The Sensitivity of Spin-Precession Axion Experiments

arXiv: 22XX.XXXXX

J.A. Dror, S. Gori, N.L. Rodd

Jacob M. Leedom

17th Patras Workshop, 11.08.2022

DESY.

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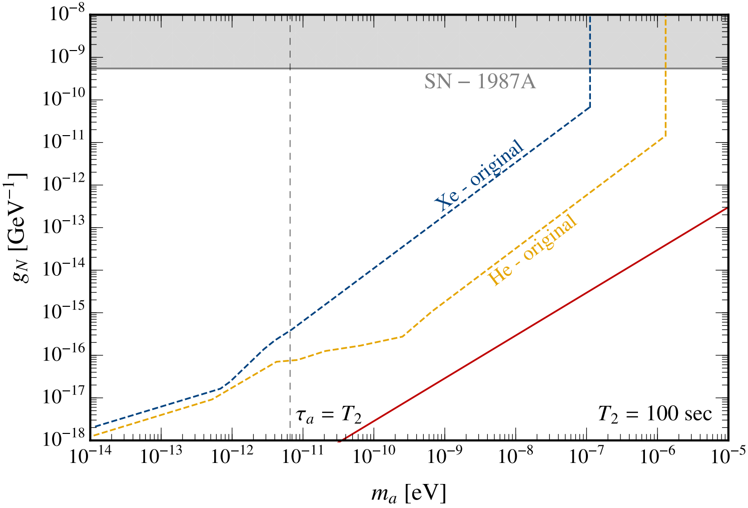
Spoiler Slide



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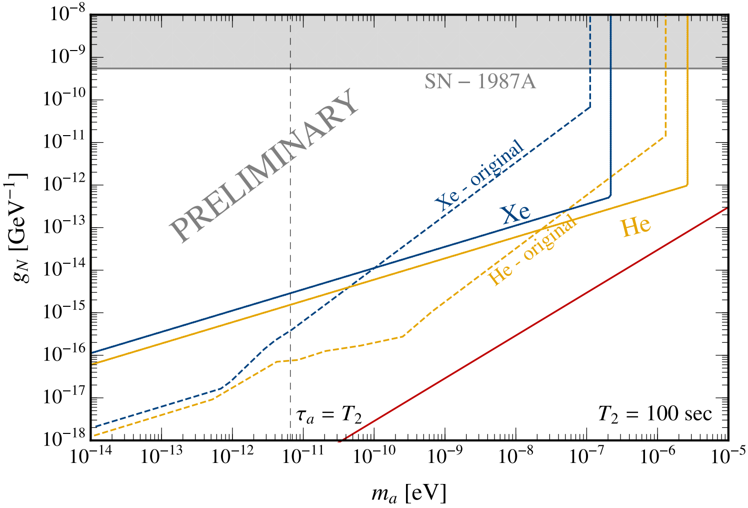

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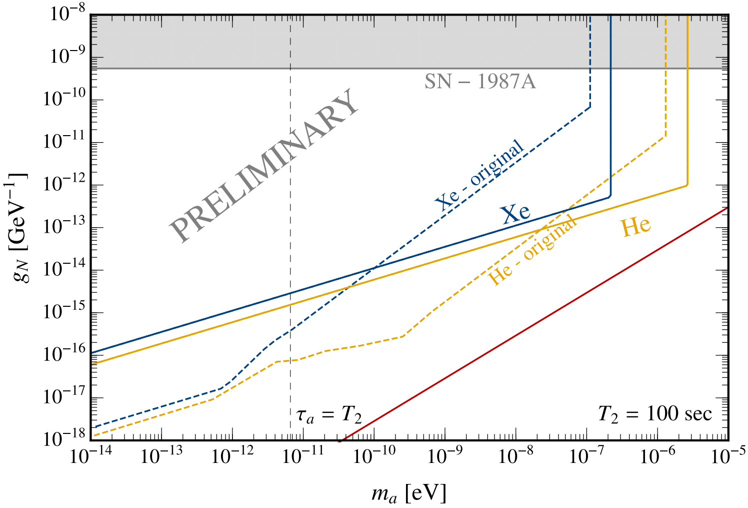

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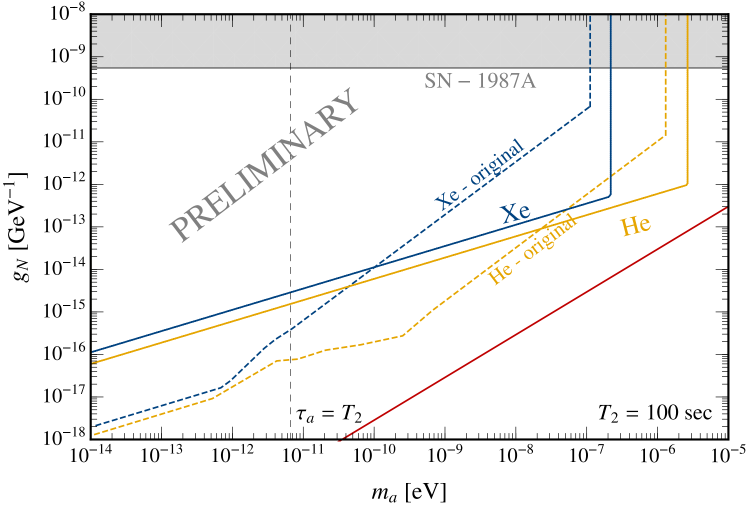

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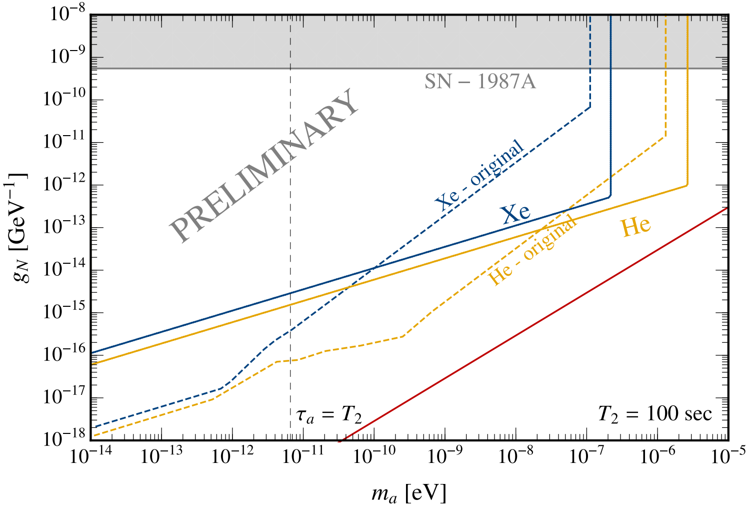

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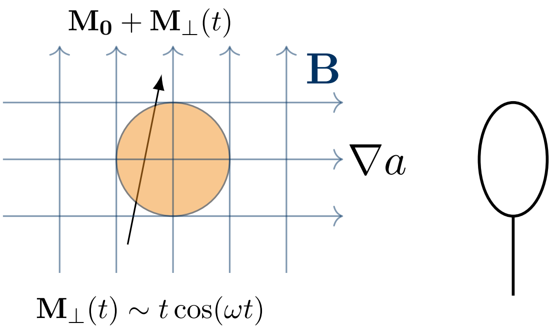
[Graham, Rajendran,'13]
[Budker,Graham, Ledbetter,Rajendran,'14]
[Padniuk,Tandon,Dogan,Ruimi, Walter,Roig,Sushkov,Bekker...]

Axion-Induced NMR: Basics

[See Sushkov's talk from yesterday]



Axion-Induced NMR: Bloch Equations

- One can start from Schrödinger equation for spins to find magnetization....or use the Bloch Equations:

$$\frac{d\mathbf{M}}{dt} = \mathbf{M} \times \gamma \mathbf{B} - \frac{M_x \hat{\mathbf{x}} + M_y \hat{\mathbf{y}}}{T_2} - \frac{(M_z - M_0)\hat{\mathbf{z}}}{T_1}$$

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- Perturbative expansion in g_n & decouple the equations

$$\ddot{M}_x + \frac{2}{T_2}\dot{M}_x + \omega_0^2 M_x = 2 g_N M_0 \omega_0 (\hat{x} \cdot \nabla a) - 2 g_N M_0 \frac{d}{dt}(\hat{y} \cdot \nabla a)$$

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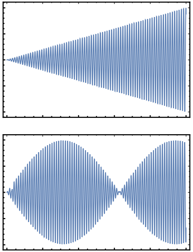

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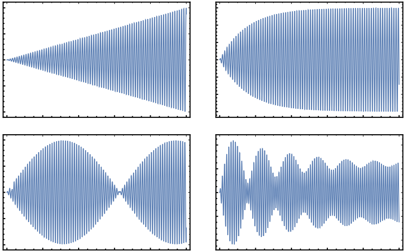

The Original Argument

[Graham, Rajendran,'13]
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- Axion DM field is not just a cosine:

$$a(t) = a_0 \cos(m_a t) \rightarrow \sum_i a_i \cos(\omega_i t + \phi_i)$$

[Related to Hendrik's talk]

one can see the difference after waiting a coherence time $\tau_a \sim v_{DM}^{-2}/m_a \sim 10^6/m_a$

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Claim: Both assumptions need to be revisited

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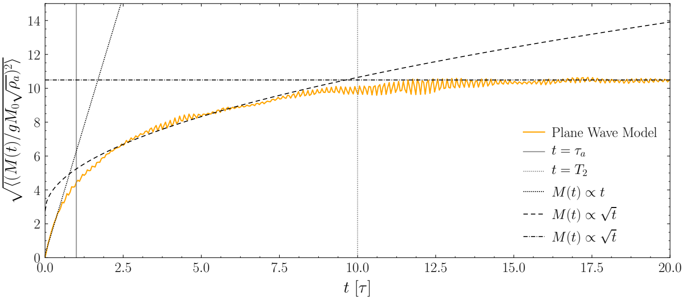

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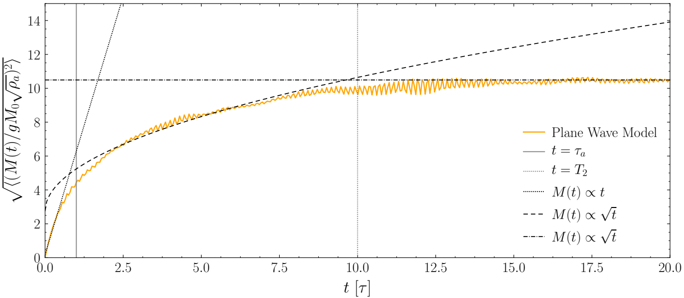

Why?

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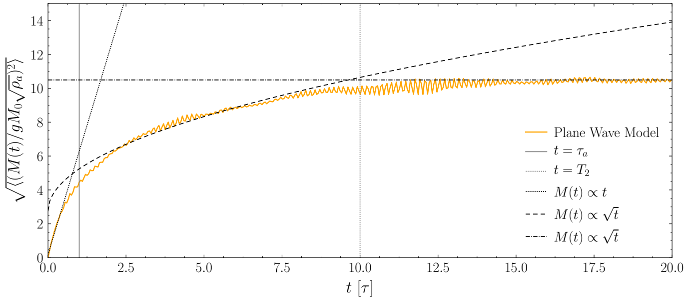

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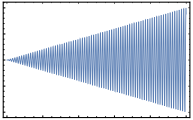

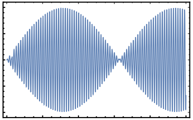

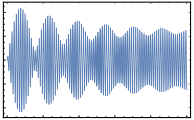

Axion-Induced NMR: Master Equation

- Take Discrete Fourier Transform of Bloch equation:

$$M_x^{(k)} = \sum_{n=0}^{N-1} M_x(n\Delta t) e^{-2\pi i k/N} \qquad T = N\Delta t$$

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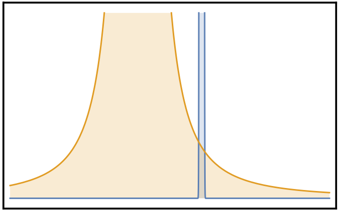

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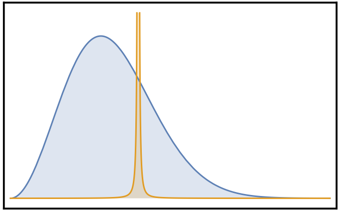

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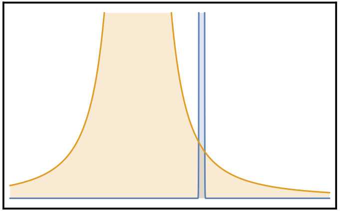

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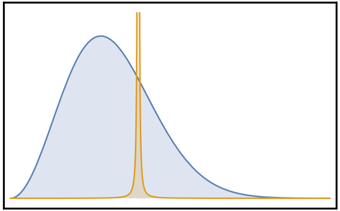

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In both regimes, the PSD is sharply peaked and power is primarily in a single bin

Axion DM Models & Master Equation

$$S_x(\omega) = \begin{cases} \dfrac{\rho_{DM} g_N^2 M_0^2 T_2^2}{m_a} \delta_{k,k_s} & \text{when } \tau_a < T_2 \\ \dfrac{\rho_{DM} g_N^2 M_0^2 v_{DM}^2 T T_2^2}{12} \delta_{k,k_s} & \text{when } T_2 < \tau_a \end{cases}$$

$$T = T_2 \times \max\left[1, \frac{\alpha v_{DM}^{-2}}{m_a T_2}\right]$$

Noise

> Magnetometer Noise:

$$\lambda_M \sim 1.6 \times 10^5 \text{ eV}^{-1}$$

> Spin Projection Noise

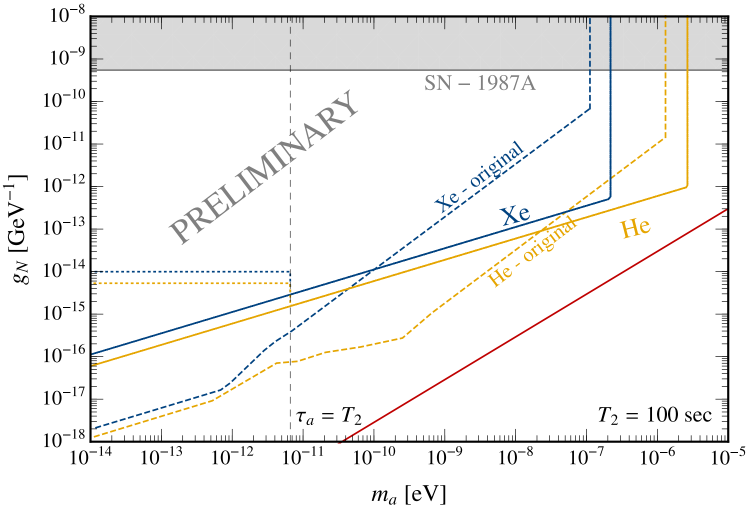
$$\lambda_{SP}(\omega) = \frac{\gamma^2 n}{4\pi V T_2} \frac{1}{T_2^{-2} + (\omega_0 - \omega)^2}$$

[Braun, König,'06]
[Aybas+,'21]

> There is also thermal noise, but we assume the sample is cooled sufficiently that is it subdominant.

> These are fed into a likelihood framework & 95% limits are derived via the Asimov dataset

Conclusions



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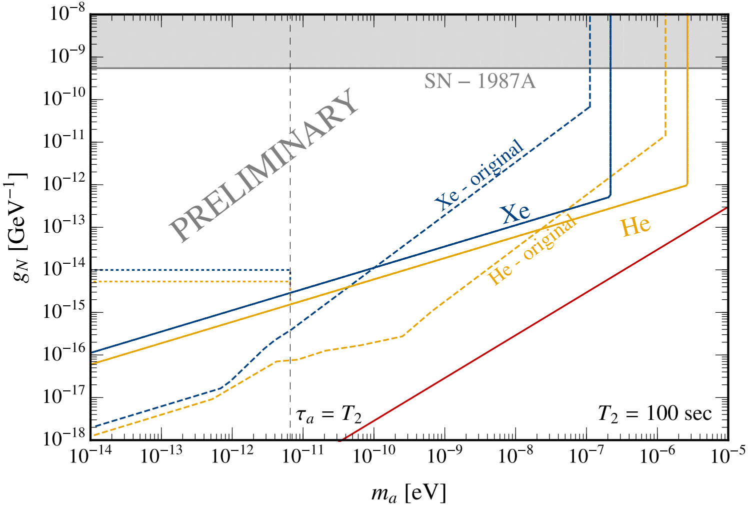

- Effects lead to enhanced sensitivity at higher masses
- Applicable to any experiment utilizing NMR or related techniques. In particular, also true for CASPEr-electric (will be in paper)
- Note: If SP noise can be mitigated, He contour approaches QCD axion line

Thank you!

Contact

DESY. Deutsches
Elektronen-Synchrotron

www.desy.de

Jacob M. Leedom
0000–0003–4911–2188
Theory - Cosmology
jacob.michael.leedom@desy.de
Office O1.142

Back Up Slide: Axion DM Models

- Jumping Phase Model

$$\nabla a(t) = a_0 \vec{k}(t) \cos(\omega_a t + \phi(t))$$

wavenumber $\vec{k}$ & phase ϕ sampled every coherence time $\tau_a \sim Q_a / m_a$

- Plane Wave Model

$$\nabla a(t) = \sum_{i=1}^{N_a} \vec{k}_i a_0^{(i)} \cos(\omega_i t + \phi_i)$$

- If the total sampling time $T << \tau_a$, both models collapse to

$$\nabla a(t) \sim \vec{k} a_0 \cos(\omega_a t + \phi)$$