Axiverse Statistics

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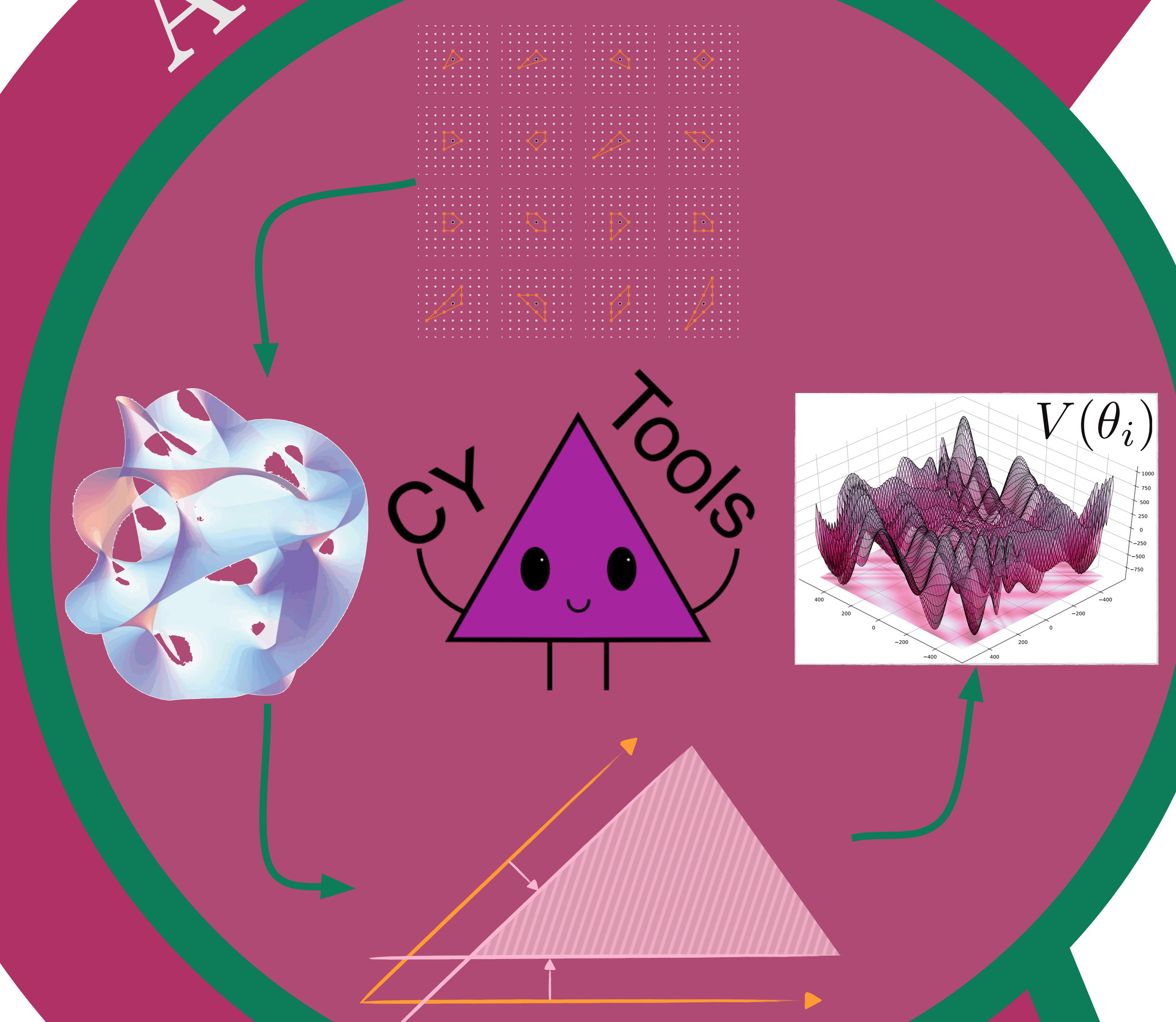
Abstract

We introduce new tools for model builders that allow for the exploration of a corner of the string axiverse, namely axions arising from Kähler moduli of Type IIB string theory compactified on Calabi-Yau 3-folds, $X/\mathcal{O}$. These tools generate axion spectra and give physical statistics for axions derived from geometries originating in the database of 4-dimensional toric varieties: the infamous Kreuzer-Skarke database.

CYTools

juCYAxiverse

Building on previous works, including the computing of spectra in a large ensemble of geometries and the calculation of vacua statistics in random axion systems, we present a new method to calculate the number of vacua in these multi-axion systems.



Axiverse

Using `CYTools` and a wrapper `juCYAxiverse` we are able to generate a variety of axion statistics originating from Type IIB string theory compactified on an orientifolded CY_3.

The multi-axion potentials are generated by ED3-brane instantons wrapping 4-cycles in the geometry, where their energy scales Λ_a^4 are determined by these cycle volumes and charge matrix, $\mathcal{Q}_i^a$, in some chosen basis given by the Kähler metric, K_{ij}.[1]

This results in the axion Lagrangian,

$$\mathcal{L} = -\frac{1}{8\pi^2} M_{\rm pl}^2 K_{ij} g^{\mu\nu}\partial_\mu\theta^i\partial_\nu\theta^j + \sum_{a=1}^{P}\Lambda_a^4\left\{1-\cos\left(\sum_i \mathcal{Q}_i^a\theta^i+\delta^a\right)\right\}$$

where i,j run over the number of Kähler moduli.

From this string data, `juCYAxiverse` then computes the physical axion spectra including: masses, decay constants, quartic couplings, [2] and *vacua statistics.*

`CYTools` accesses our largest known source of Calabi-Yau manifolds, the *Kreuzer-Skarke Database.* Recently [3], an approximation was computed showing the number of possible string geometries in this database is $\sim 10^{400}$. Thus, understanding axion physics arising from such a huge dataset requires both robust algorithms for uniform geometry sampling and highly parallelised code – which the combination of `CYTools` and `juCYAxiverse` are able to deliver.

Interesting inherited properties of these systems are, *e.g.*

- Λ_a^4 - sorted P-length vector of instanton scales with $\Lambda_a^4 \gg \Lambda_b^4$ for $a<b$ where $N \ll P$
- K_{ij} - $N\times N$ non-diagonal Hermitian matrix determining axion dynamics
- $\mathcal{Q}_i^a$ - $P\times N$ instanton charge matrix (integer entries, sparse)
- δ^a - unknown string origin – randomly generated or set to zero

Optima

Understanding vacua statistics in axion potentials can be instructive for a variety of physical applications, *e.g.* inflationary dynamics, CC problem, *etc.* In particular, studying such data in string-derived models can be particularly illuminating due to the large number of geometries now accessible due to technological and methodological progress.

In previous works [4], pseudo-random axion models were generated, however these techniques are unsuitable for our purposes due to

a) the exponential hierarchies in Λ_a^4 b) the sparsity of $\mathcal{Q}_i^a$.

We can impose these properties by defining

$$\varepsilon_j = -\sum_{a=1}^{P-N}\frac{\bar\Lambda_a^4}{\bar\Lambda_j^4}A_a\alpha_j^a\,,\quad \forall j\in\{1,\cdots,N\}$$

$$\text{where } A_a \equiv \sin\left(2\pi\sum_{i=1}^{N}n_i\alpha_a^i\right);\qquad \bar{\boldsymbol{\mathcal{Q}}}_a = \boldsymbol{\alpha}_a\tilde{\boldsymbol{\mathcal{Q}}}^a\,,\quad \forall a\in\{1,\cdots,P-N\}.$$

Here, $\tilde{\mathcal{Q}}_i^a$ are the matrices that define the "leading potential", *i.e.* corresponding to the most dominant, linearly-independent instanton contributions, and $\bar{\mathcal{Q}}_i^a$ are subleading instantons.

If

$$|\varepsilon_i| \ll 1\,,\quad \forall i\in\{1,\cdots,N\}\,,$$

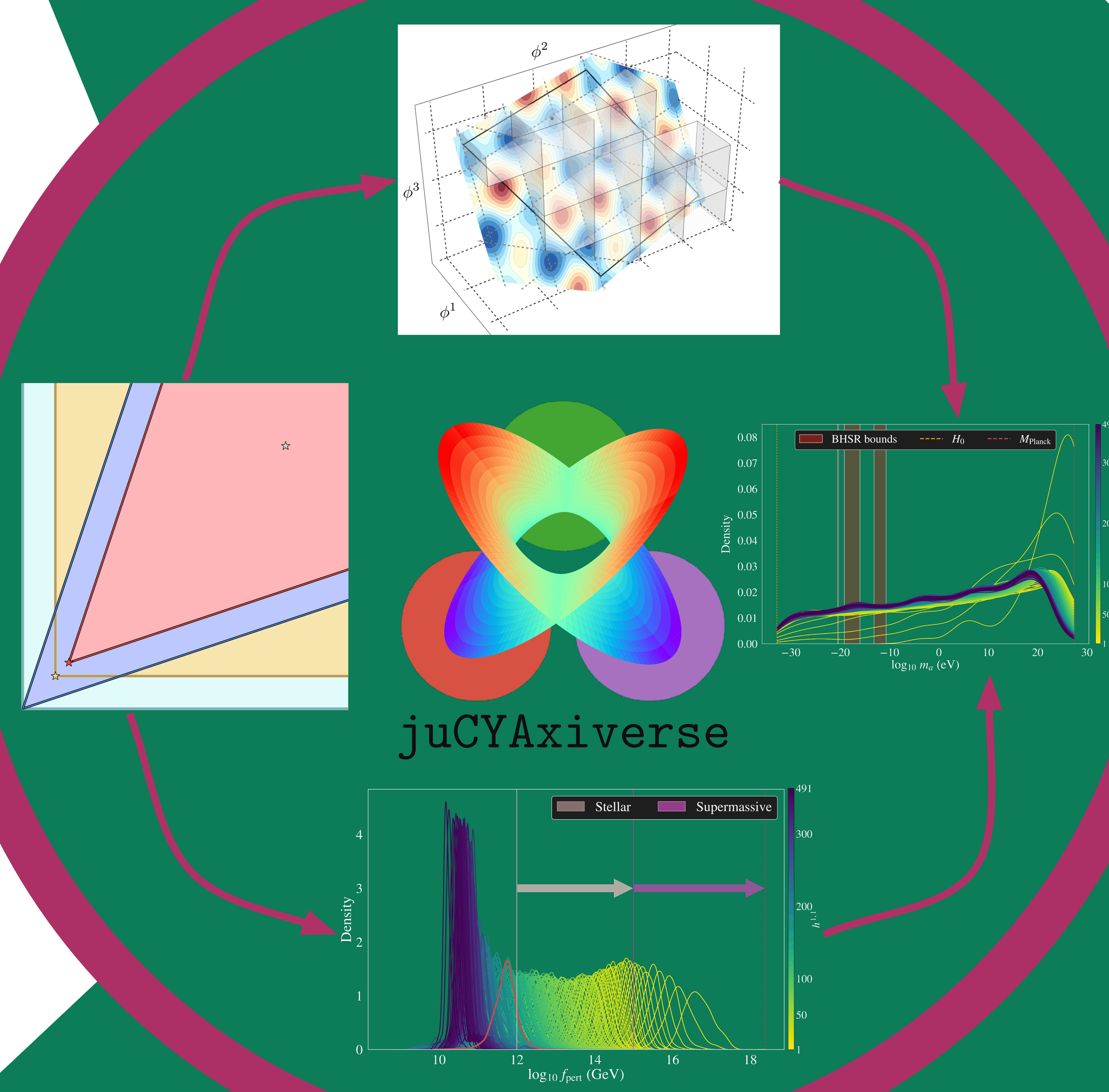
$$\left|\sum_{i=1}^{N}\varepsilon_i\alpha_a^i\right| \ll 1\,,\quad \forall a\in\{1,\cdots,P-N\}\,,$$

$$|\varepsilon_j| \gg \left|\sum_{i=1}^{N}\left(\sum_{a\in\mathcal{S}}\frac{\bar\Lambda_a^4}{\bar\Lambda_j^4}\alpha_a^i\alpha_j^a\right)\varepsilon_i\right|\,,\quad \forall j\in\{1,\cdots,N\},$$

are satisfied then we can show that the number of vacua in such a system is simply given by the ratio of volumes of the fundamental domain of the leading potential and the overall potential,

$$\mathcal{N}_{\rm vac} = \frac{\mathrm{vol}\left(\mathcal{F}_{\rm full}\right)}{\mathrm{vol}\left(\mathcal{F}_{\rm leading}\right)}.$$

In geometries that satisfy these conditions we found there to be very few vacua. However, due to the vast number of geometries in the database, we expect an alternative solution to the CC problem may be found.



Outlook

Developing both technological – *i.e.* code – and analytical –*i.e.* theory – methods gives us access to previously unobtainable structures and phenomena. Accessing string data at these unprecedented scales permits a deeper understanding of the 4d physics, in this case: **axion dynamics**.

Ongoing projects building on these tools include:

- incorporation of experimental constraints
- using ML for landscape exploration
- further axion physics, *e.g.* DM abundance, *etc.*
- applications to geometries beyond CY_3s in KS database

[1] arXiv:1808.01282
[2] arXiv:2103.06812
[3] arXiv:2008.01730
[4] arXiv:1709.01080

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