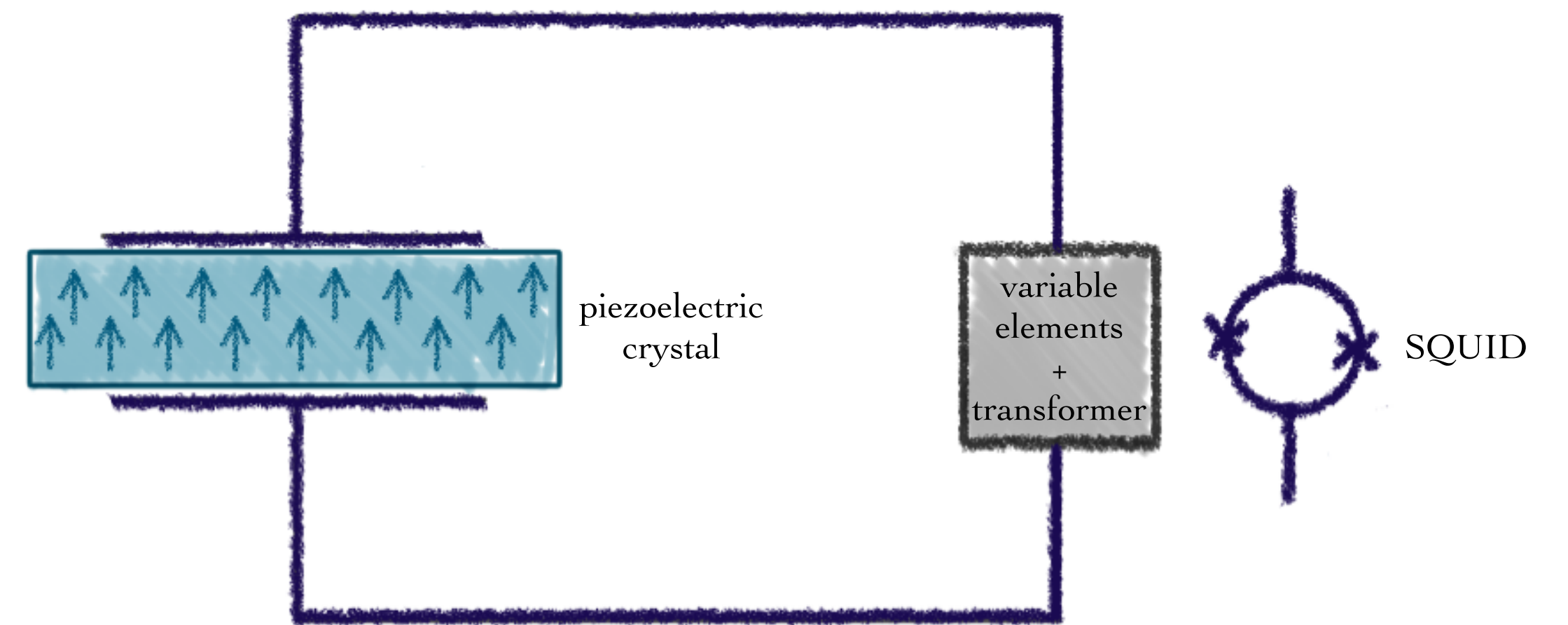
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Journey from microscopic to macroscopic

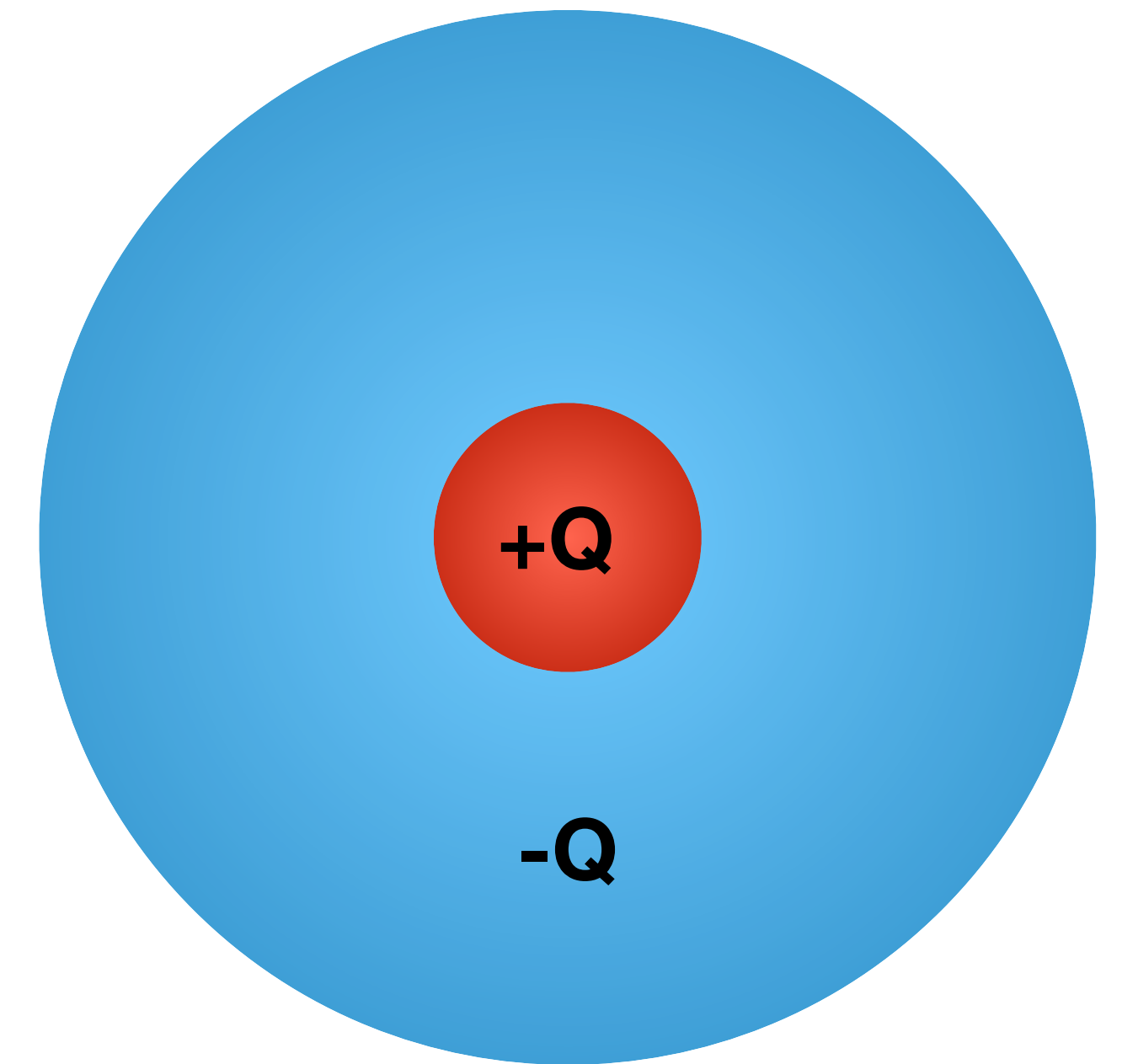
START HERE



Schiff's Theorem

QCD axion DM induces an *oscillating* neutron EDM:

$$d_n \sim 10^{-16} \frac{\sqrt{\rho_{DM}}}{m_a f_a} \cos m_a t \cdot e \cdot \text{cm}$$

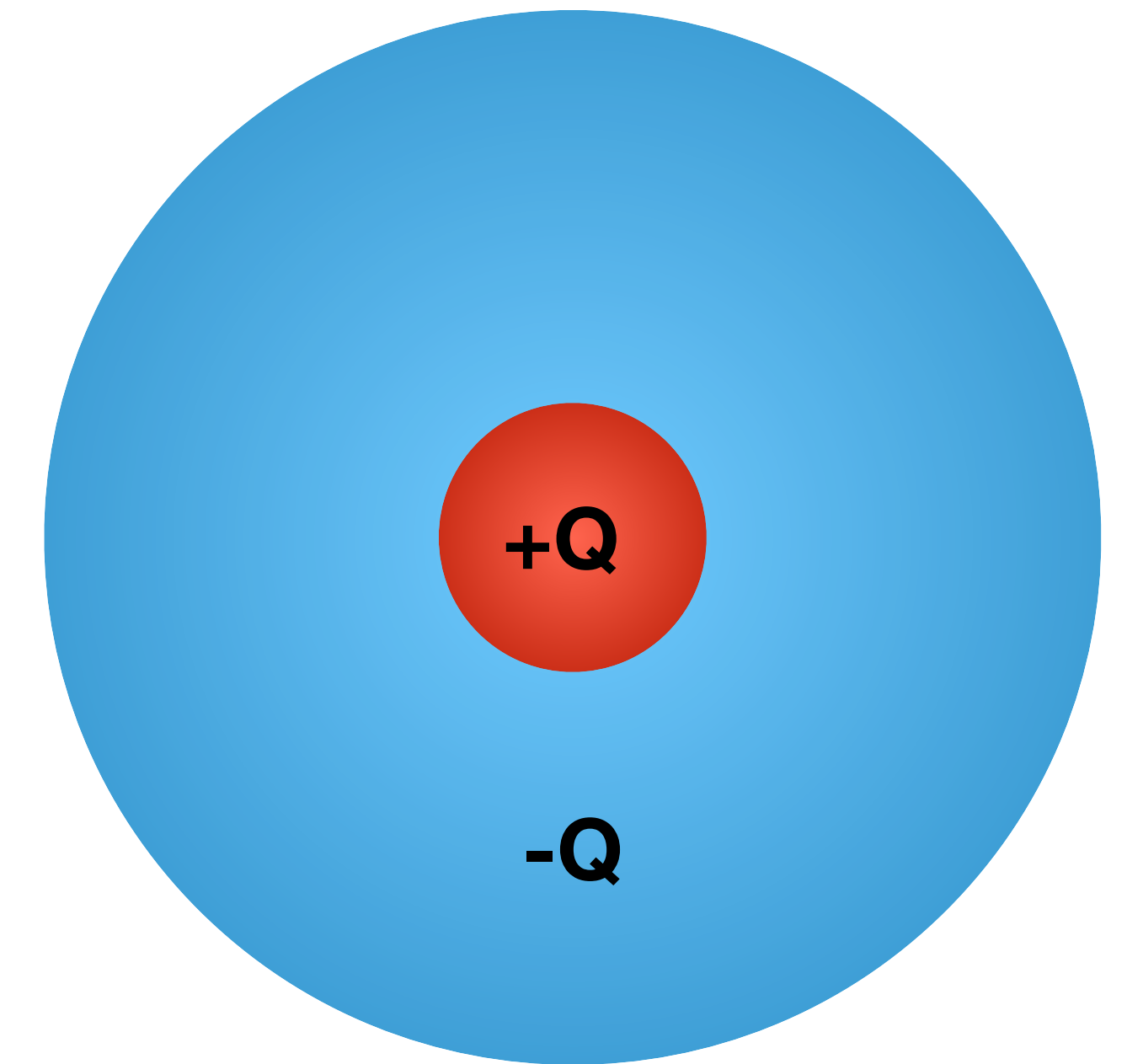


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If we treat an atom as a system of *point* particles, nuclear EDM is perfectly shielded by electron cloud [Schiff 1963].

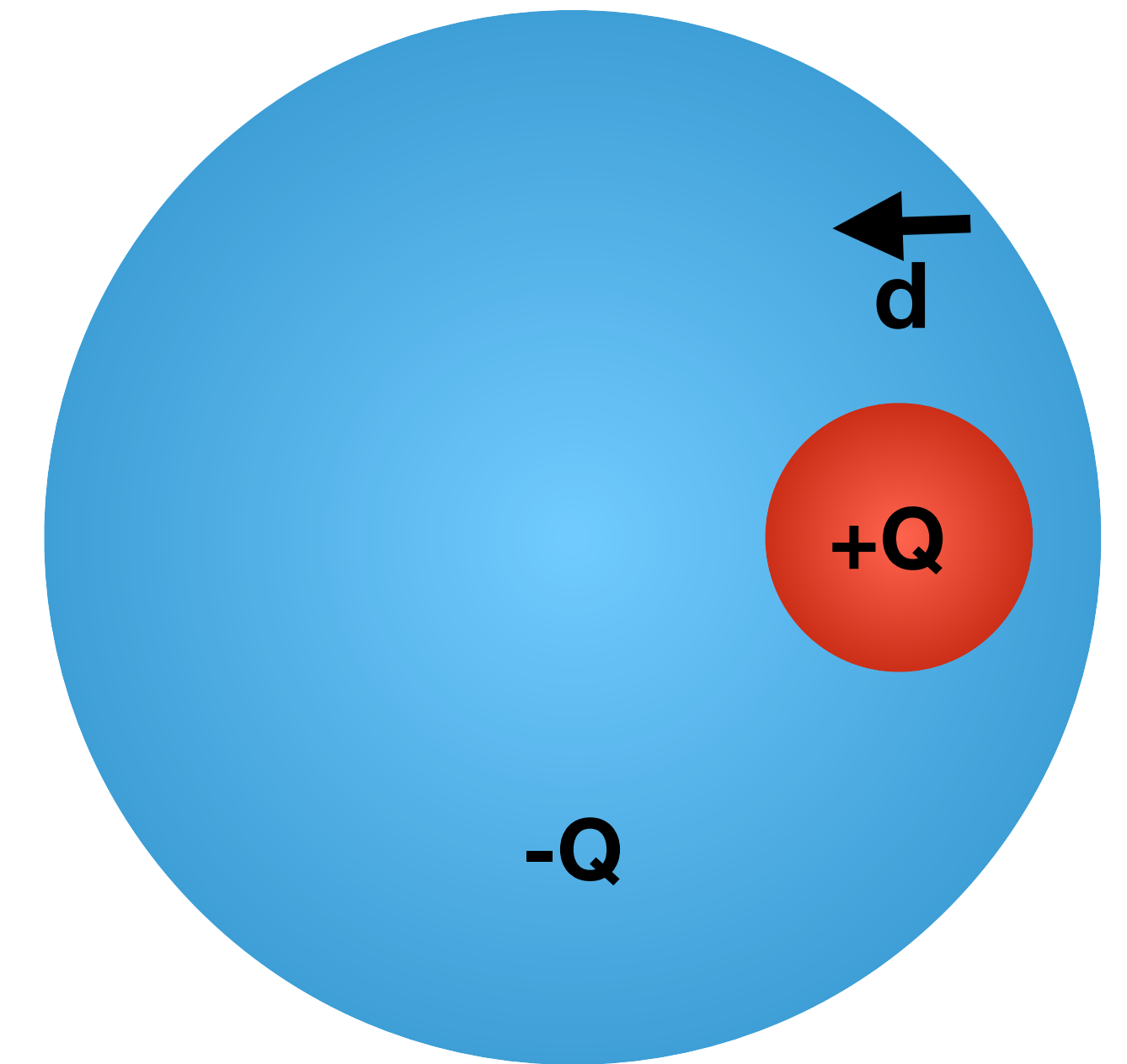


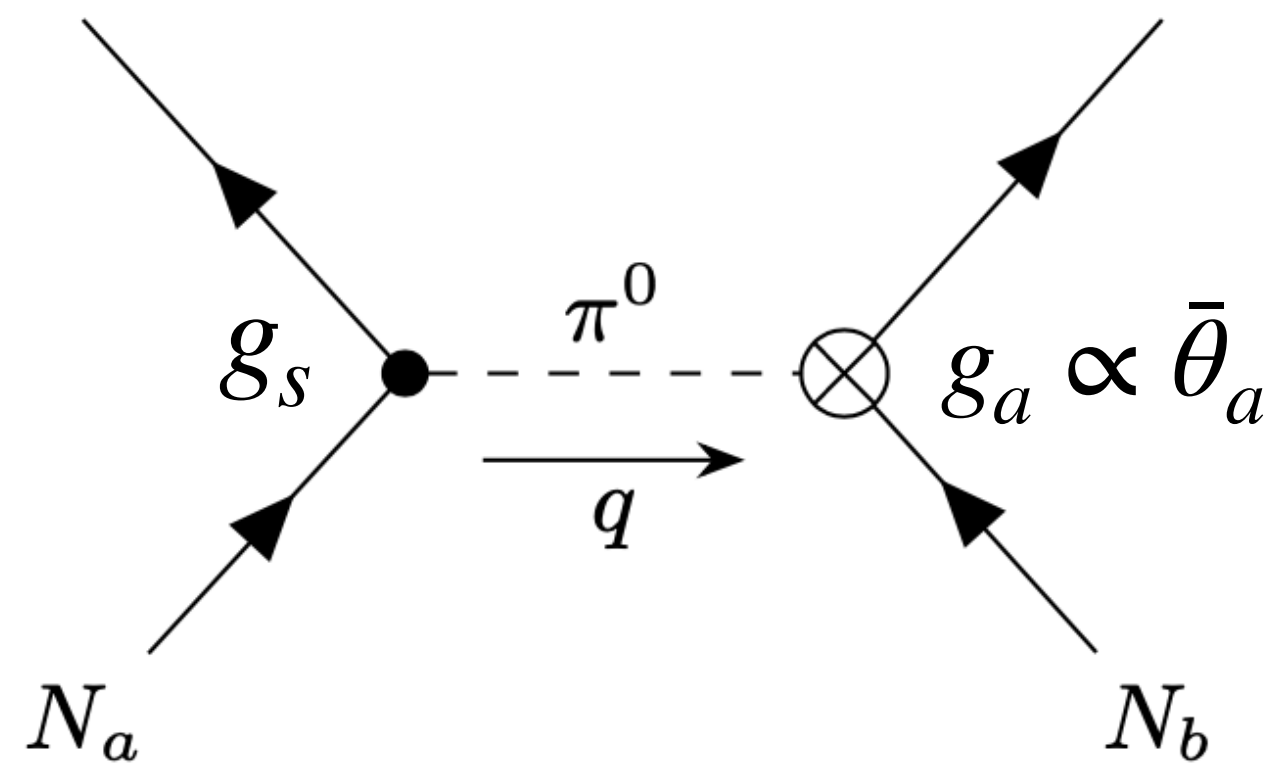
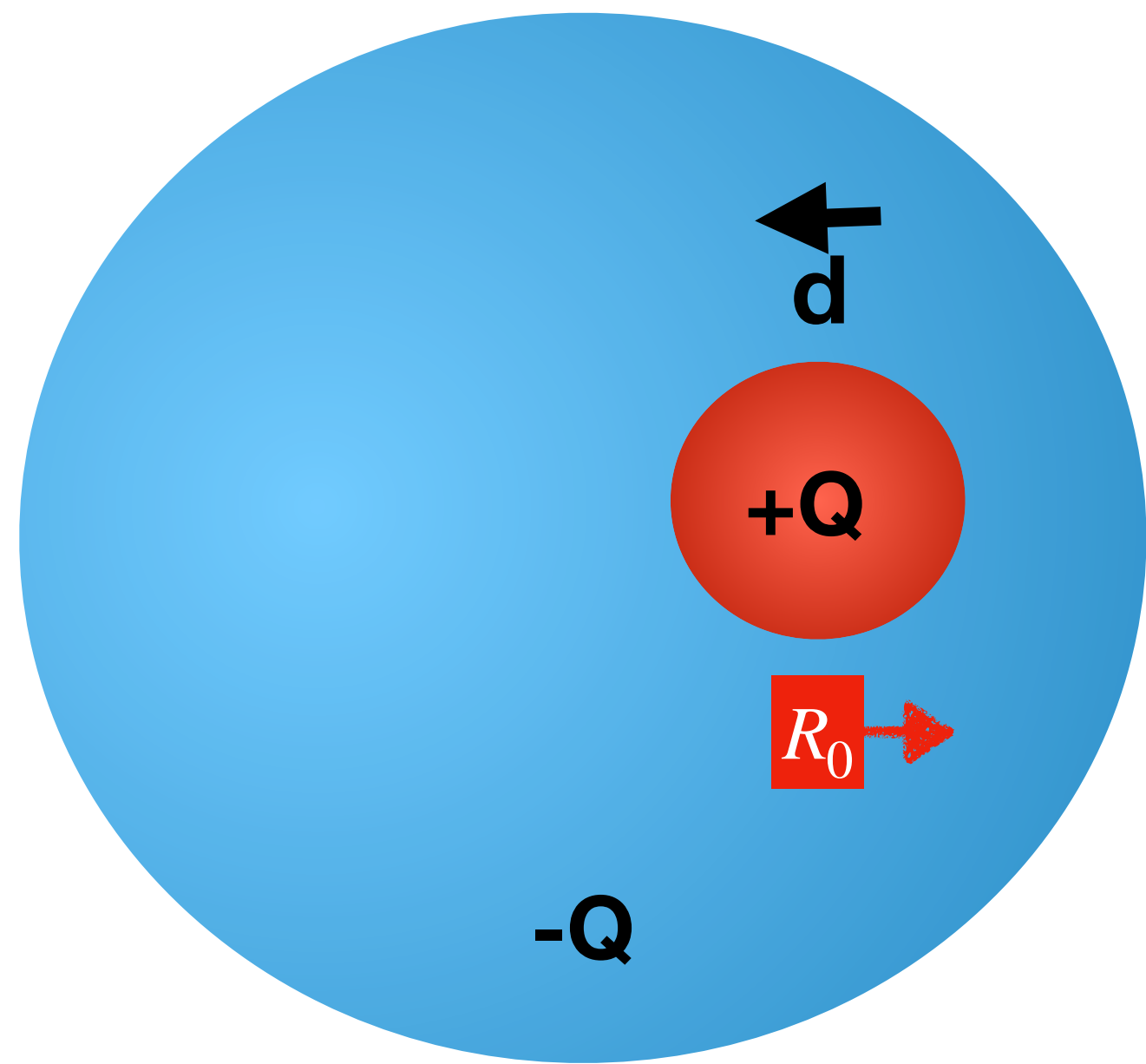
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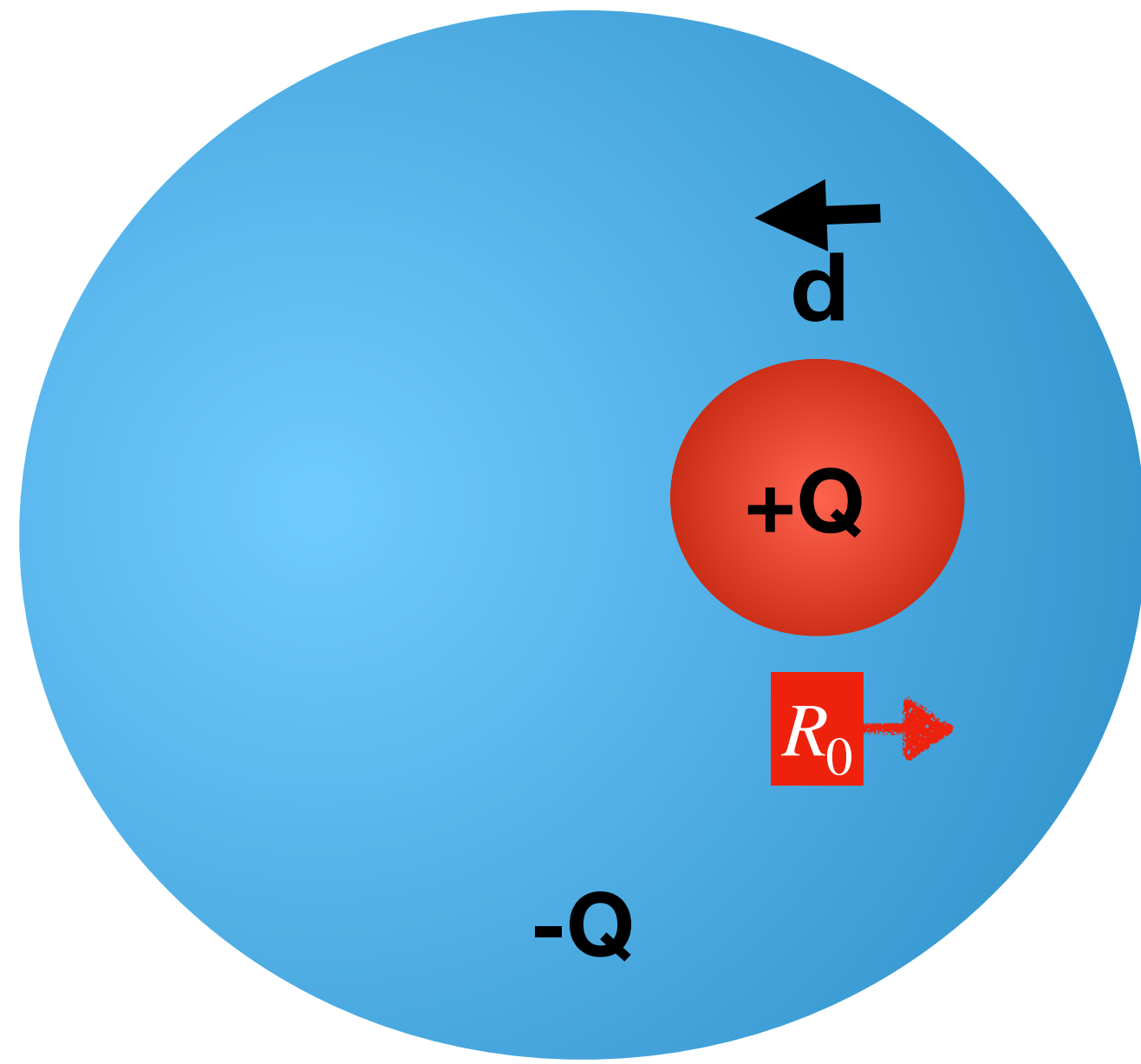
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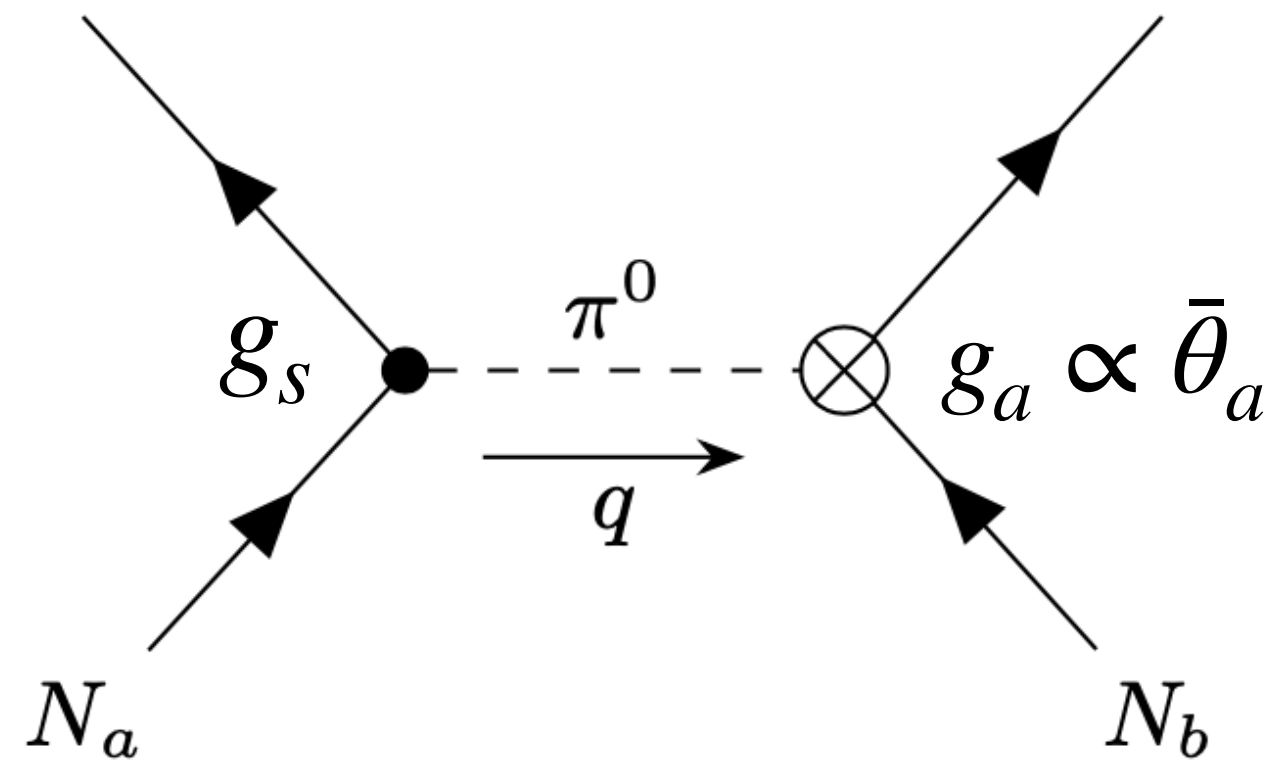


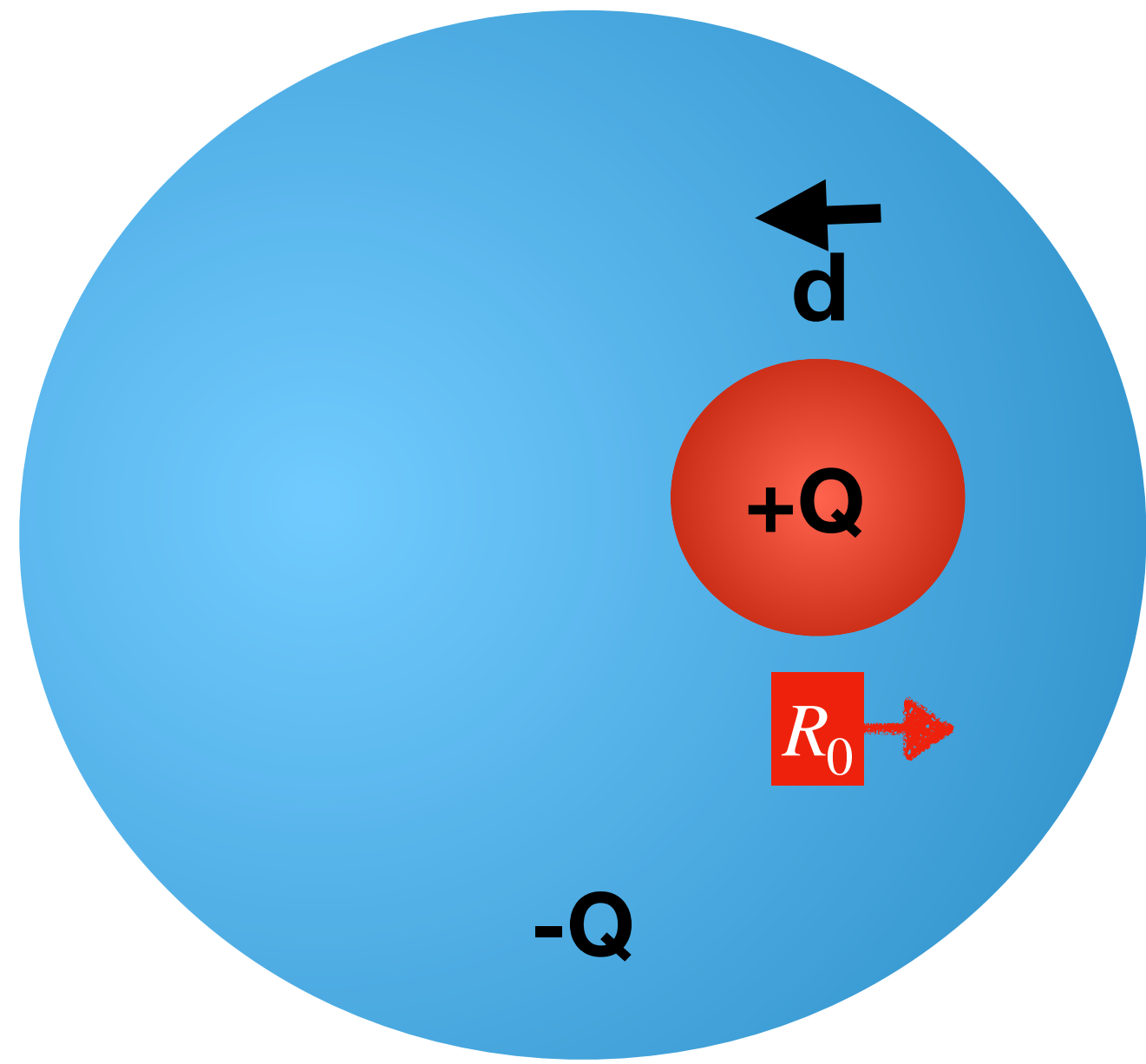


Resolution: Schiff's theorem violated by finite size effects:

$$V_{\text{Schiff}} = 4\pi e \mathcal{S} \cdot \nabla(\delta_e(\mathbf{r}))$$

"Schiff moment \mathcal{S} "





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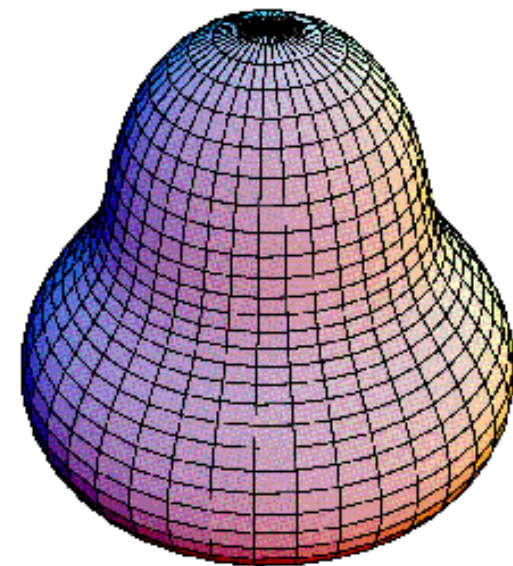
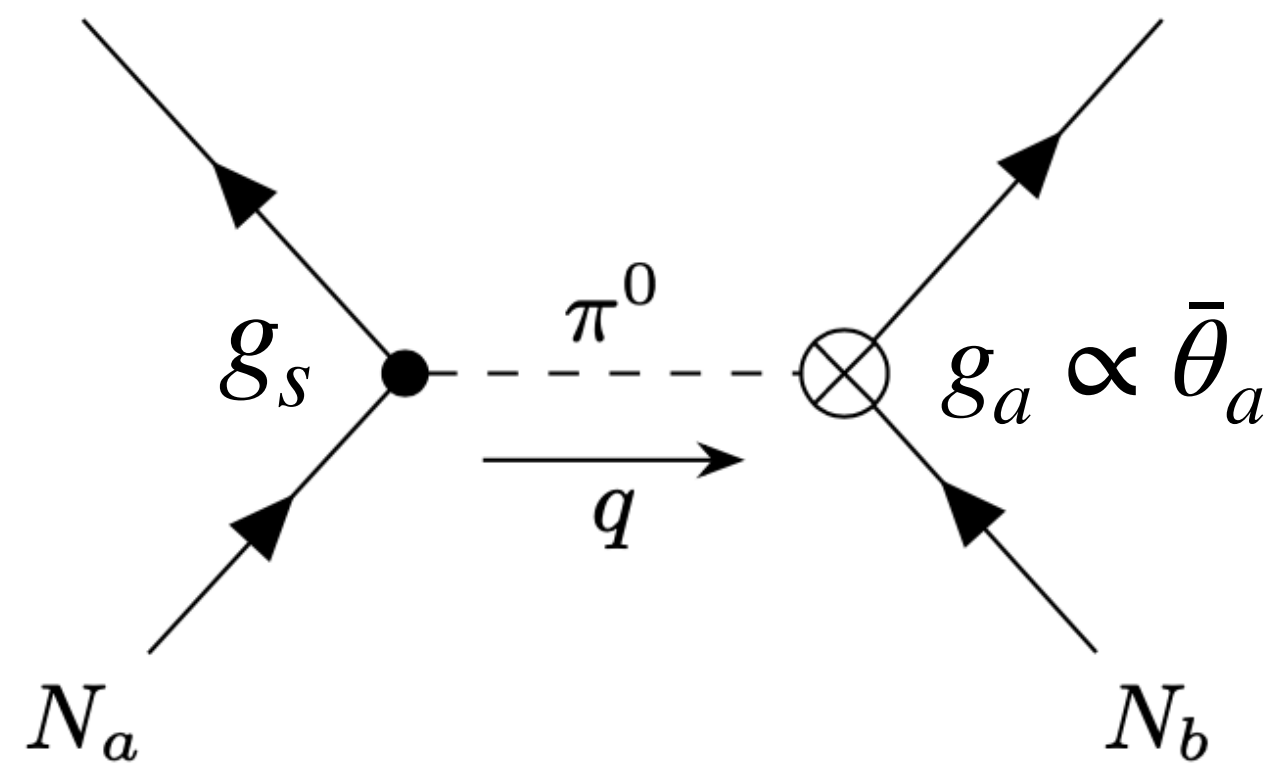
“Schiff moment \mathcal{S} ”

$$\mathcal{S} \sim e \frac{\bar{\theta}_a}{m_N} R_0^2 \propto A^{2/3}$$

for non-deformed nuclei

$$\mathcal{S} \sim e Z \frac{\bar{\theta}_a}{m_N} R_0^2 \propto Z A^{2/3}$$

for deformed (pear shaped) nuclei



- $V_{electrons} = V_{crys} + V_{schiff}$

V_{cell} = unit cell volume, \mathcal{S} = Schiff moment magnitude



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This gives an energy shift:

$$\langle H_{schiff} \rangle \simeq \sum_{s,p} \epsilon_s \epsilon_p^* \langle s | V_{schiff} | p \rangle + c.c$$

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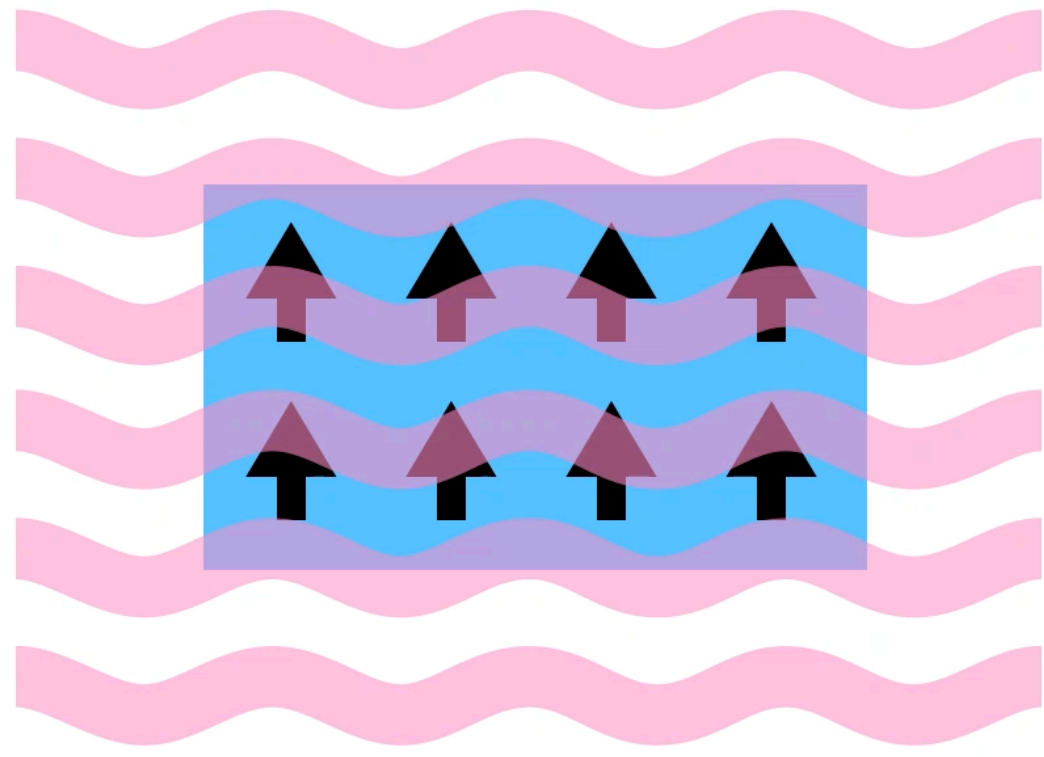
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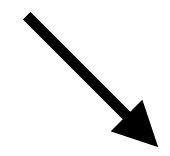
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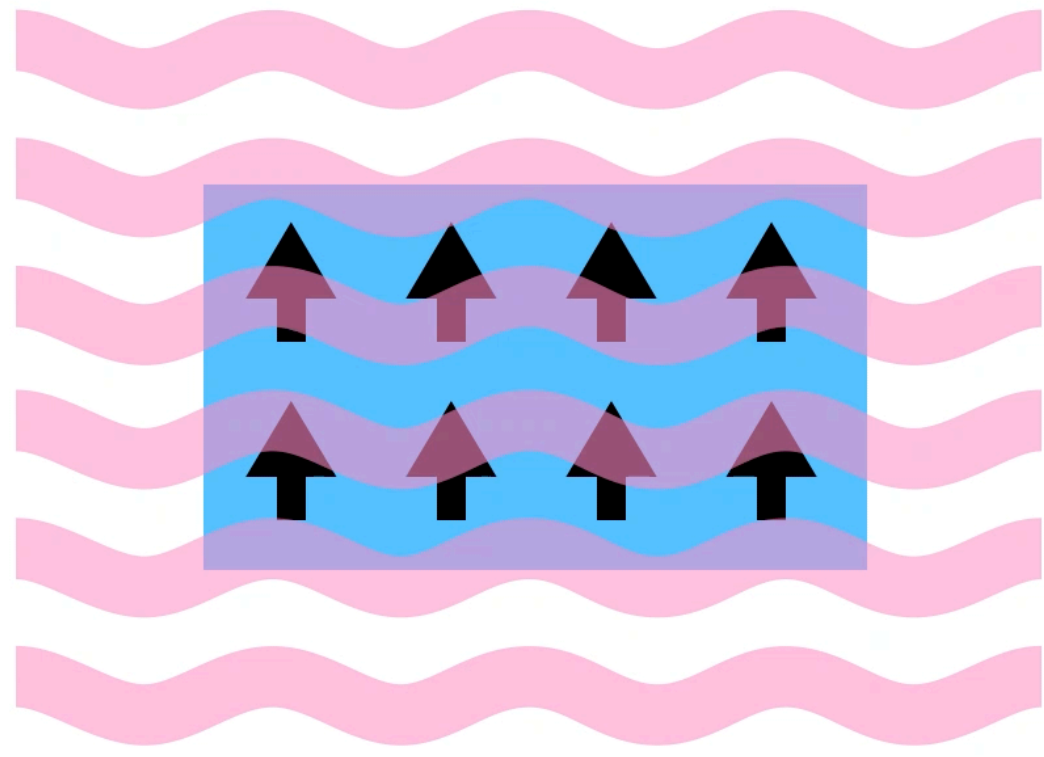




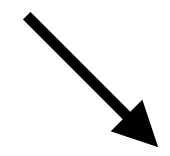
$$S = \text{strain} = \frac{\Delta L}{L}$$



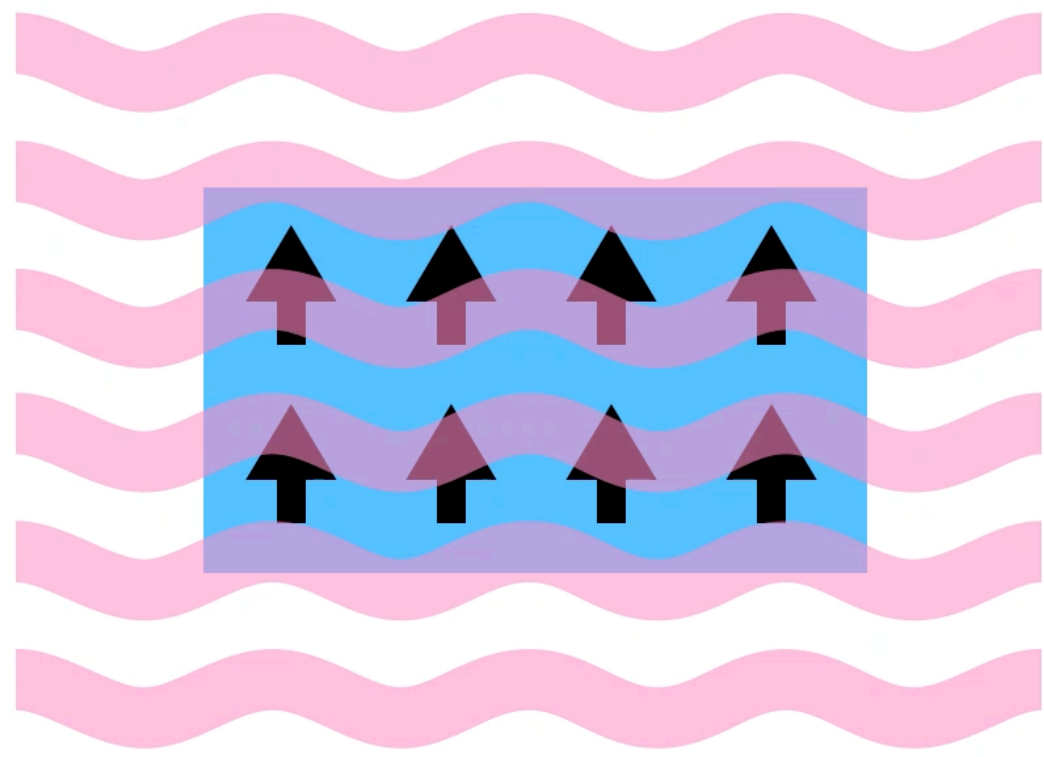
$$S = |\xi c^{-1} \hat{I} \bar{\theta}_a|$$



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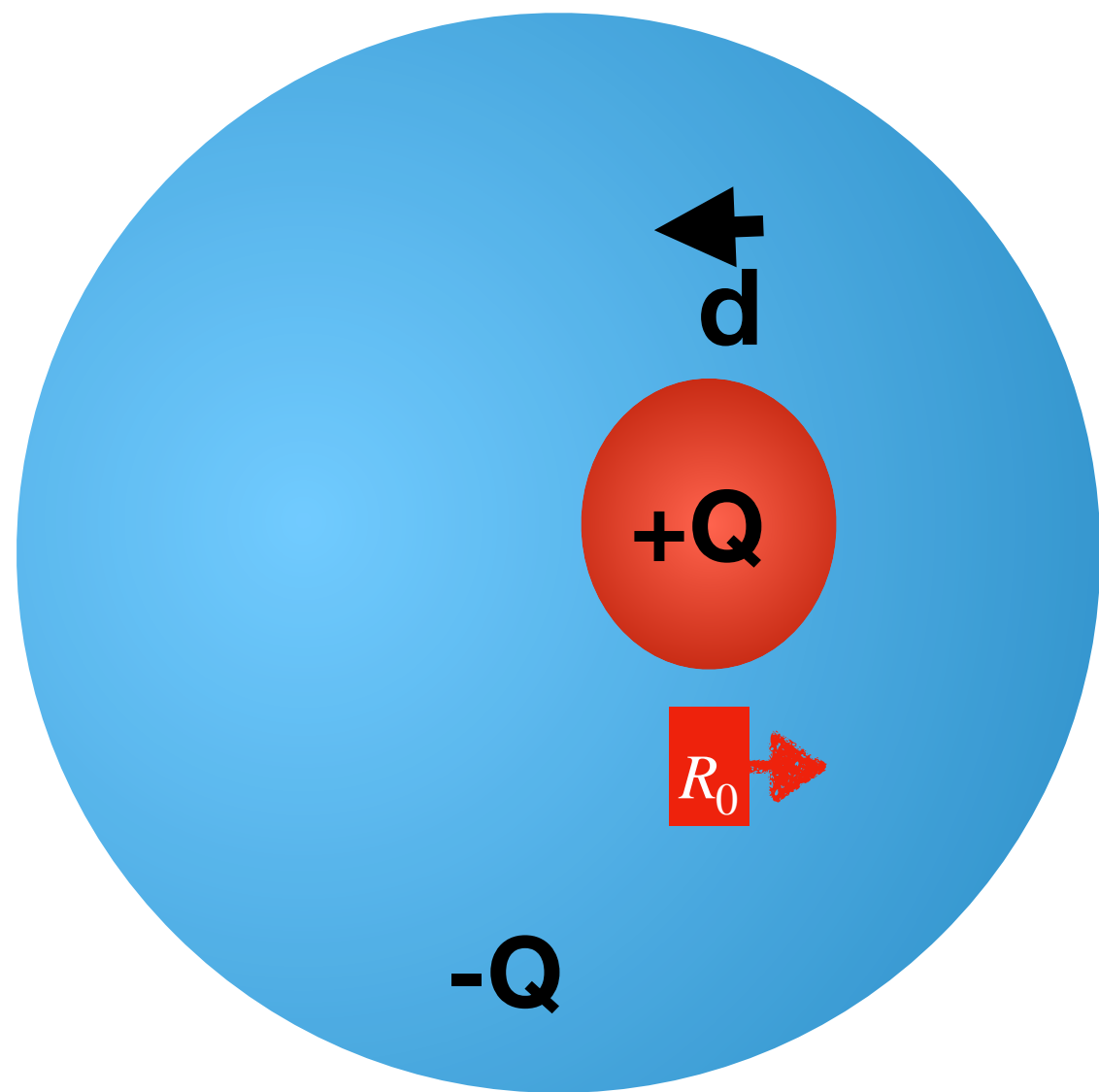
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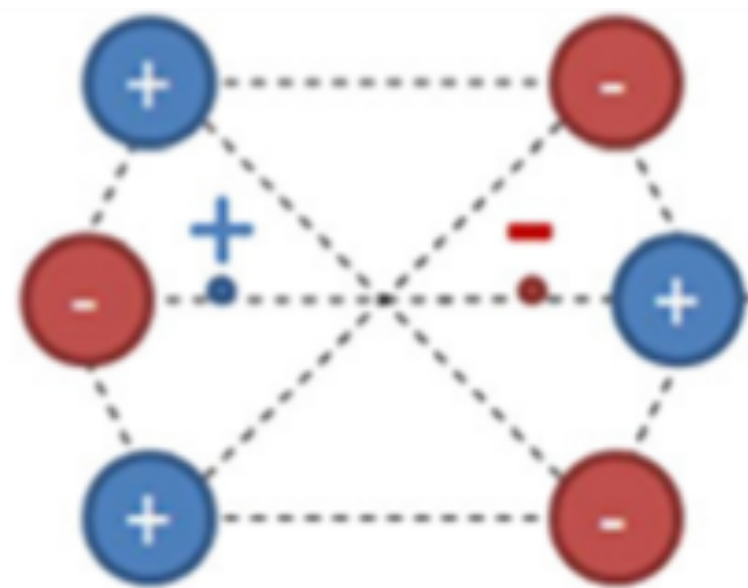
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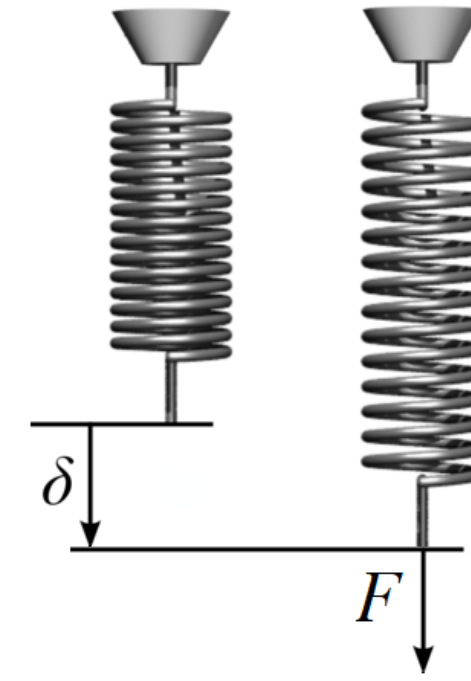
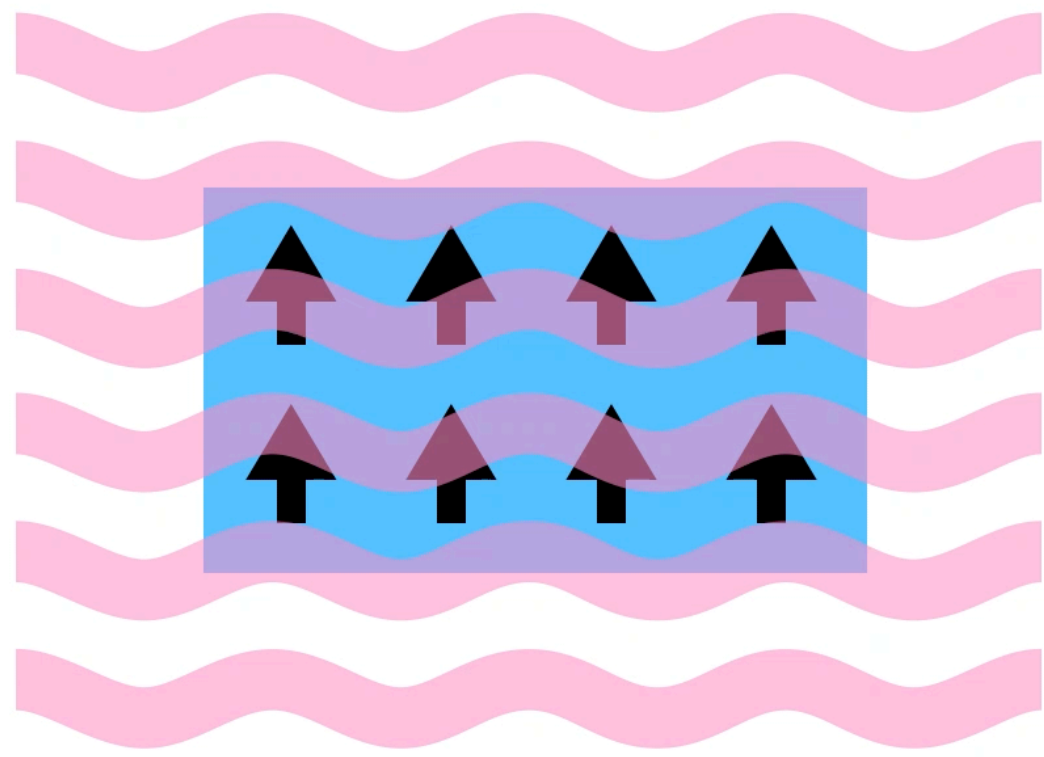


Schiff Moment

×



Piezoelectric Crystal

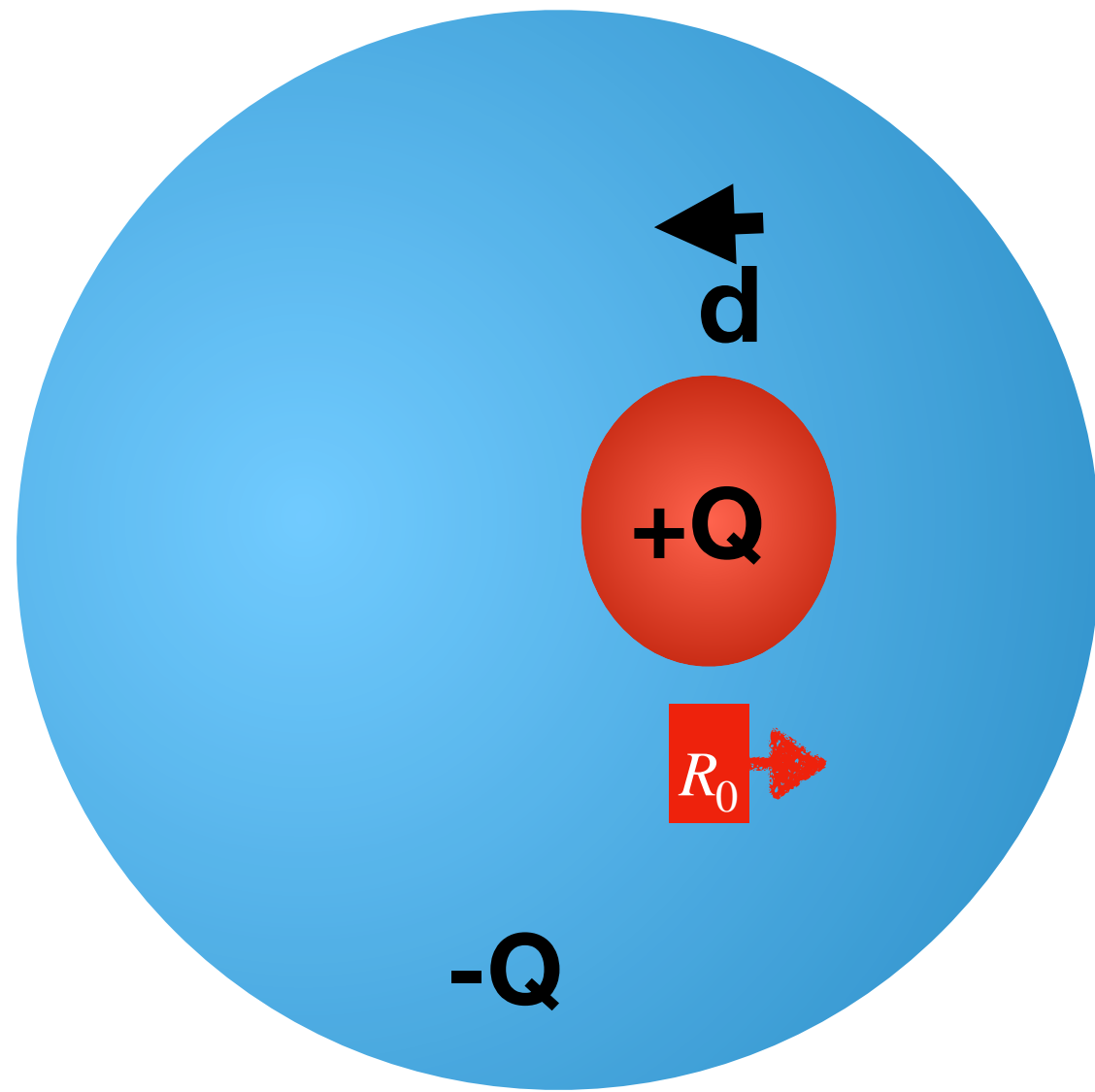


C = elastic stiffness tensor

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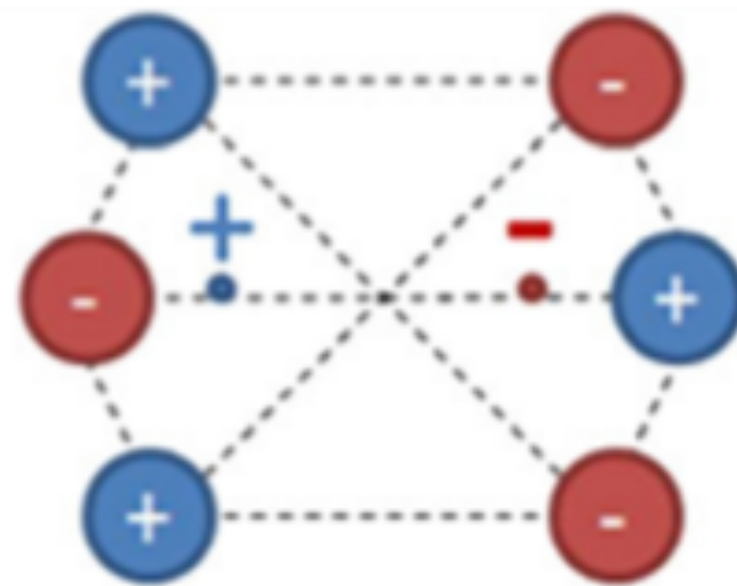
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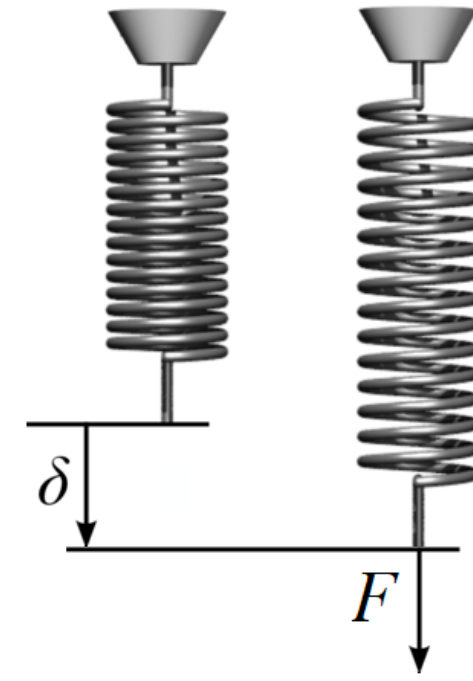
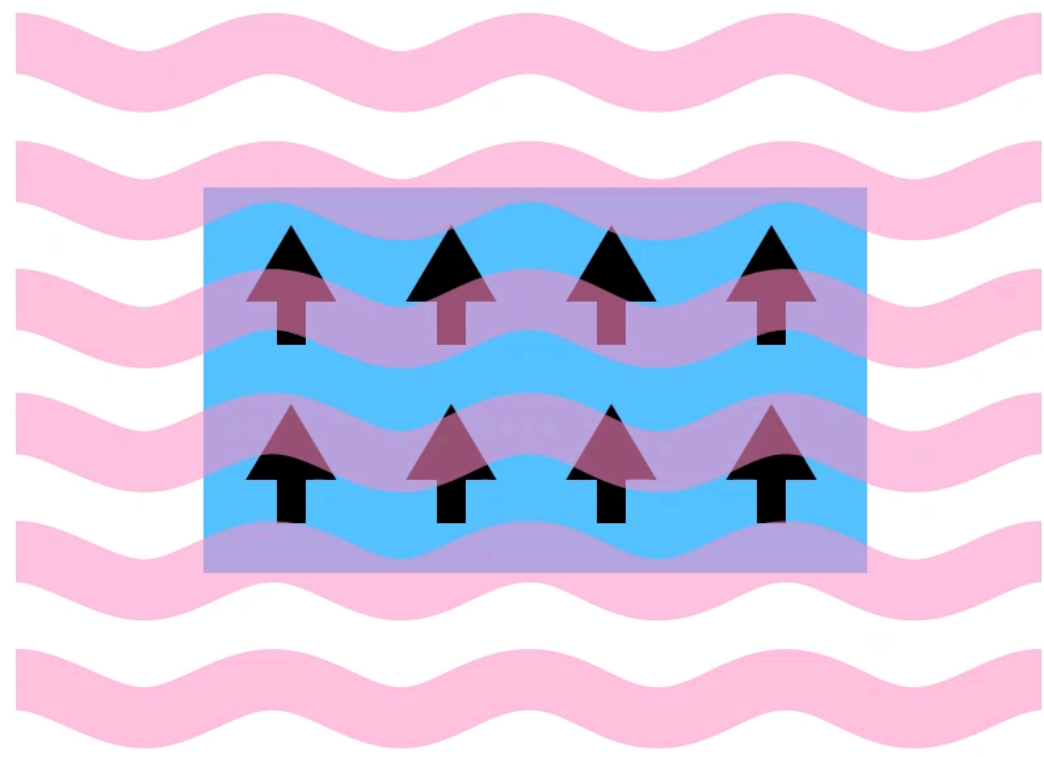


Schiff Moment

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**Piezoelectric
Crystal**



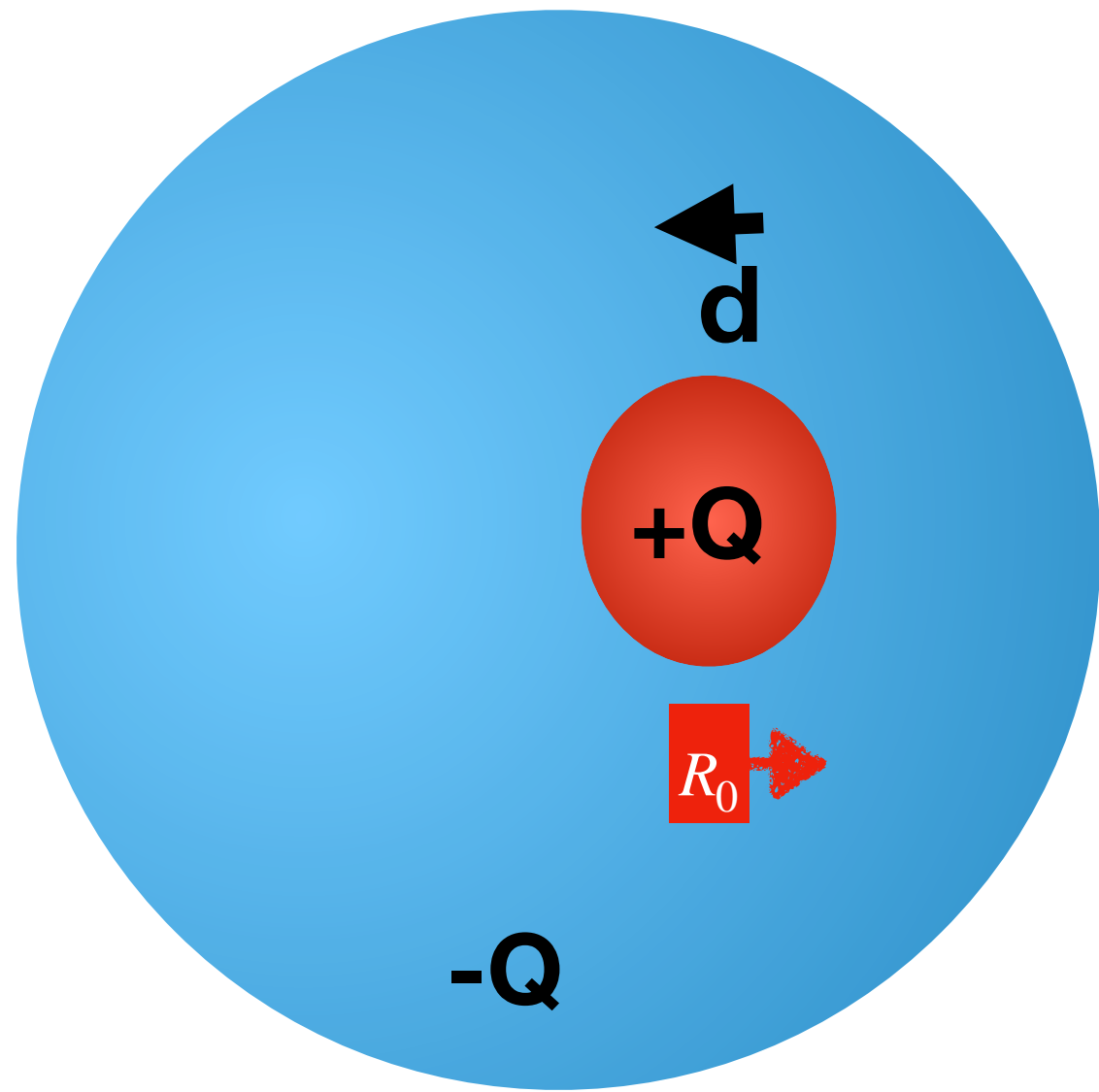
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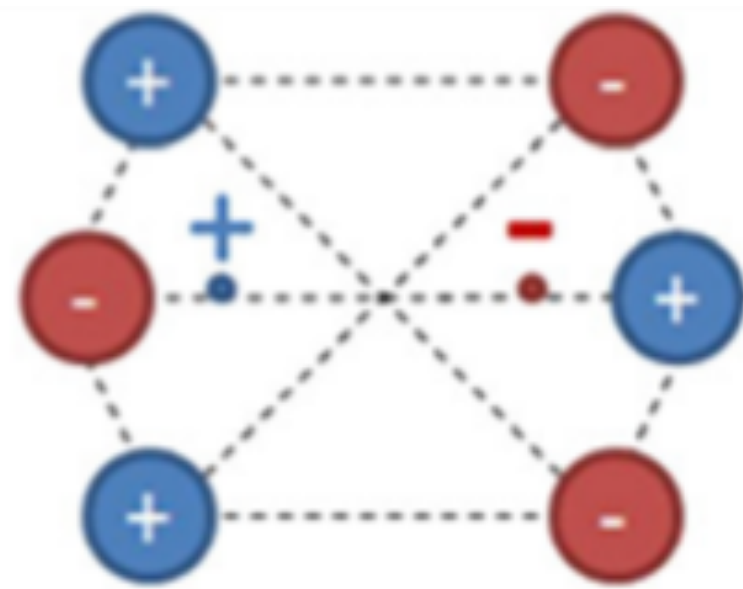
ξ = Piezoaxionic tensor

\hat{I} = nuclear spin direction

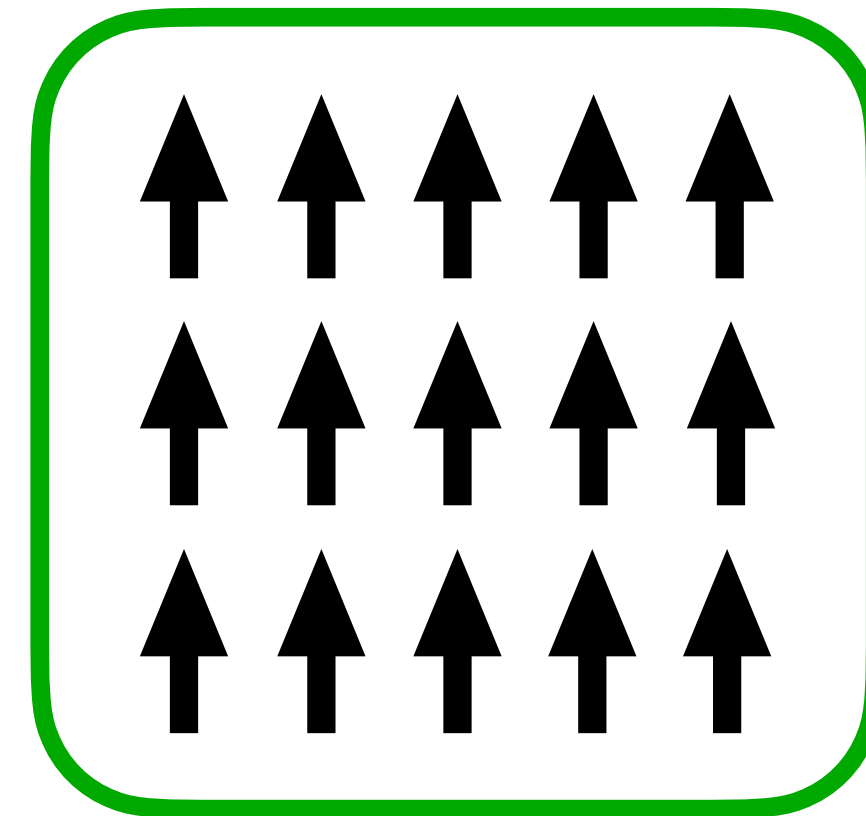


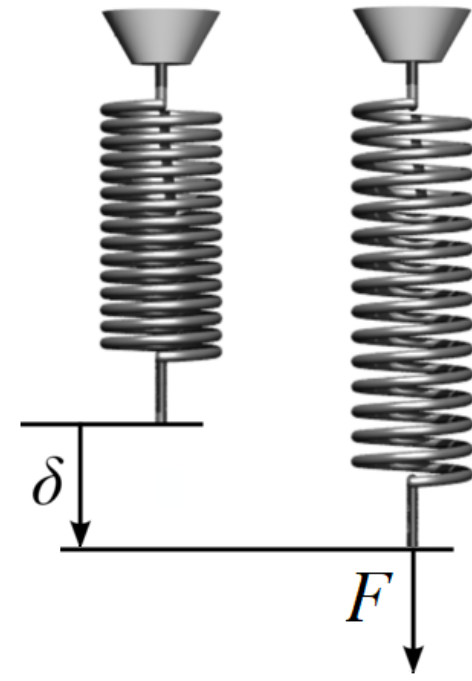
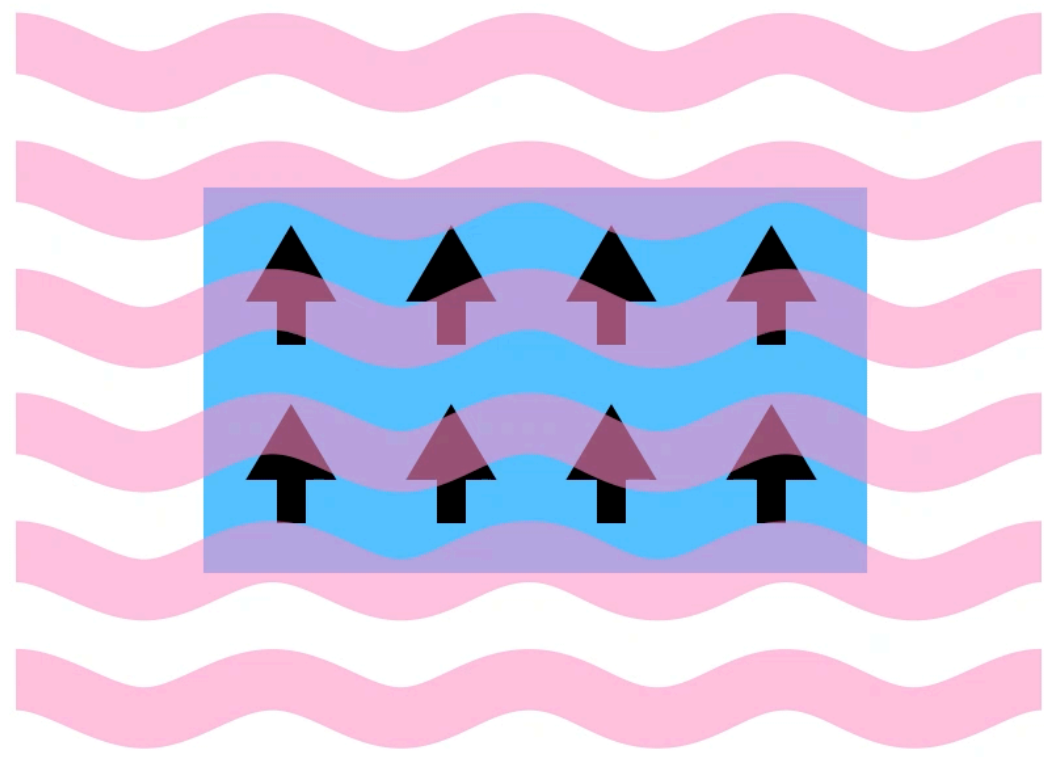
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×



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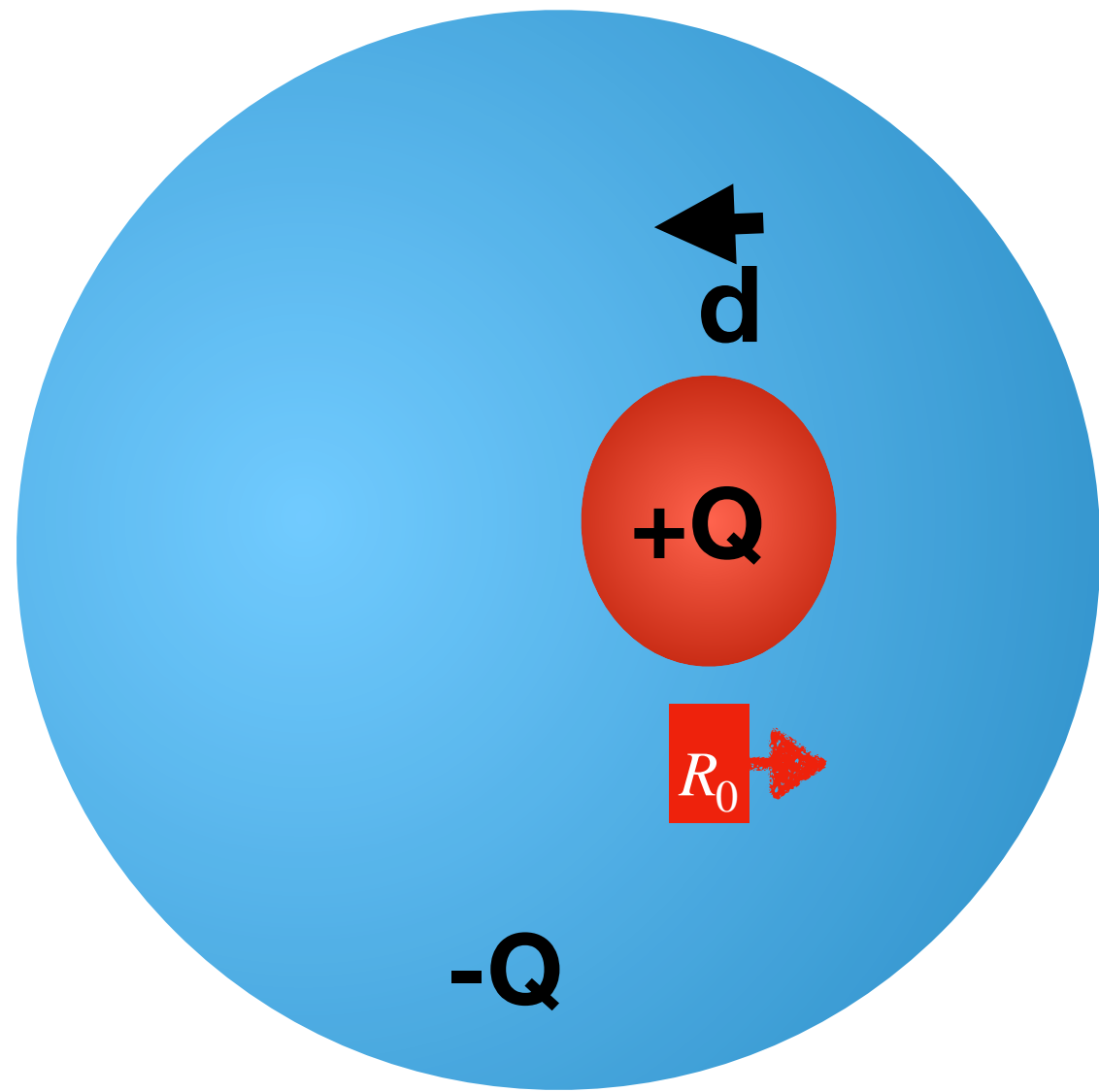
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Axion theta angle $\propto \frac{\sqrt{\rho_a}}{m_a f_a}$

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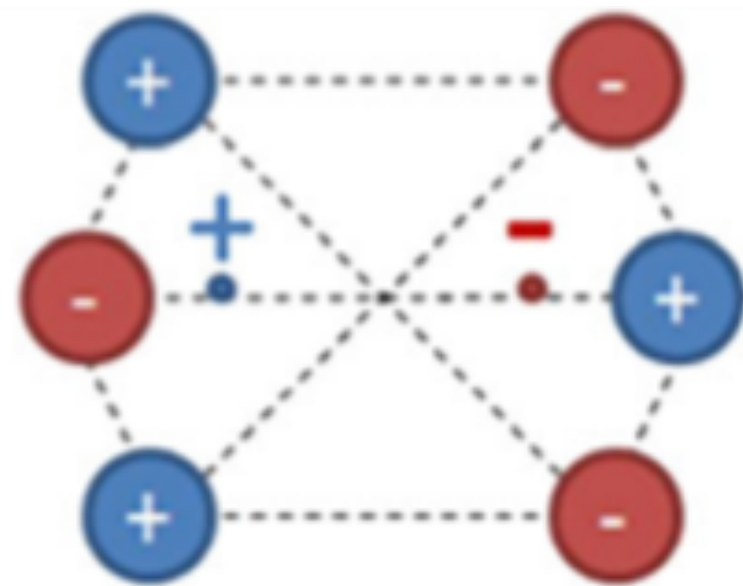
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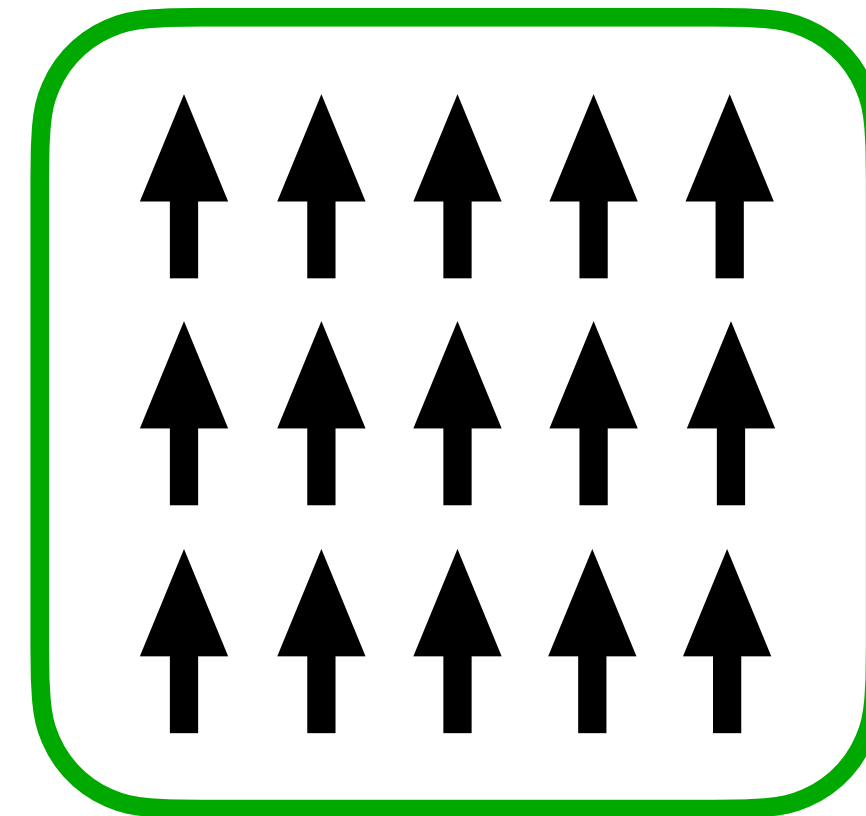


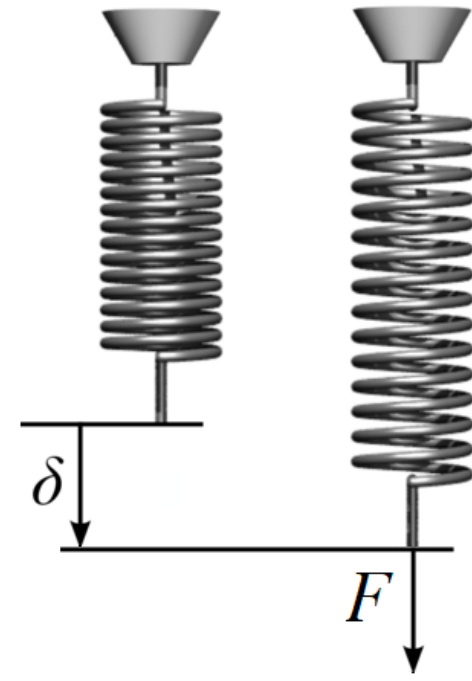
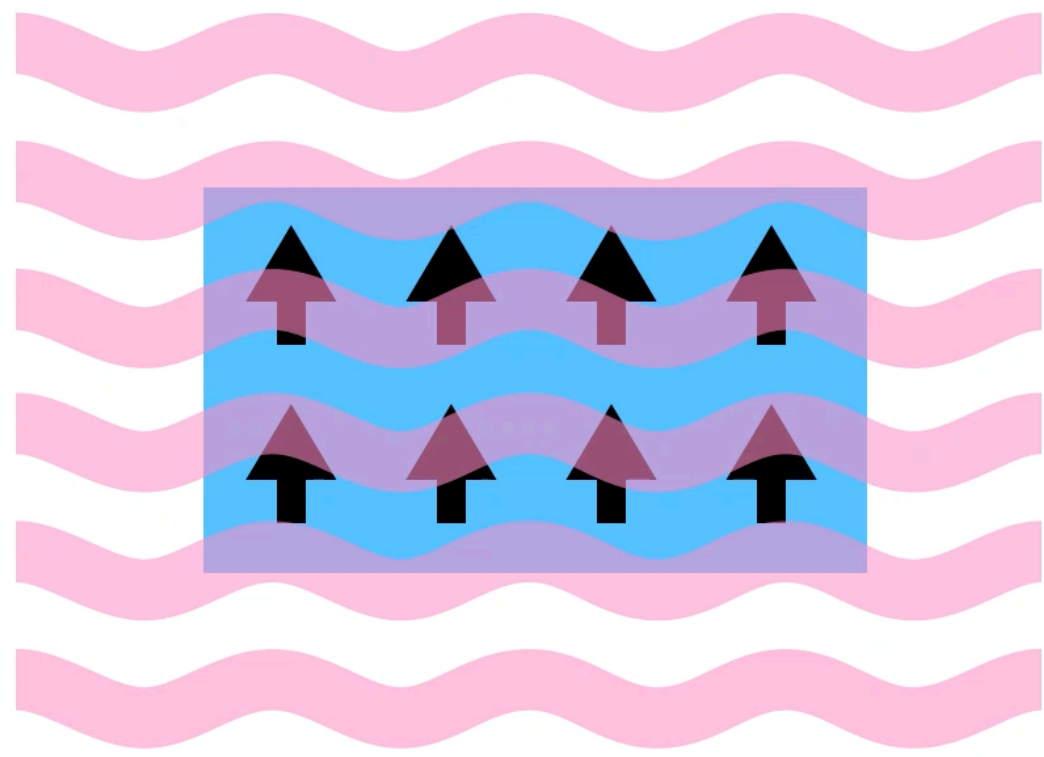
Schiff Moment

\times



Piezoelectric Crystal





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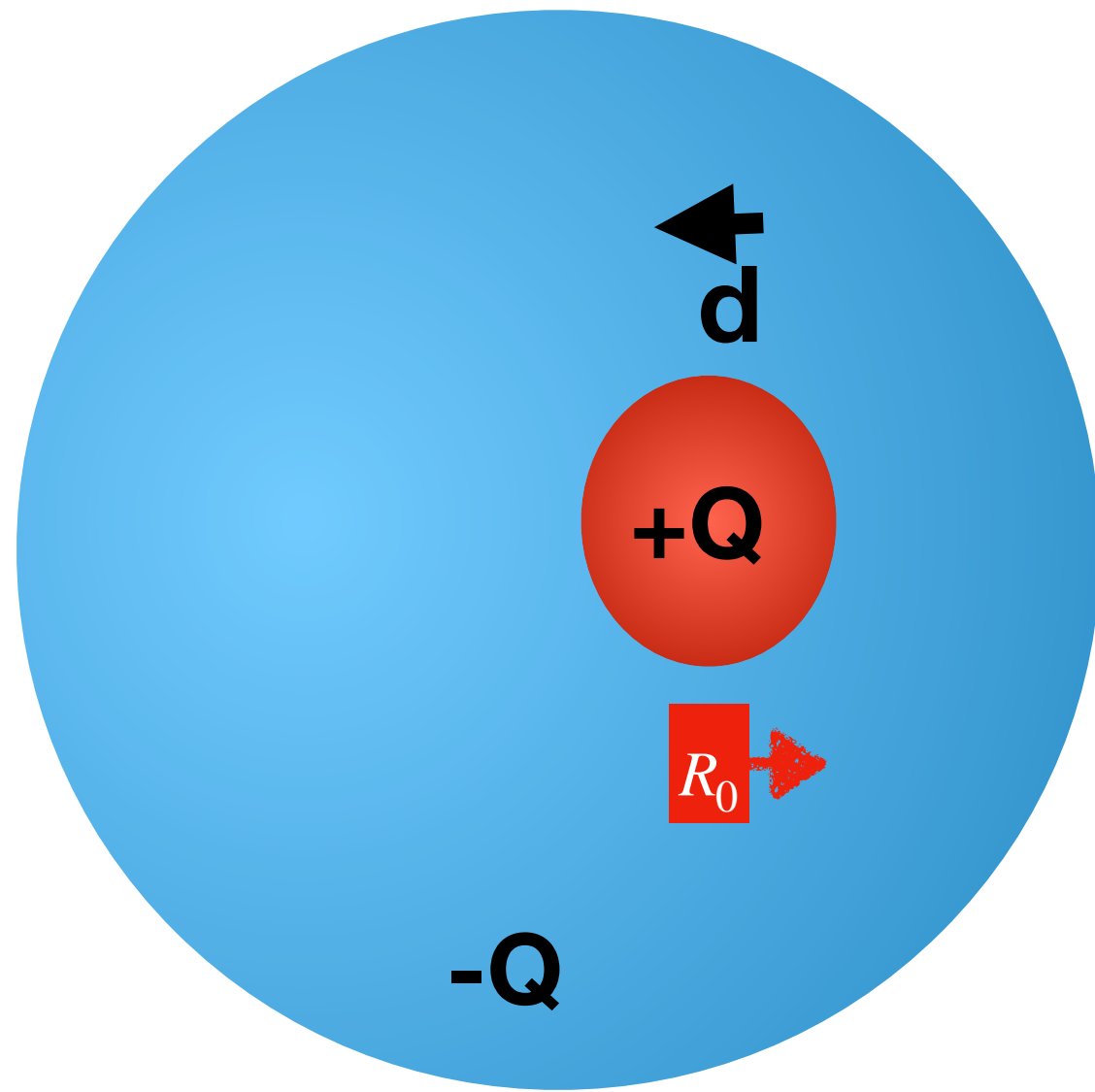
$$S = \text{strain} = \frac{\Delta L}{L}$$

$$S = |\xi c^{-1} \hat{I} \bar{\theta}_a| \sim 10^{-26}$$

Axion theta angle $\propto \frac{\sqrt{\rho_a}}{m_a f_a}$

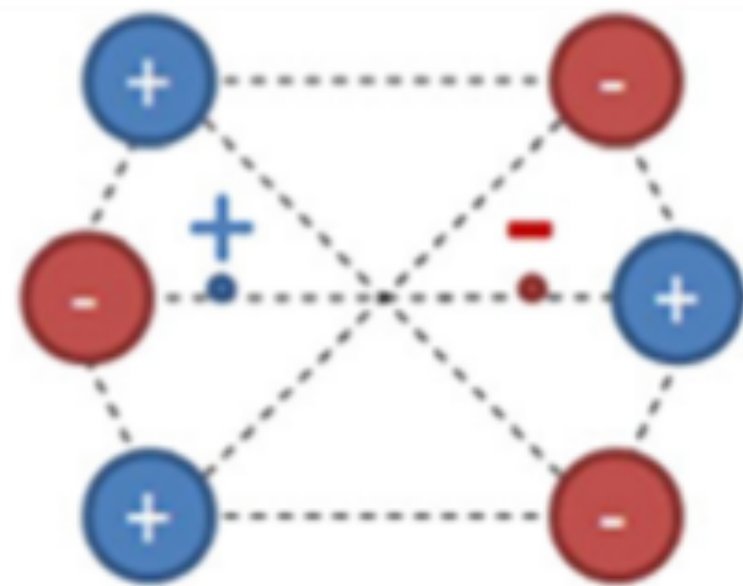
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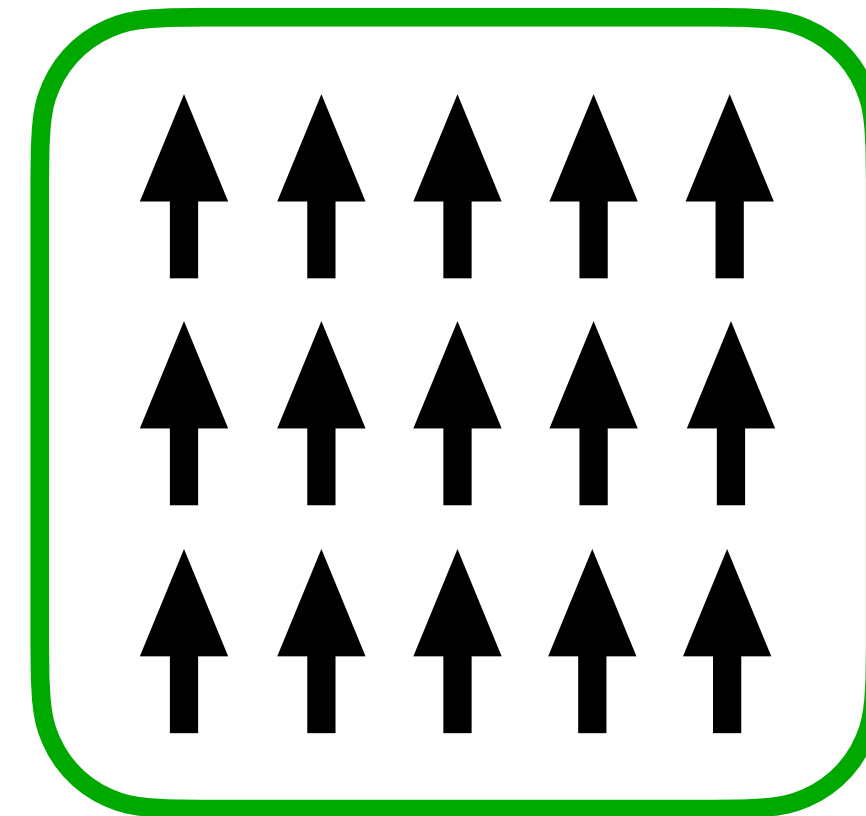


Schiff Moment

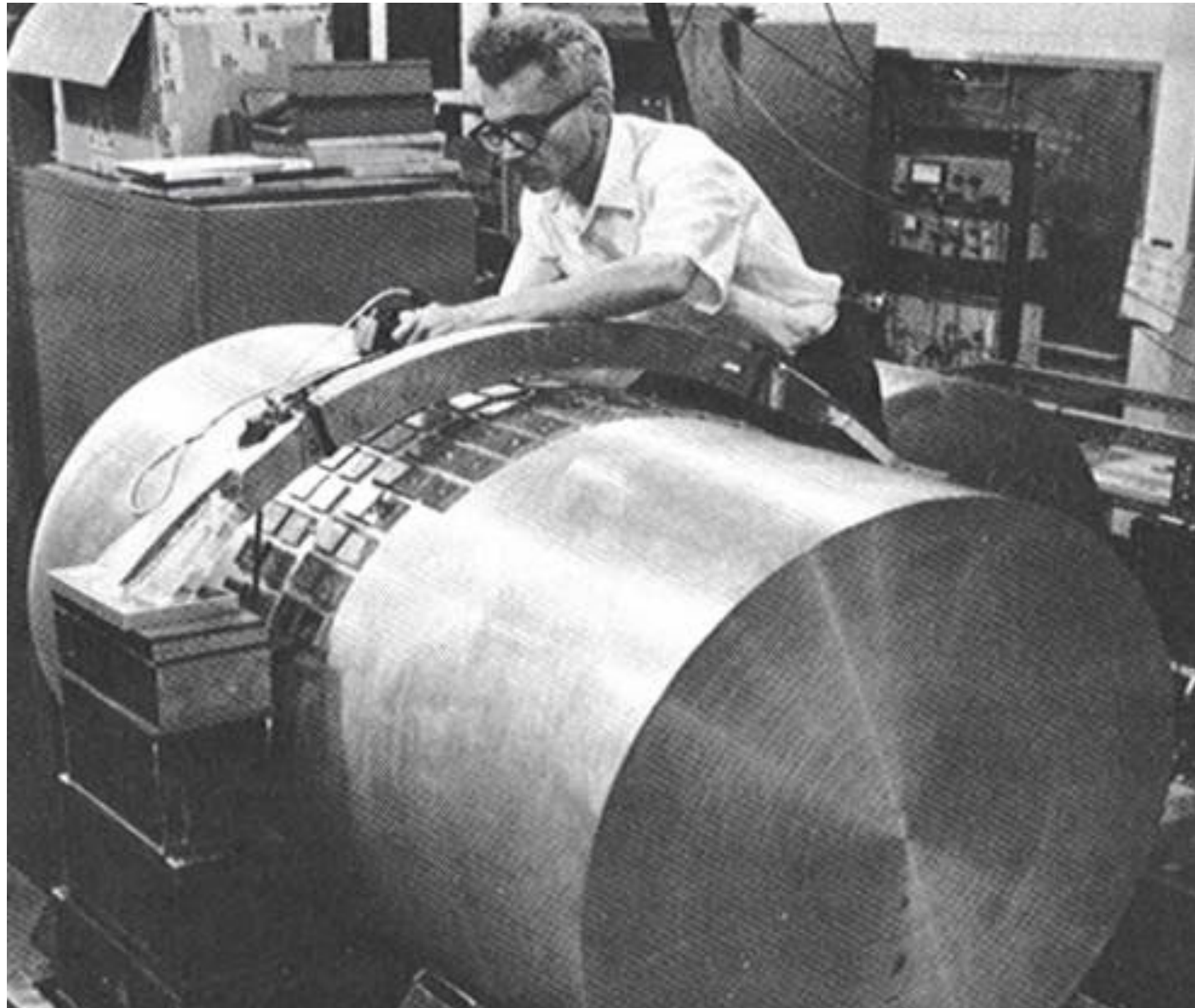
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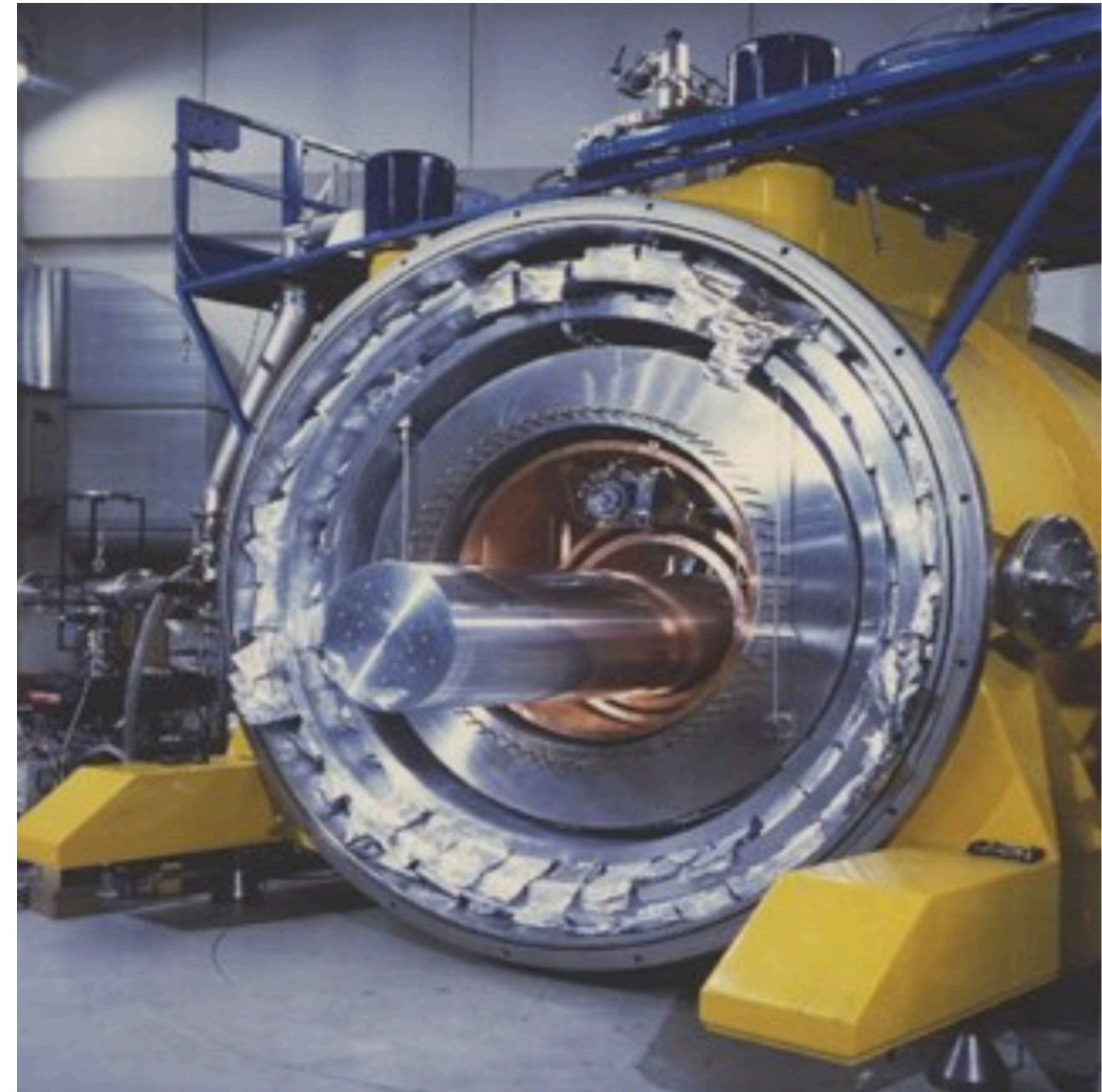
Piezoelectric Crystal



Resonant Mass Detectors



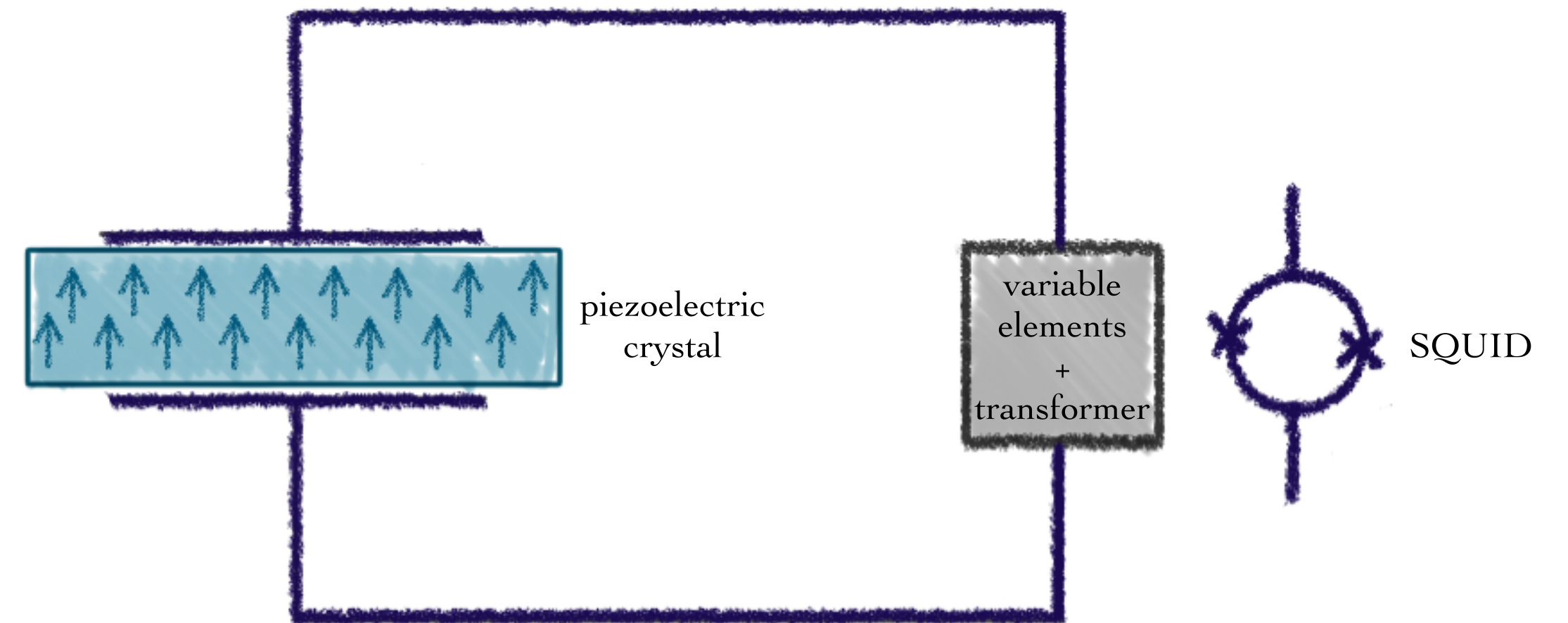
In the 1960's: The Weber Bar,
 $S \sim 10^{-17}$



Today: AURIGA, NAUTILUS,
MiniGrail, $S \sim 10^{-25}$

Experimental Setup

1. Find a piezoelectric material with low mechanical noise and big Schiff moments
2. Align nuclear spins using a magnetic field
3. Cool to $\sim 1 \text{ mK}$ to reduce thermal noise
4. Oscillating voltage across crystal generates a tiny AC, measured using a SQUID



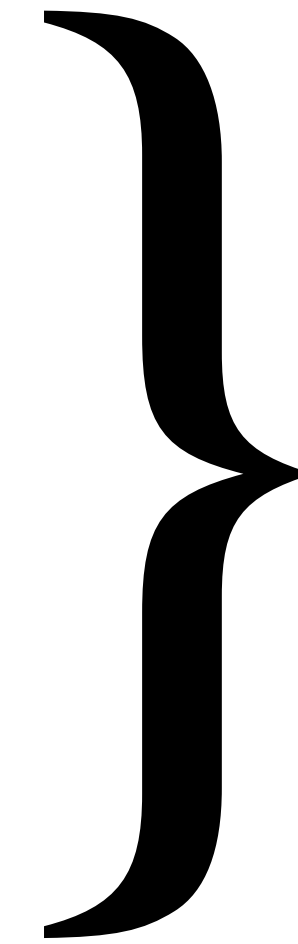
Backgrounds:

Fluctuating nuclear spins

Small effect, NMR frequency \neq
mechanical resonance frequency

Fluctuating magnetic impurities in
material

\lesssim ppm



Magnetization noise
→ fictitious EMF

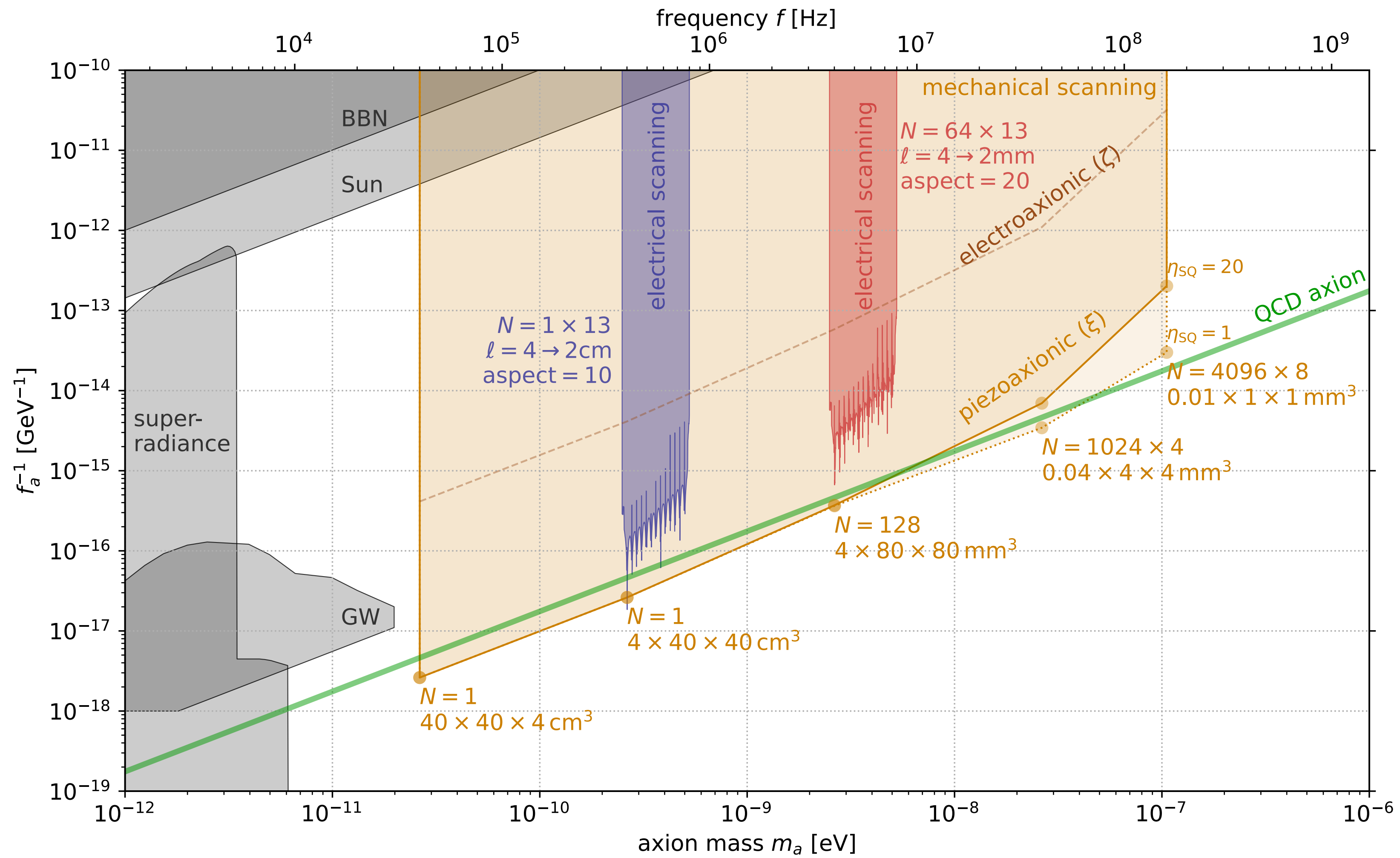
Vibrational noise

Systematic, demonstrated at AURIGA

Noise:

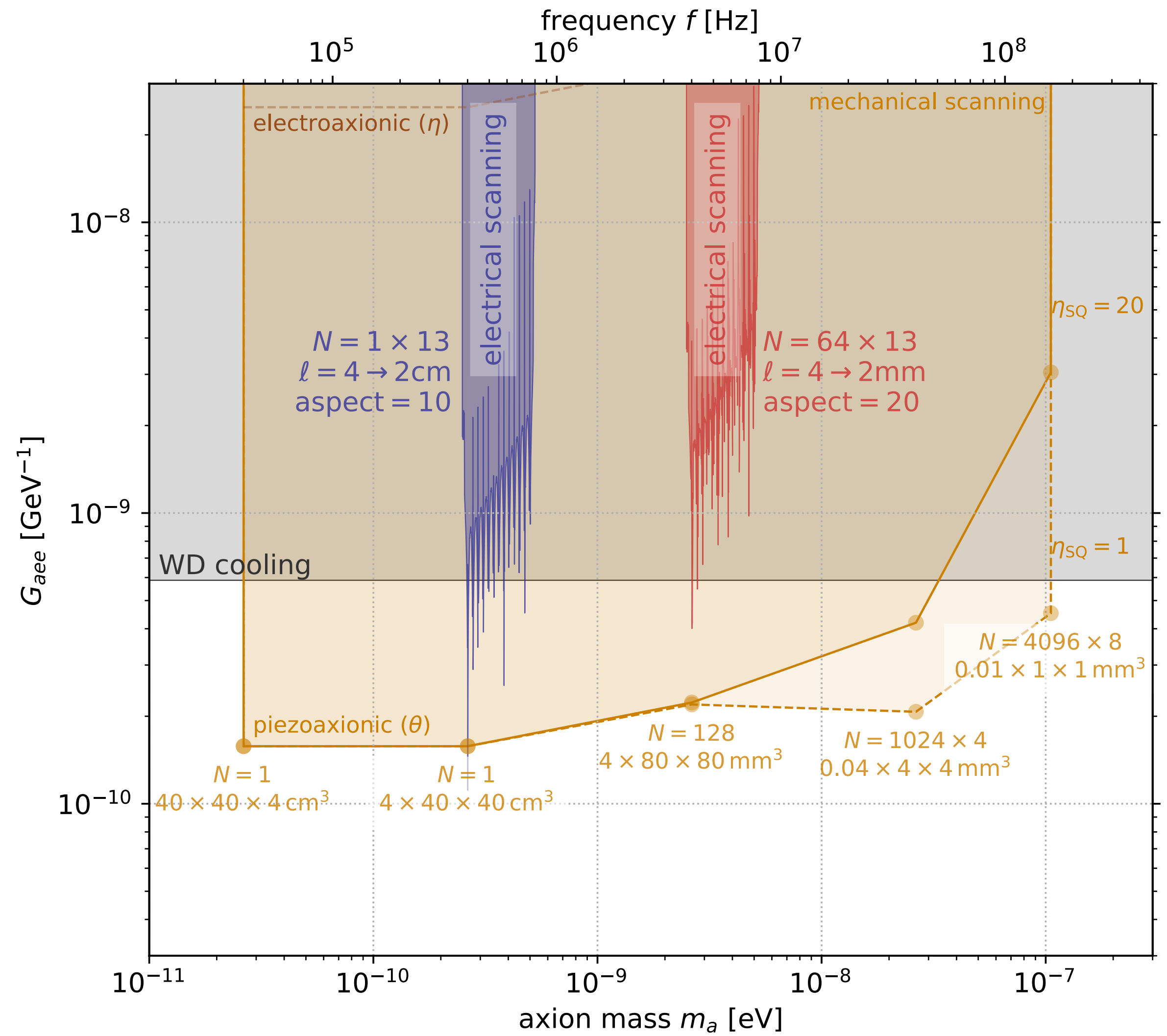
Thermal noise limited, main sources: crystal mechanical noise and SQUID noise

Idealized Forecast



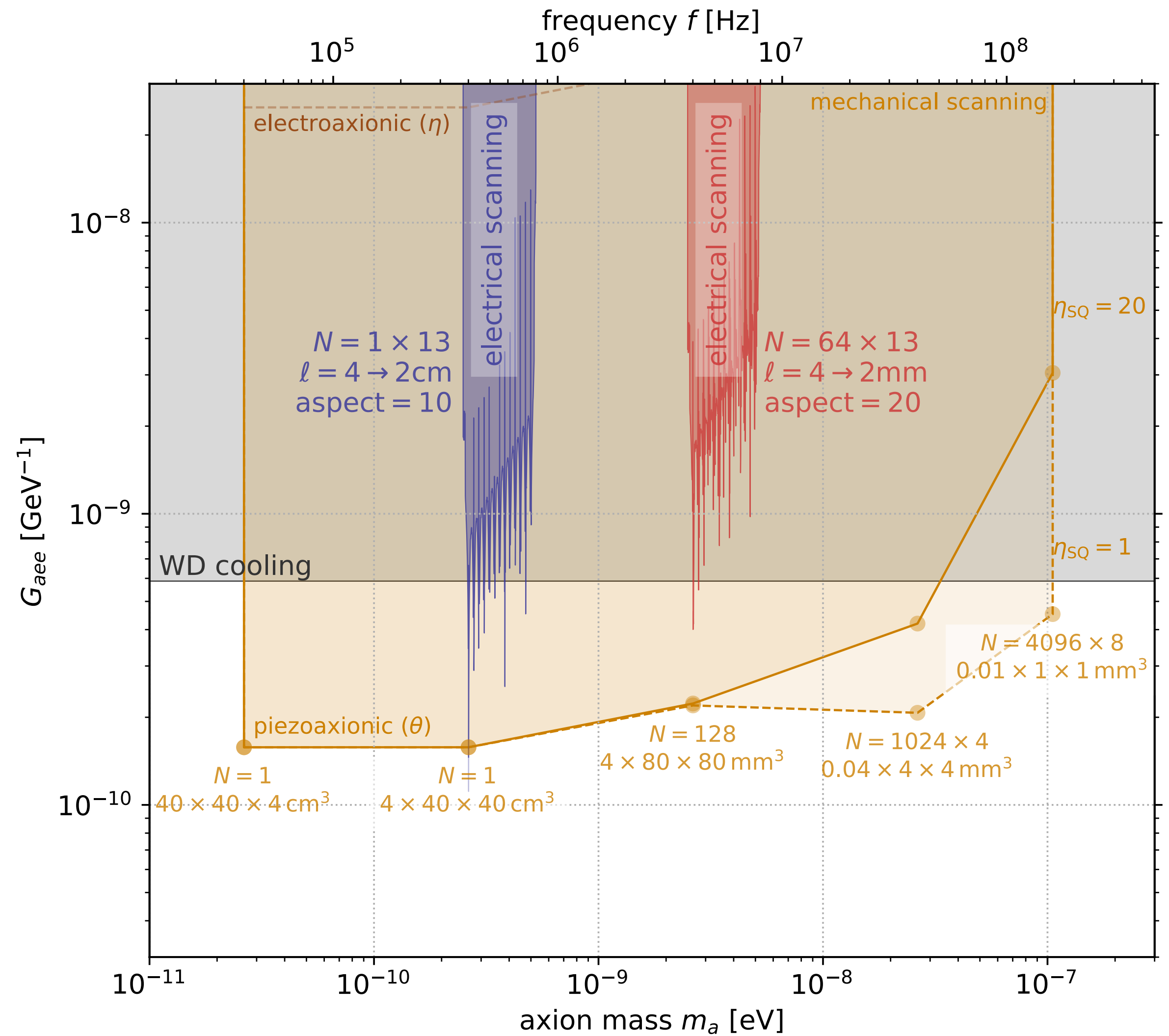
<https://github.com/kenvantilburg/piezoaxionic-effect>

Axion-Electron Coupling



Axion-Electron Coupling

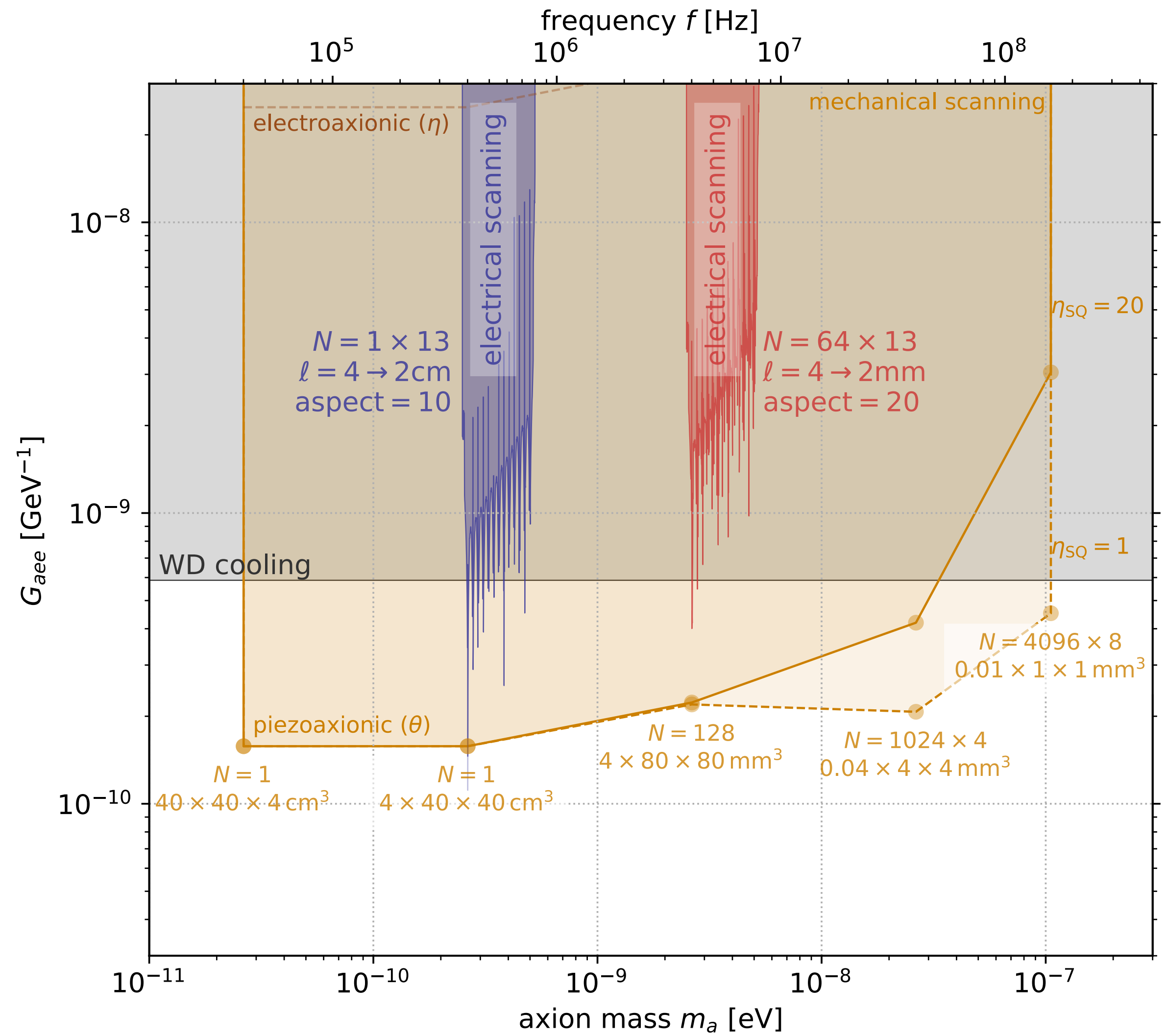
$$H_f \simeq -\frac{G_{aff}}{2} \sigma_f \cdot \left(\nabla a + \dot{a} \frac{\mathbf{p}_f}{m_f} \right)$$



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P EVEN
T ODD
P ODD
T EVEN



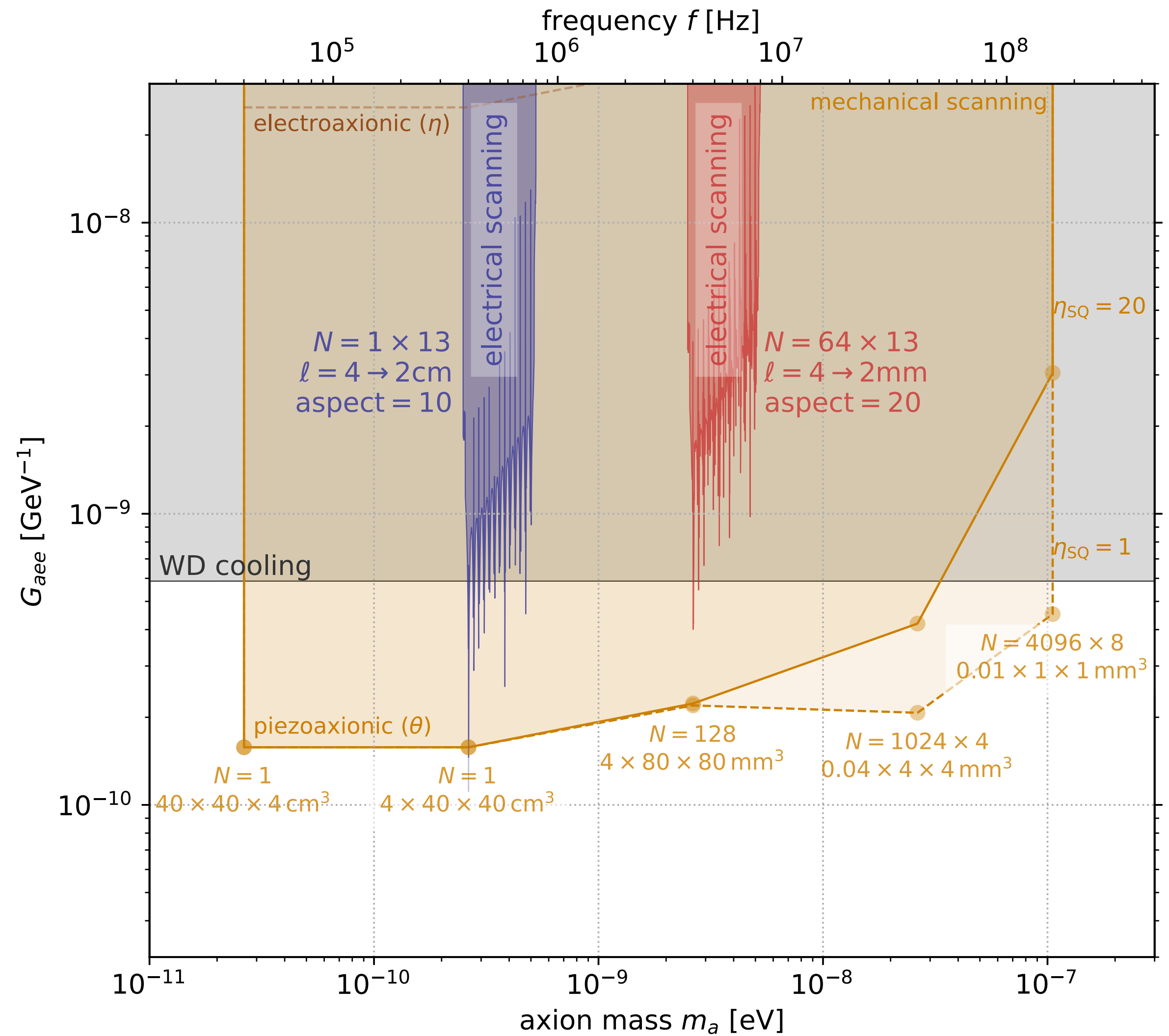
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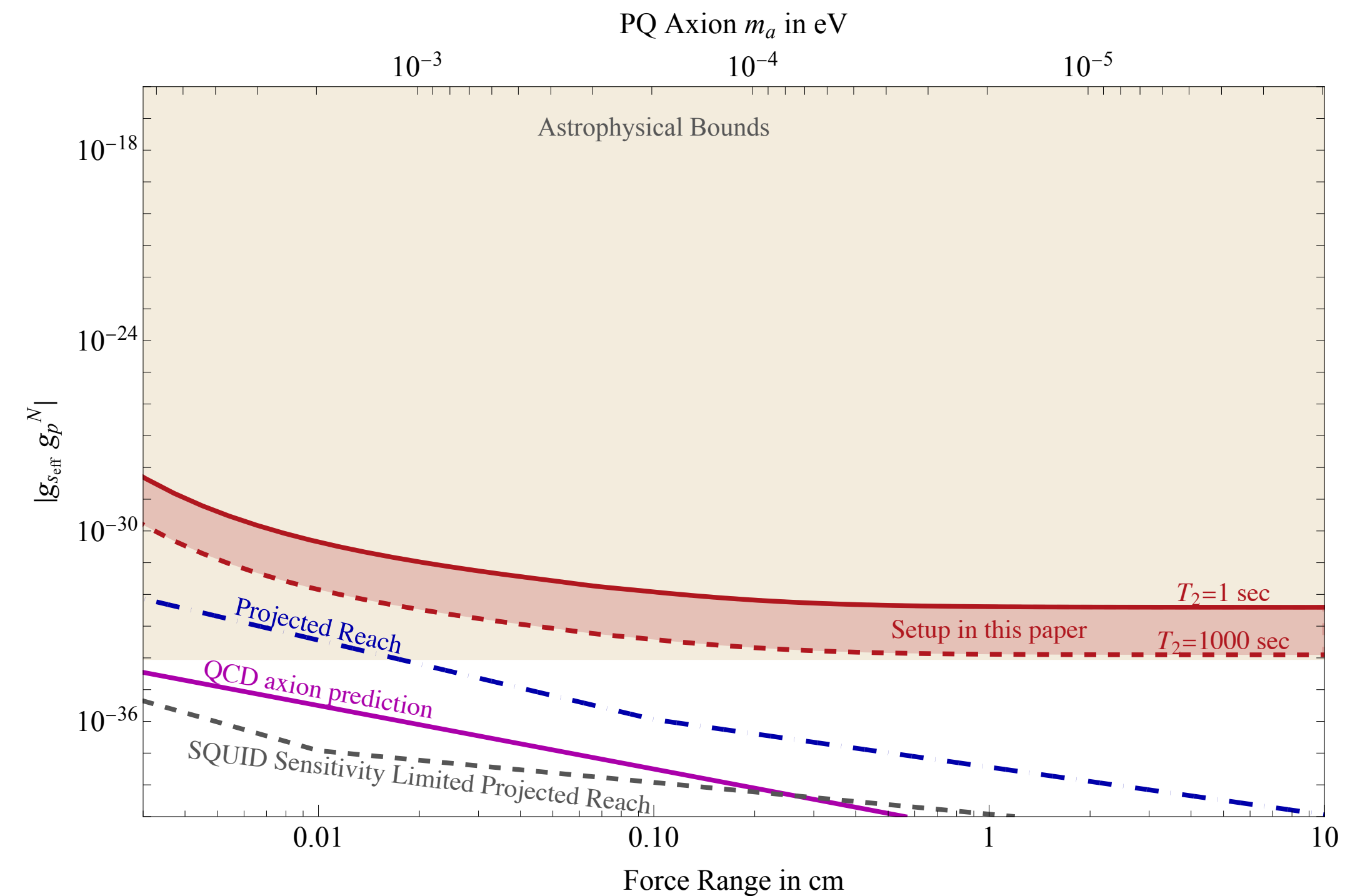
P ODD
T EVEN

$$U \supset \sum_e \frac{\langle H_e \rangle}{V_c} = -\frac{G_{aee} \dot{a}}{2V_c} \sum_e \left\langle \psi_e \left| \frac{\mathbf{p}_e \cdot \sigma_e}{m_e} \right| \psi_e \right\rangle$$



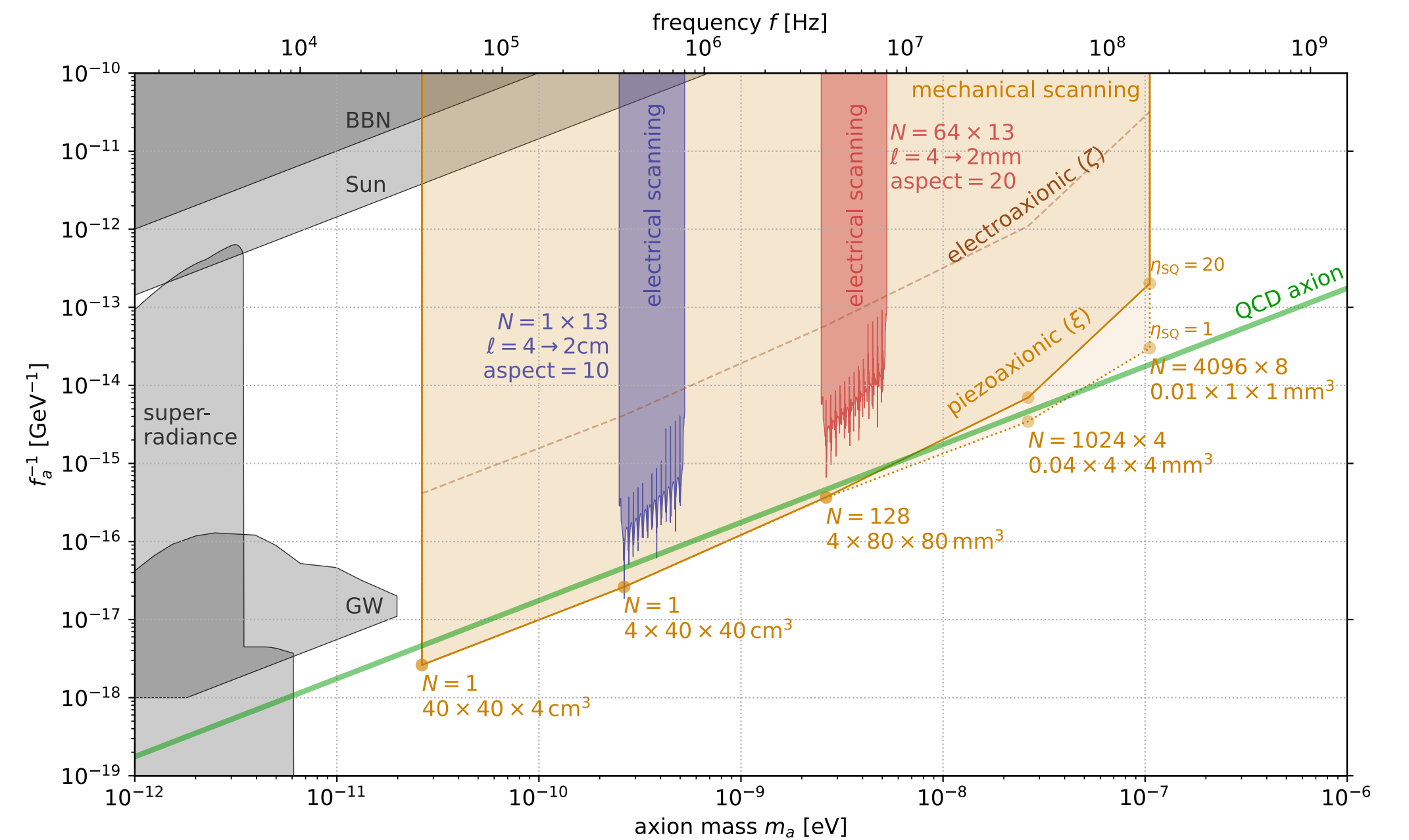
Future Directions

- Precise Schiff moment calculations for stable, octupole deformed nuclei
- Density functional theory (DFT) calculations for ξ and ζ
- Further experimental investigation into suitable materials.
- Axion mediated force experiments from piezoelectric sources (figure)



Summary

- New observable for the QCD axion that probes its model independent coupling
- Experimental set-up with sensitivity for axion masses in range $10^{-11} eV$ to $10^{-7} eV$
- Complementary to cavity experiments



Materials

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- Structural similarity to well-known bulk resonator crystals.

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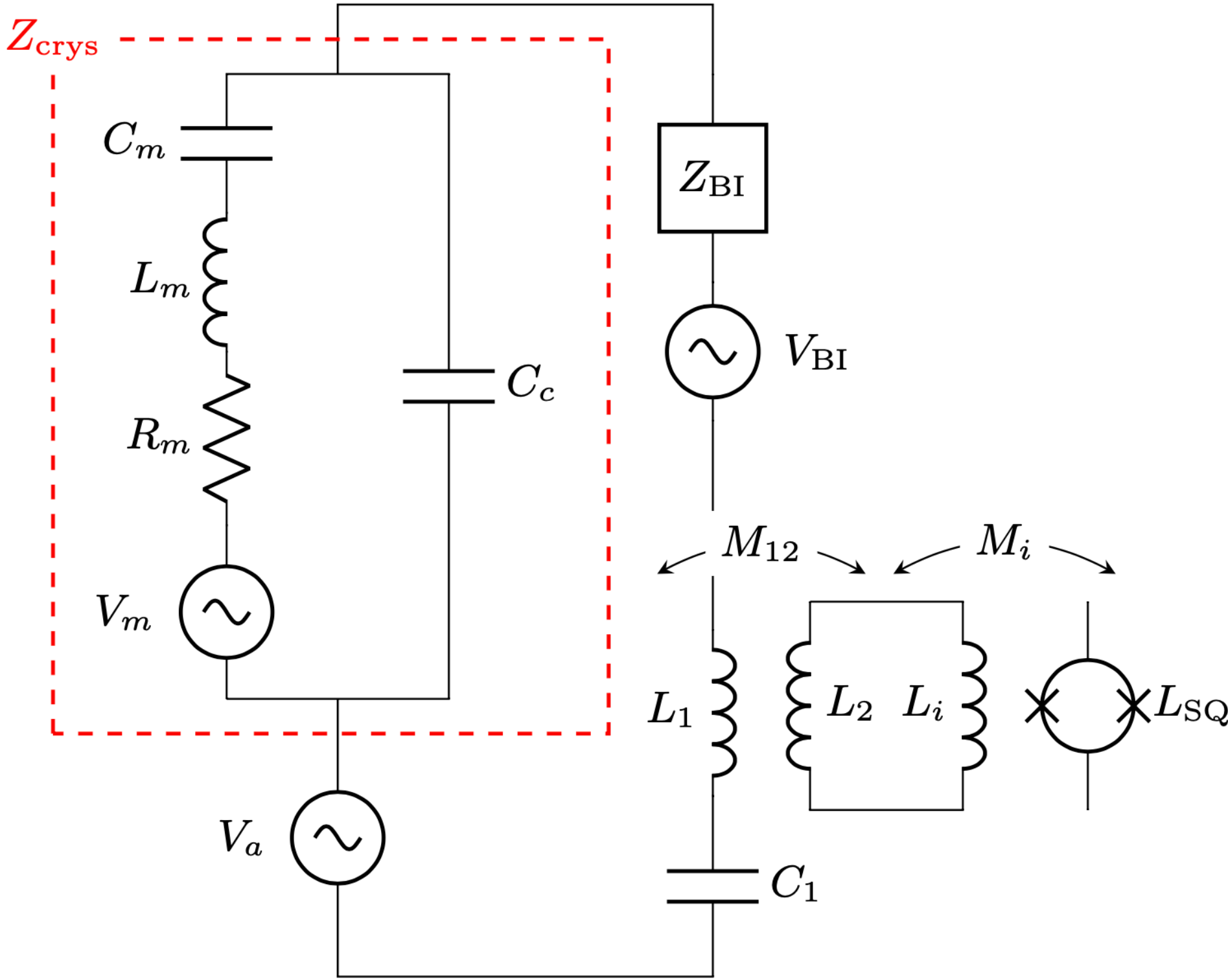
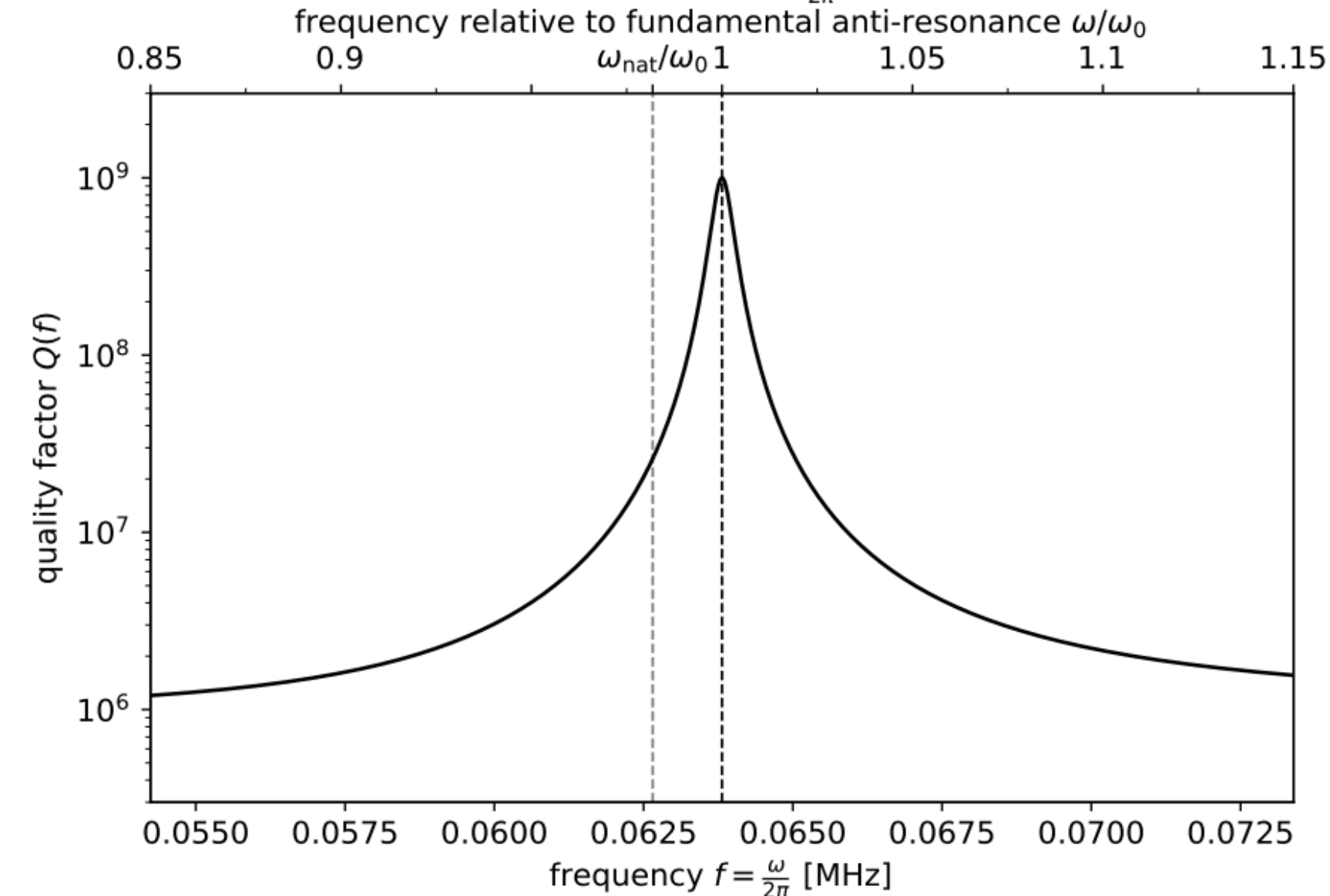
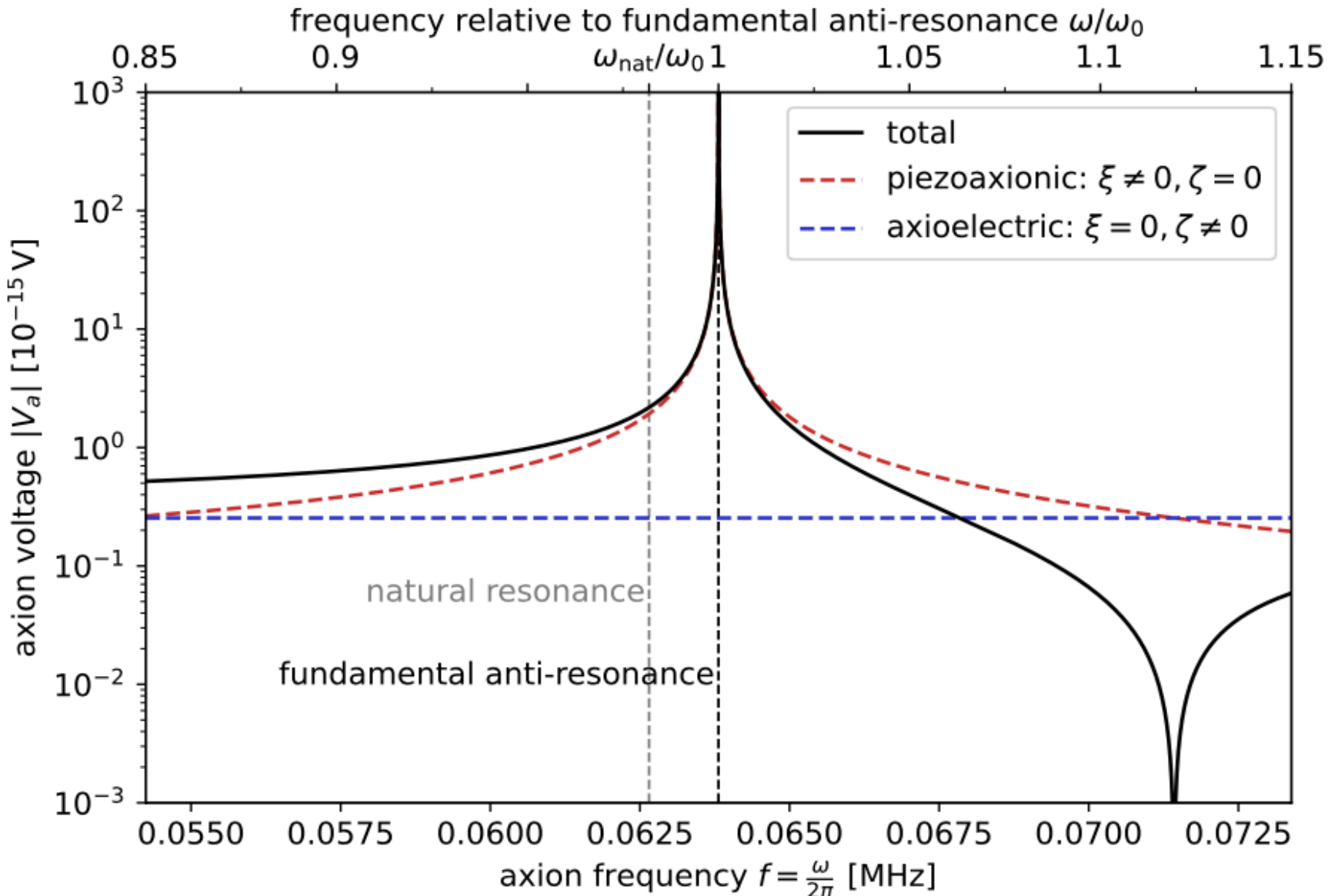
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Class	Candidates	Similar Crystals
32	NaDyH ₂ S ₂ O ₉	SiO ₂ (quartz)
	BiPO ₄	Ga ₅ La ₃ SiO ₁₄ (langasite) GaPO ₄ (gallium orthophosphate)
$\bar{3}m$	U(CuAs) ₂	tourmaline
	Dy ₂ SO ₂ DyOF	LiNbO ₃ (lithium niobate) LiTaO ₃ (lithium tantalate)
4mm	DySi ₃ Ir DyAgSe ₂	Li ₂ B ₄ O ₇ (lithium tetraborate)
$\bar{4}2m$	DyAgTe ₂	NH ₆ PO ₄ (ADP)
	Dy ₂ Be ₂ GeO ₇	KH ₂ PO ₄ (KDP)
mm2	UCO ₅	Ba ₂ NaNb ₅ O ₁₅ (barium sodium niobate)

Database: <https://materialsproject.org/>

Scanning



- Monitor all harmonics of a given mode
- Vary electrical resonance frequency using capacitor and inductor