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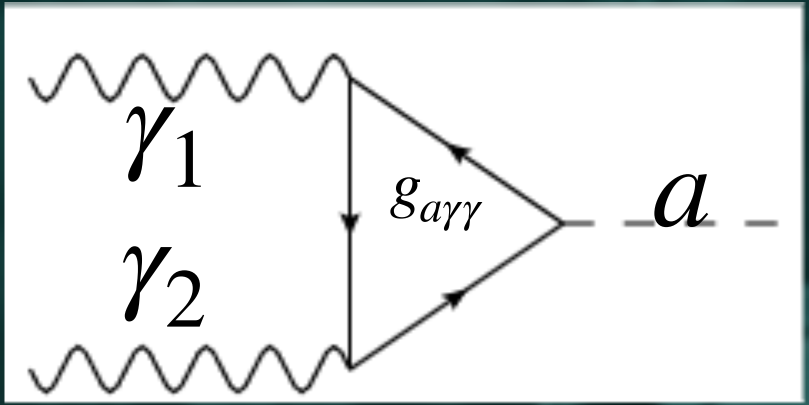

ARC CENTRE OF EXCELLENCE FOR
DARK MATTER

New Axion Dark Matter Search Techniques

Michael Tobar

QDM
Quantum Technologies and Dark Matter Research Lab

THE UNIVERSITY OF
WESTERN
AUSTRALIA

SEEK WISDOM

17TH PATRAS WORKSHOP ON
AXIONS, WIMPS AND WISPS
08 - 12 August 2022

JOHANNES GUTENBERG UNIVERSITÄT MAINZ
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The QDM Lab: https://www.qdmlab.com/

QUANTUM TECHNOLOGIES AND DARK MATTER RESEARCH LAB

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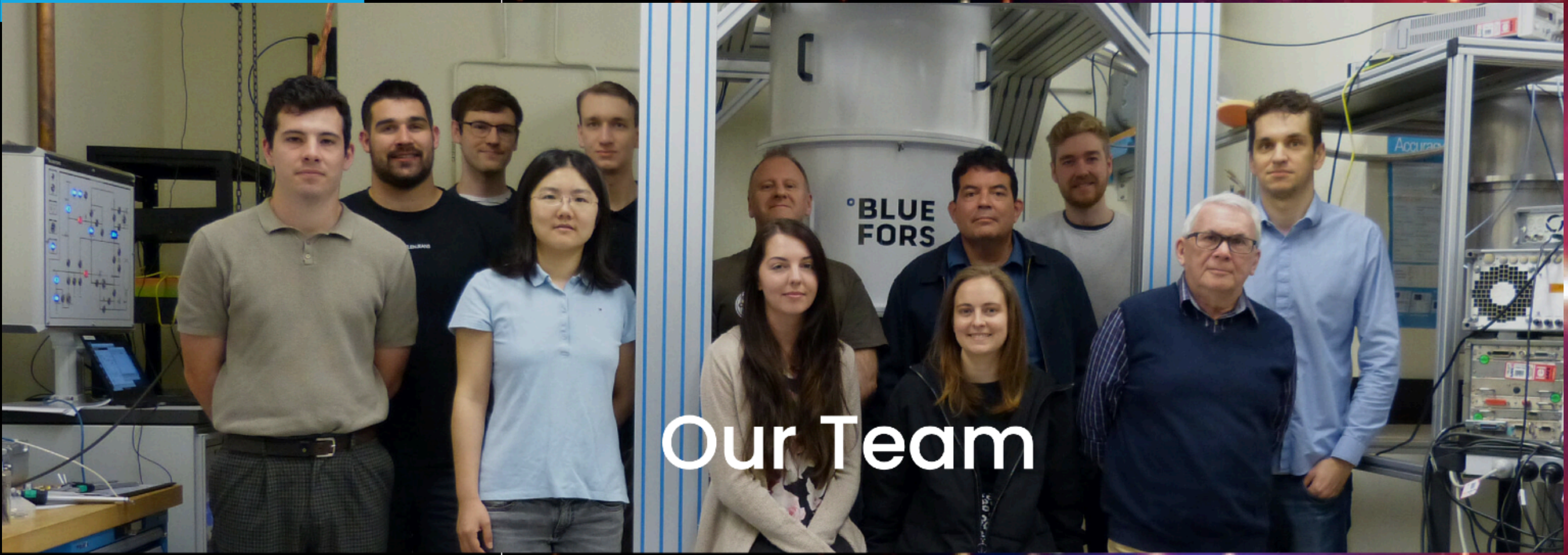

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Outline

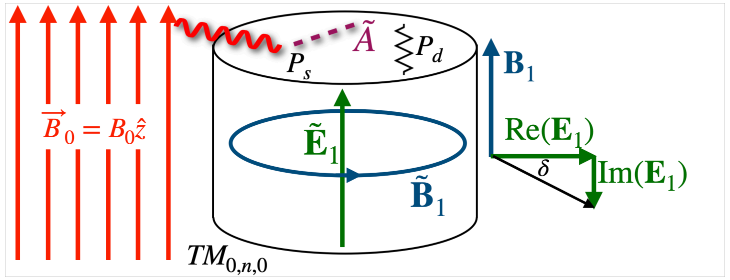
- Poynting Theorem; a systematic way to calculate resonant haloscope sensitivity, generalised to include QEMD (Sokolov and Ringwald arXiv:2205.02605 [hep-ph])
- Sensitivity of AC and DC Haloscopes
- Anyon Cavity Haloscope for ultra-light dark matter
- Sensitivity of Axion Haloscopes to GWs and Comparing Dissimilar Axion Haloscopes
- Low-mass sensitivity

Sensitivity of a Resonant Haloscope

PHYSICAL REVIEW D **105**, 045009 (2022)

$$P_{av} = \frac{1}{2}\mathrm{Re}\oint_{S_c}(\mathbf{E}\times\mathbf{H}^*)\cdot ds$$

Average radiated power



Poynting vector controversy in axion modified electrodynamics

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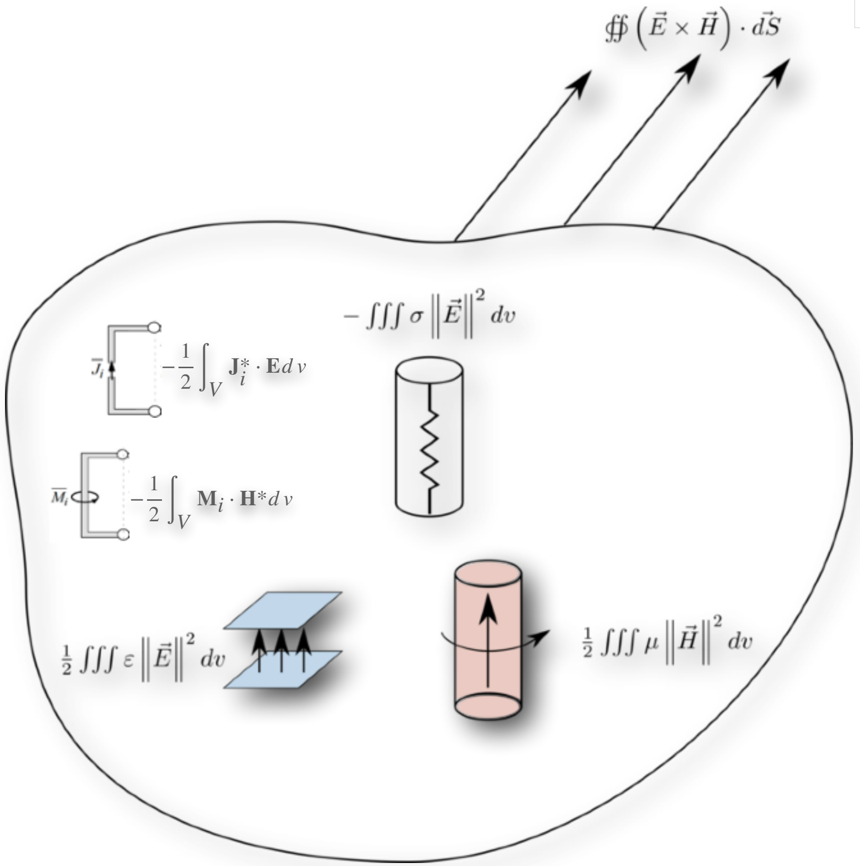

$$\mathbf{S} = \frac{1}{2\mu_0}\mathbf{E}_1\times\mathbf{B}_1^* \text{ and } \mathbf{S}^* = \frac{1}{2\mu_0}\mathbf{E}_1^*\times\mathbf{B}_1$$

$$\nabla\cdot\mathbf{S} = \frac{1}{2\mu_0}\nabla\cdot(\mathbf{E}_1\times\mathbf{B}_1^*) = \frac{1}{2\mu_0}\mathbf{B}_1^*\cdot(\nabla\times\mathbf{E}_1) - \frac{1}{2\mu_0}\mathbf{E}_1\cdot(\nabla\times\mathbf{B}_1^*)$$

$$\nabla\cdot\mathbf{S}^* = \frac{1}{2\mu_0}\nabla\cdot(\mathbf{E}_1^*\times\mathbf{B}_1) = \frac{1}{2\mu_0}\mathbf{B}_1\cdot(\nabla\times\mathbf{E}_1^*) - \frac{1}{2\mu_0}\mathbf{E}_1^*\cdot(\nabla\times\mathbf{B}_1)$$

On resonance: Real part of Complex Poynting Theorem = 0 for closed system

$$\oint\mathrm{Re}\,(\mathbf{S})\cdot\hat{n}\,ds = \frac{j\omega_a g_{a\gamma\gamma}\epsilon_0 c}{4}\int(\mathbf{E}_1\cdot\tilde{a}^*\mathbf{B}_0^* - \mathbf{E}_1^*\cdot\tilde{a}\mathbf{B}_0)\,d\tau - \frac{1}{4}\int(\mathbf{E}_1\cdot\mathbf{J}_{e1}^* + \mathbf{E}_1^*\cdot\mathbf{J}_{e1})\,d\tau$$

P_s Axion power input

P_d Cavity power distribution

$$P_d = \frac{1}{4}\int(\mathbf{E}_1\cdot\mathbf{J}_{e1}^* + \mathbf{E}_1^*\cdot\mathbf{J}_{e1})\,d\tau = \frac{\omega_1\epsilon_0}{2Q_1}\int\mathbf{E}_1\cdot\mathbf{E}_1^*\,dV = \frac{\omega_1 U_1}{Q_1}$$

$$P_s = \frac{\omega_a g_{a\gamma\gamma}a_0\epsilon_0 c}{2Q_1}\int(Re(\mathbf{E}_1)\cdot Re(\mathbf{B}_0))\,d\tau = P_d = \frac{\omega_1 U_1}{Q_1}$$

Anton Solokov: Electromagnetic couplings of axions 12 Aug 2022, 10:20

Electromagnetic Couplings of Axions

Anton V. Sokolov, Andreas Ringwald

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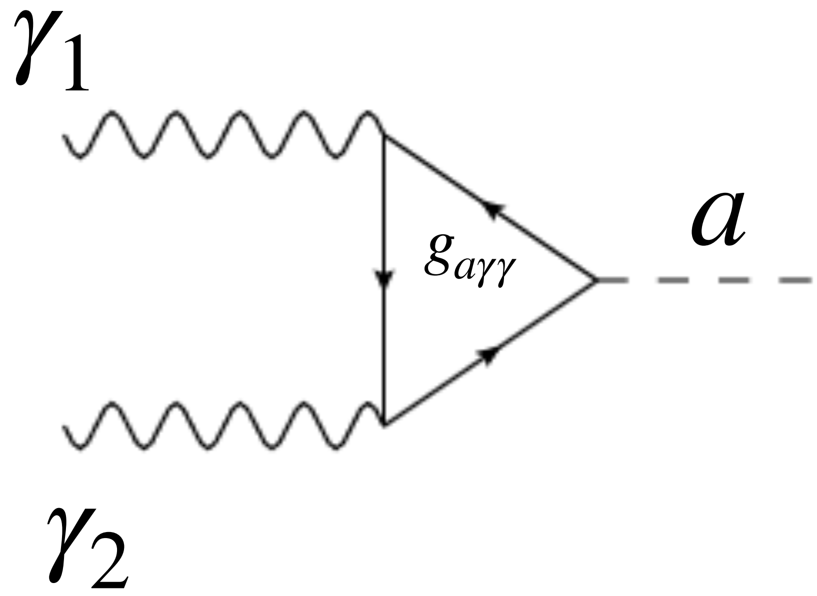

QUANTUM ELECTROMAGNETODYNAMICS

1971 ZWANZIGER

A_μ and B_μ $\longleftrightarrow$ photon

$$\mathcal{L} = \mathcal{L}_{\text{kin}}\left(A_\mu, B_\mu, n_\mu\right) - j_e^\nu A_\nu - \boxed{j_m^\nu B_\nu}$$

1977 ZBN

$$Z\left(a, b, \cancel{n_\mu}\right) = \int \exp\left\{i\left(\mathcal{S}[\mathbf{A}_\mu, \mathbf{B}_\mu, \mathbf{n}_\mu, \chi, \bar{\chi}] + j_e a + j_m b\right)\right\} \times \mathcal{D}\mathbf{A}_\mu \mathcal{D}\mathbf{B}_\mu \mathcal{D}\chi \mathcal{D}\bar{\chi}$$

- TWO vector-potentials describe ONE particle - photon
- partition function is Lorentz-invariant
- theory is generally not CP-invariant

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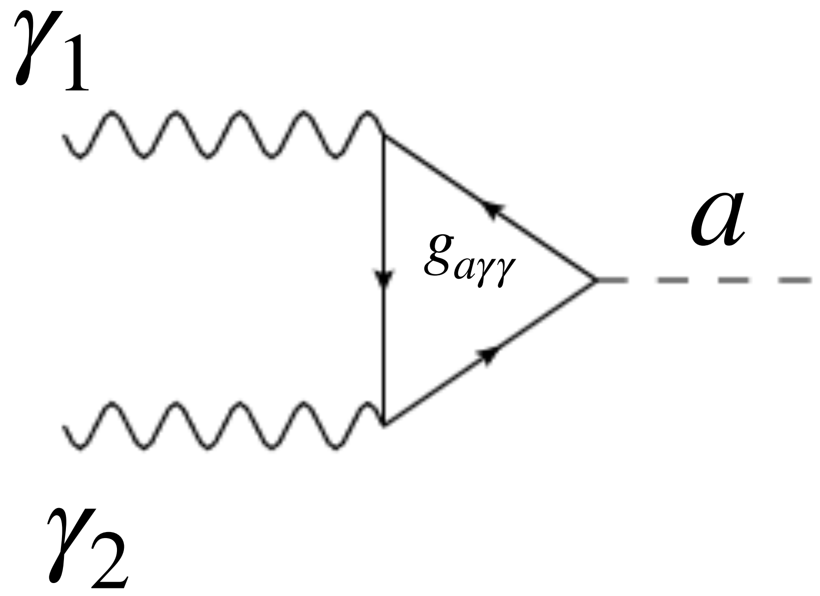

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Two other axion couplings

g_{aAB} $\quad$ g_{aBB}

- TWO vector-potentials describe ONE particle - photon
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DESY.

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Constant DC Background Magnetic field

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$$+\frac{j\omega_a g_{aAB}}{4\mu_0}\int\left(\mathbf{B}_1\cdot\tilde{a}^*\vec{B}_0 - \mathbf{B}_1^*\cdot\tilde{a}\vec{B}_0\right)d\tau$$
$$-\frac{1}{4}\int\left(\mathbf{E}_1\cdot\mathbf{J}_{e1}^* + \mathbf{E}_1^*\cdot\mathbf{J}_{e1}\right)d\tau$$

$$\int\left(\mathbf{B}_1\cdot\tilde{a}^*\vec{B}_0 - \mathbf{B}_1^*\cdot\tilde{a}\vec{B}_0\right)d\tau = 0$$

Electromagnetic Couplings of Axions

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arXiv:2207.14437 [pdf, other]

Searching for Scalar Field Dark Matter using Cavity Resonators and Capacitors

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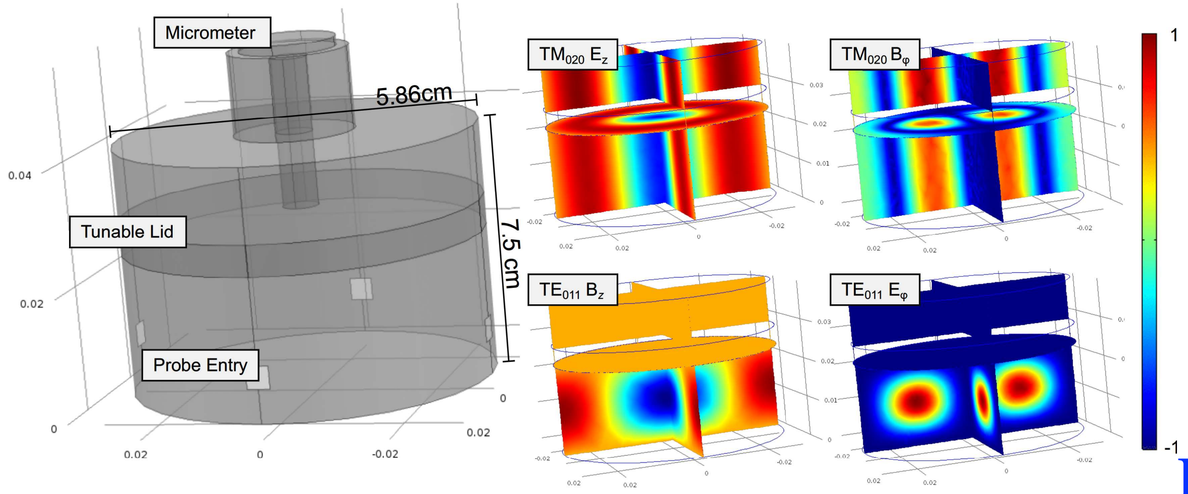
Dual Mode Upconversion: UPLOAD



Catriona Thomson

Frequency Technique Applying Perturbation Theorem

$$\langle\frac{\delta\omega_1}{\omega_1}\rangle \approx \frac{\omega_a\epsilon_0\langle a_0\rangle}{4\omega_1 U_1}\int_V \left(g_{a\gamma\gamma}c\mathbf{E}_1^*\cdot\mathbf{B}_0 - g_{aBB}c\mathbf{B}_1^*\cdot\mathbf{E}_0\right) dV$$

Power Technique Applying Poynting Theorem

$$\oint \mathrm{Re}\left(\mathbf{S}_1\right)\cdot\hat{n}ds = \int\Big(-\frac{1}{4}(\mathbf{E}_1\cdot\mathbf{J}_{e1}^* - \mathbf{E}_1^*\cdot\mathbf{J}_{e1})$$
$$+\frac{j\omega_a\epsilon_0 c g_{aBB}\sqrt{2}\langle a_0\rangle}{4}(\mathbf{B}_1^*\cdot\mathbf{E}_0 - \mathbf{B}_1\cdot\mathbf{E}_0^*)$$
$$+\frac{j\omega_a\epsilon_0 c g_{a\gamma\gamma}\sqrt{2}\langle a_0\rangle}{4}(\mathbf{E}_1\cdot\mathbf{B}_0^* - \mathbf{E}_1^*\cdot\mathbf{B}_0)\Big)\, dV$$

$$\xi_{10} = \frac{1}{V}\int \mathbf{e}_1\cdot\mathbf{b}_0\, dV \qquad \xi_{01} = \frac{1}{V}\int \mathbf{e}_0\cdot\mathbf{b}_1\, dV,$$

$$SNRp_{a\gamma\gamma} \sim g_{a\gamma\gamma}|\xi_{10}|\sqrt{\frac{2Q_{L0}Q_{L1}P_{0inc}\rho_a c^3}{\omega_1\omega_0 k_B(T_1+T_{amp})}}\left(\frac{10^6 t}{f_a}\right)^{\frac{1}{4}},$$

$$SNRp_{aBB} \sim g_{aBB}|\xi_{01}|\sqrt{\frac{2Q_{L0}Q_{L1}P_{0inc}\rho_a c^3}{\omega_1\omega_0 k_B(T_1+T_{amp})}}\left(\frac{10^6 t}{f_a}\right)^{\frac{1}{4}},$$

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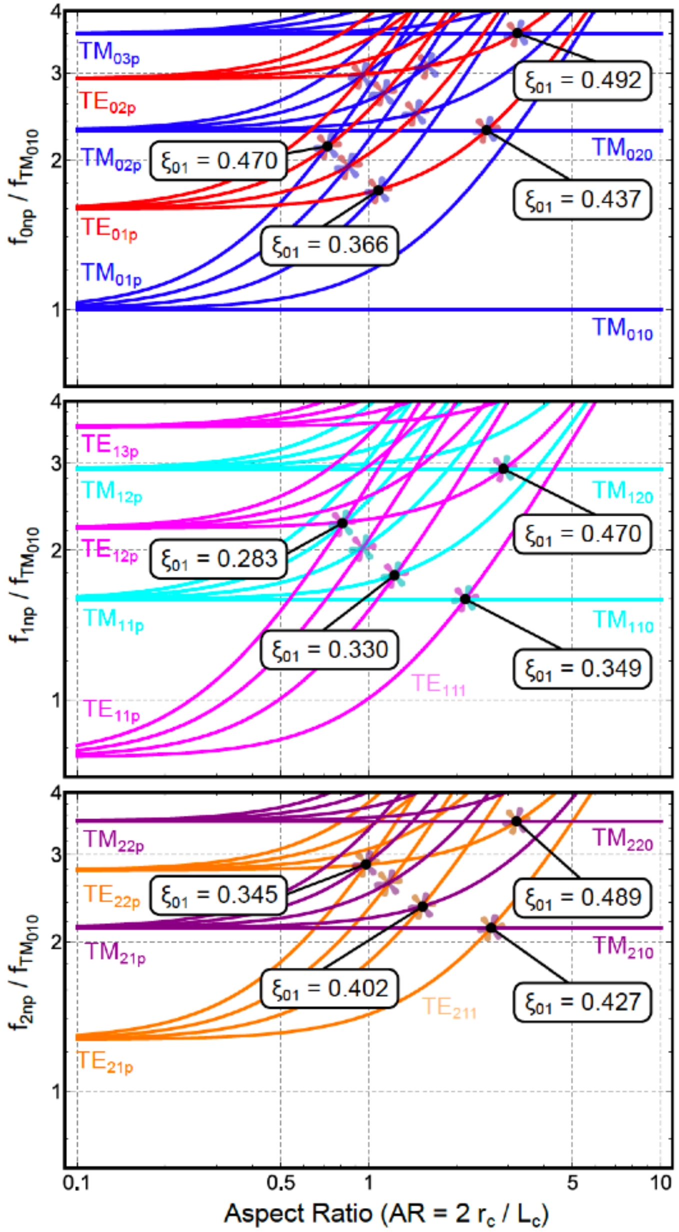
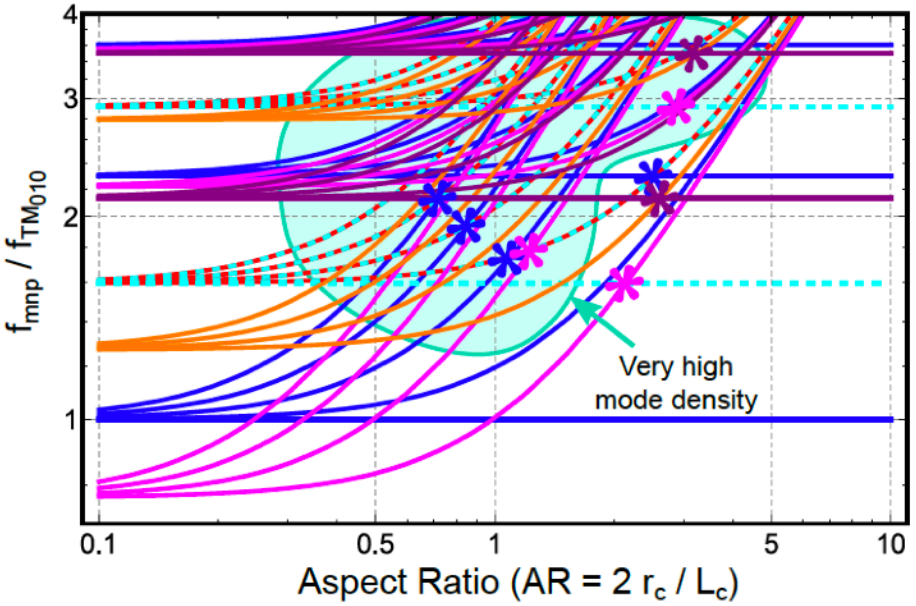
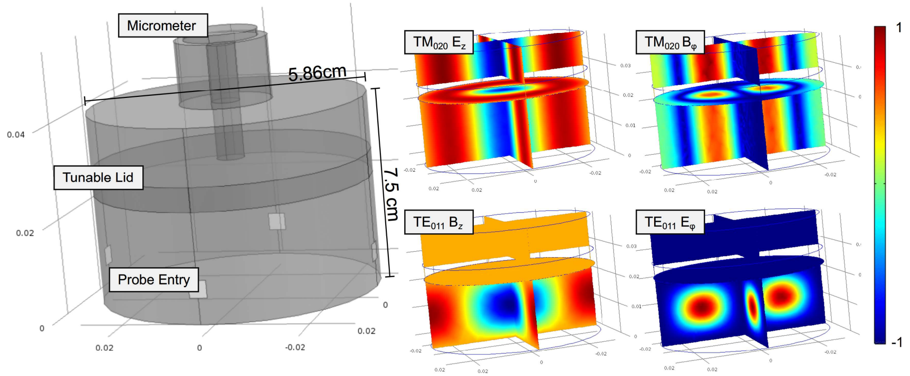
Mode Pairs: choose m=0: For upconversion $\xi_{10} \sim \xi_{01}$







$$TE_{0,1,1} - TM_{0,2,0}$$

$$\xi_{01} = -0.437$$

$$\xi_{10} \sim -0.451 \rightarrow -0.449.$$

Sensitivity ~ $\xi_{01}(g_{a\gamma\gamma}\dfrac{\omega_1}{\omega_0} + g_{aBB})$

arXiv:2208.01640 [pdf, other]

Twisted Anyon Cavity Resonators with Bulk Modes of Chiral Symmetry and Sensitivity to Ultra-Light Axion Dark Matter

J. F. Bourhill, E. C. I. Paterson, M. Goryachev, M. E. Tobar

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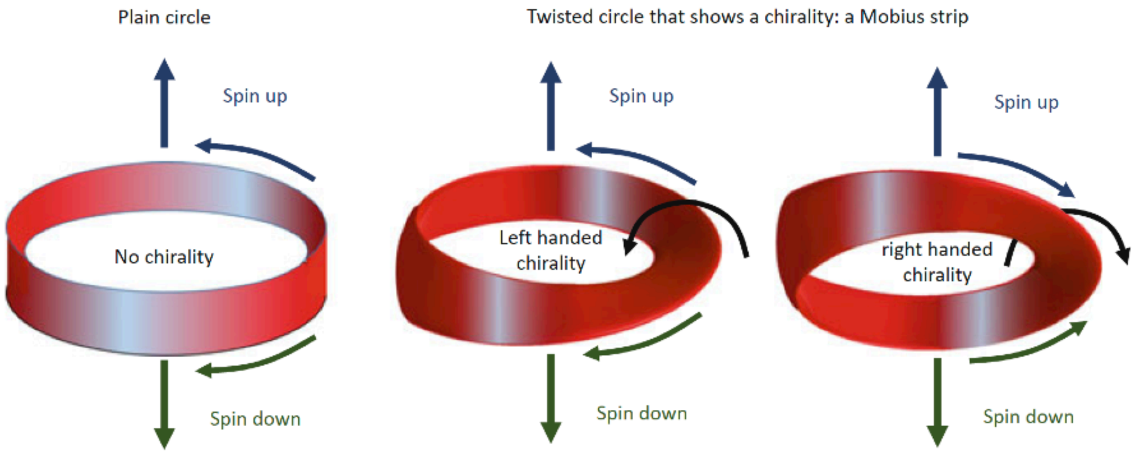
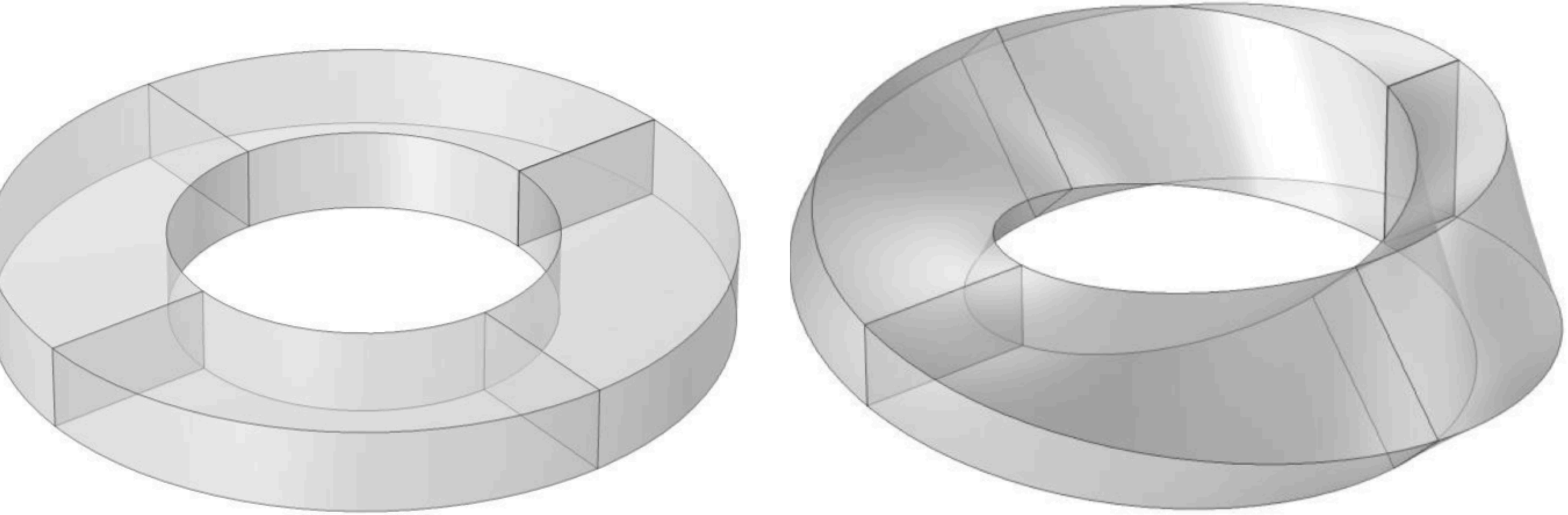
Classical Möbius-Ring Resonators Exhibit Fermion-Boson Rotational Symmetry

Douglas J. Ballon and Henning U. Voss

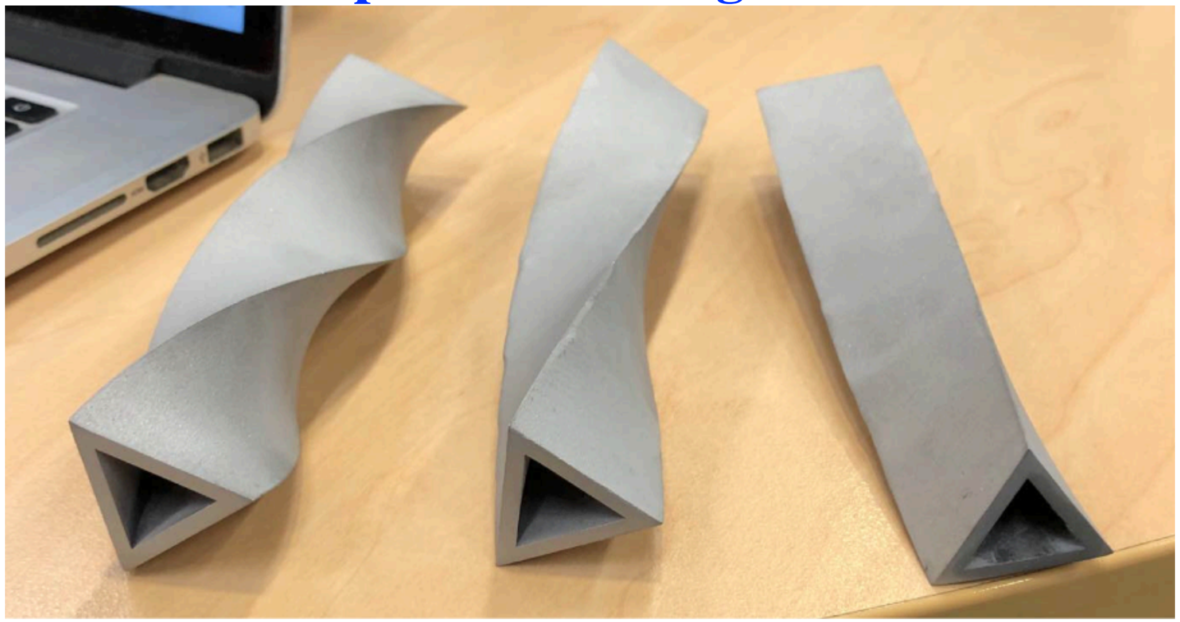
Phys. Rev. Lett. **101**, 247701 – Published 9 December 2008

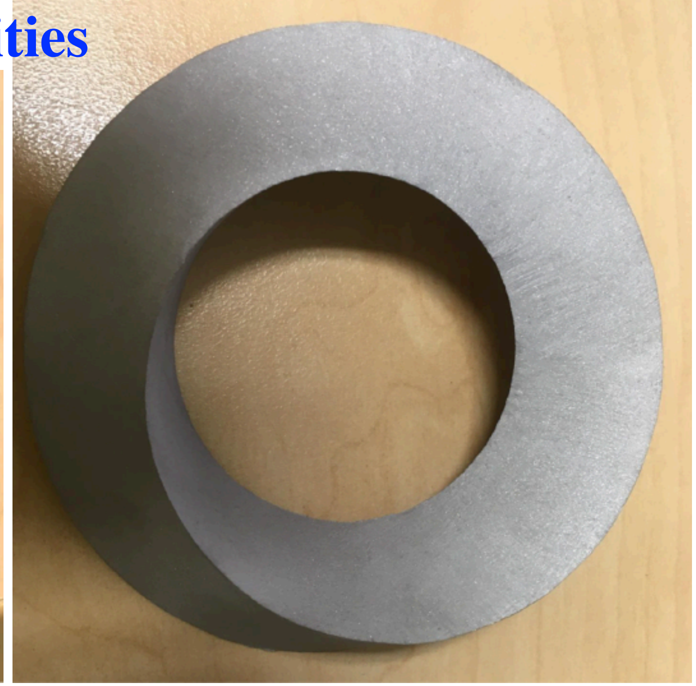




Fermions Come in Two Chiralities, Called Left and Right. Bosons Do Not

3D Printed Super Conducting Aluminium Cavities

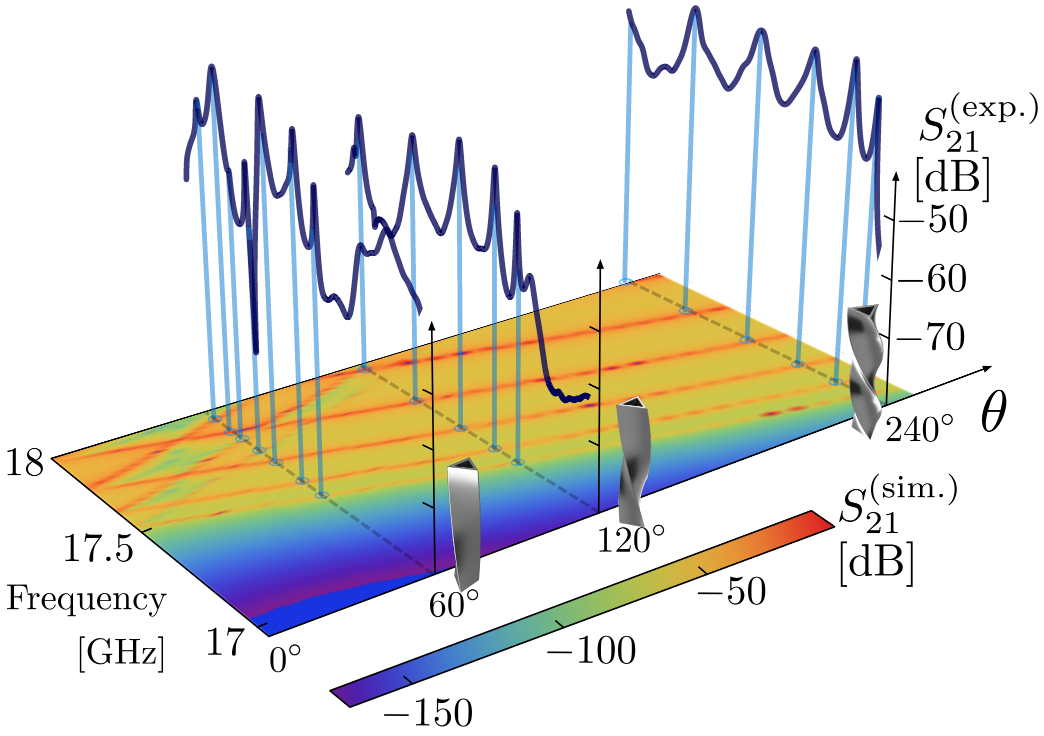




$$\mathscr{H}_p = \frac{2\,\mathrm{Im}[\int \mathbf{B}_p(\vec{r})\cdot\mathbf{E}_p^*(\vec{r})\,d\tau]}{\sqrt{\int \mathbf{E}_p(\vec{r})\cdot\mathbf{E}_p^*(\vec{r})\,d\tau \int \mathbf{B}_p(\vec{r})\cdot\mathbf{B}_p^*(\vec{r})\,d\tau}},$$

Resonator	Mode	f (GHz)	G (Ω)	$\mathscr{H}$
Ring	ψ_0^-	17.221	6200	0.933
Ring	ψ_1^-	17.297	6570	0.9166
Ring	ψ_0^+	17.895	7290	-0.820
Linear	ψ_0^-	17.214	1950	0.932
Linear	ψ_1^-	17.278	2030	0.896
Linear	ψ_0^+	17.859	1920	-0.884

TABLE I. Simulated f, G and $\mathscr{H}$ values for the lowest order $\psi^\pm$ modes for $l = 150$ mm, $v = 20$ mm and $\theta = 120°$ ring and linear resonators.



Zilch (electromagnetism)

From Wikipedia, the free encyclopedia

In physics, **zilch** is a conserved quantity of the electromagnetic field.

Daniel M. Lipkin observed that if he defined the quantities

$$Z^0 = \mathbf{E}\cdot\nabla\times\mathbf{E} + \mathbf{B}\cdot\nabla\times\mathbf{B}$$

$$\mathbf{Z} = \frac{1}{c}\left(\mathbf{E}\times\frac{d}{dt}\mathbf{E} + \mathbf{B}\times\frac{d}{dt}\mathbf{B}\right)$$

then the Maxwell equations imply that

$$\partial_0 Z^0 + \nabla\cdot\mathbf{Z} = 0$$

which implies that the total "zilch" $\int Z^0\, d^3x$ is constant ($\mathbf{Z}$ is the "zilch current").

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On the natures of the spin and orbital parts of optical angular momentum

Stephen M Barnett[1], L Allen[2], Robert P Cameron[1], Claire R Gilson[3], Miles J Padgett[1], Fiona C Speirits[1] and Alison M Yao[2]

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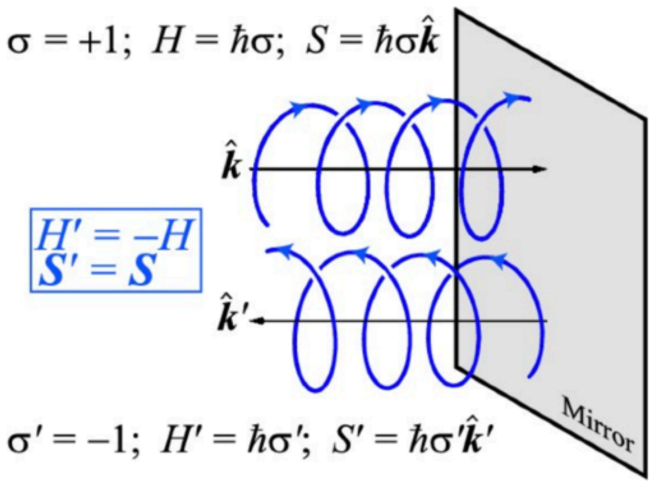
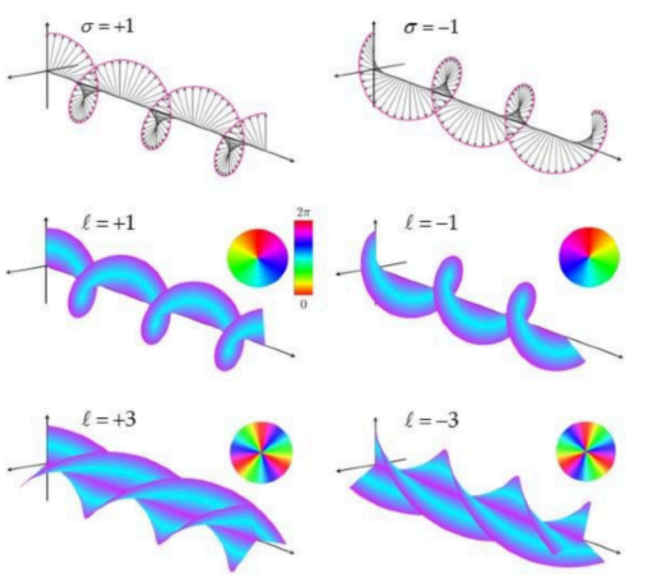
[3] School of Mathematics and Statistics, University of Glasgow, Glasgow G12 8QW, UK

Optical chirality: Twisted light



Circularly polarized light

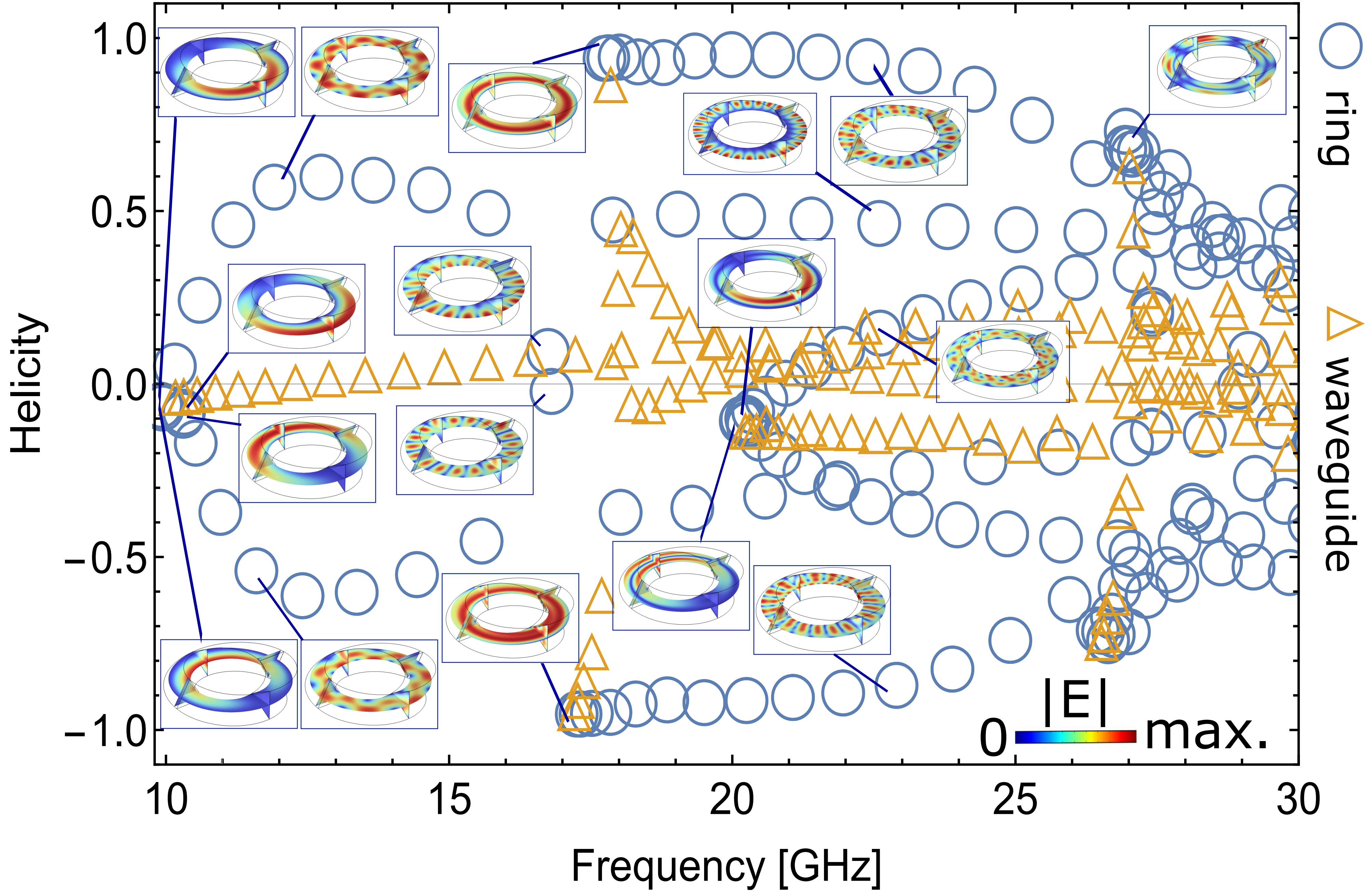


optical vortex beams

Helicity of light plays an important part in the coupling between electromagnetic fields and chiral objects

Axion is a Chiral Object

$$\mathscr{H}_p = \frac{2\,\mathrm{Im}[\int \mathbf{B}_p(\vec{r})\cdot\mathbf{E}_p^*(\vec{r})\,d\tau]}{\sqrt{\int \mathbf{E}_p(\vec{r})\cdot\mathbf{E}_p^*(\vec{r})\,d\tau\int \mathbf{B}_p(\vec{r})\cdot\mathbf{B}_p^*(\vec{r})\,d\tau}},$$



1.0
0.5
0.0
−0.5
−1.0
Helicity
10
15
20
25
30
Frequency [GHz]
ring
waveguide
|E|
0
max.

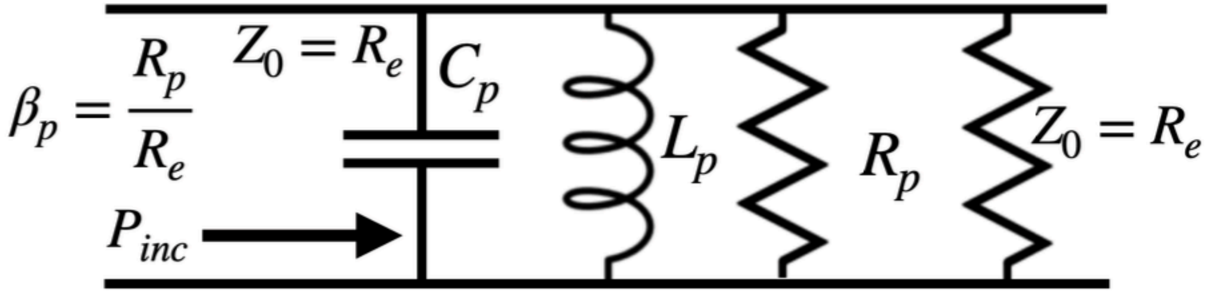
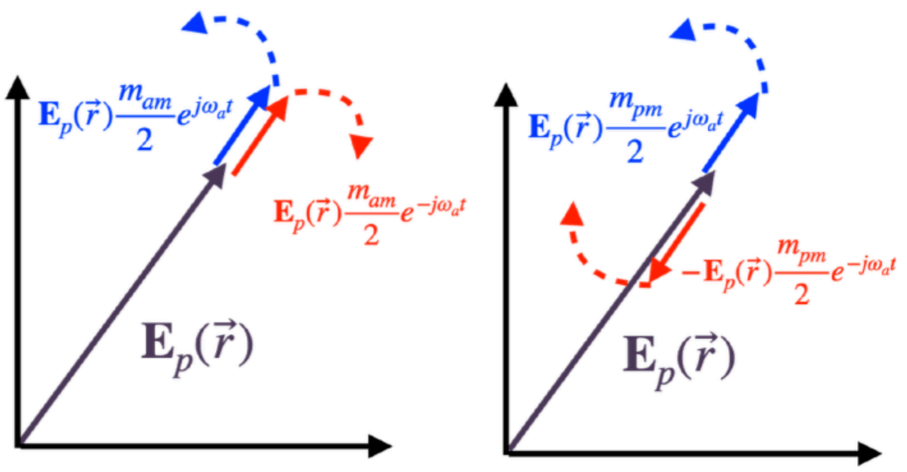


FIG. 7. Equivalent parallel LCR circuit model of a resonant mode with a coupling of β_p, when impedance matched $\beta_p = 1$.

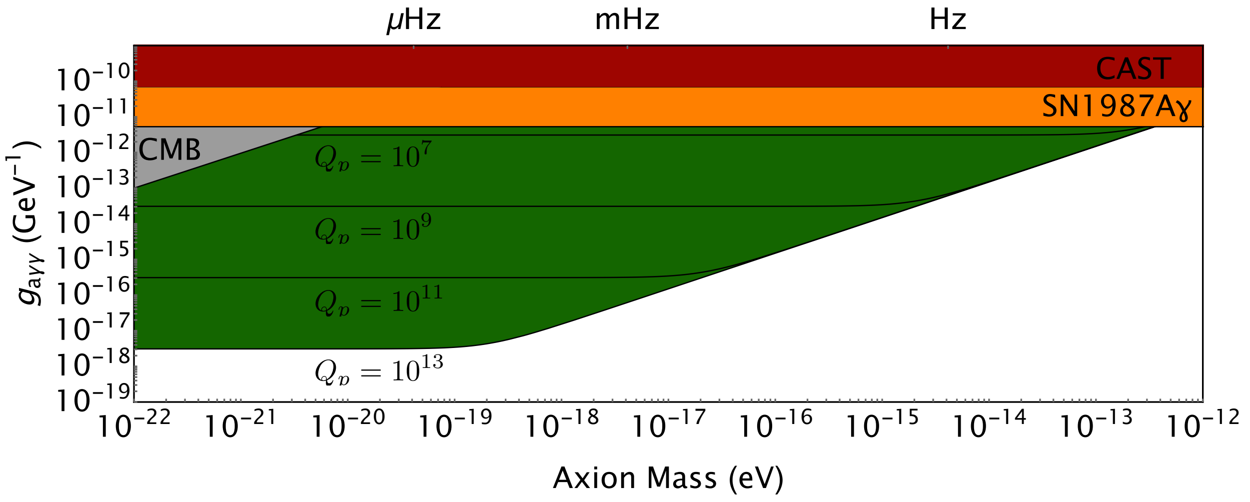


$$P_p = \frac{\beta_p P_d}{\beta_p + 1} = \frac{4\beta_p^2}{(1+\beta_p)^2}P_{inc}.$$

$$\frac{P_{am}}{P_{inc}} = \frac{m_{am}^2 P_p}{P_{inc}} = Q_p^2\frac{4\beta_p^2}{(1+\beta_p)^2}\left(\frac{\omega_a}{\omega_p}\right)^2\frac{\langle\theta_0\rangle^2}{8}\mathscr{H}_p^2.$$

$$SNR = \frac{g_{a\gamma\gamma}\beta_p|\mathscr{H}_p|}{\sqrt{2}(1+\beta_p)}\frac{Q_p}{\sqrt{1+4Q_p^2(\frac{\omega_a}{\omega_p})^2}}\frac{\left(\frac{10^6 t}{\omega_a}\right)^{\frac{1}{4}}\sqrt{\rho_a c^3}}{\omega_p\sqrt{S_{am}}}$$

$$SNRp_{a\gamma\gamma} \sim g_{a\gamma\gamma}|\xi_{10}|\sqrt{\frac{2Q_{L0}Q_{L1}P_{0inc}\rho_a c^3}{\omega_1\omega_0 k_B(T_1+T_{amp})}}\left(\frac{10^6 t}{f_a}\right)^{\frac{1}{4}},$$



Sensitivity ~ $|\mathscr{H}|(g_{a\gamma\gamma} + g_{aBB})$

High Energy Physics – Phenomenology

[Submitted on 21 Dec 2021]

Detecting High-Frequency Gravitational Waves with Microwave Cavities

Asher Berlin, Diego Blas, Raffaele Tito D'Agnolo, Sebastian A. R. Ellis, Roni Harnik, Yonatan Kahn, Jan Schütte-Engel

We give a detailed treatment of electromagnetic signals generated by gravitational waves (GWs) in resonant cavity experiments. Our investigation corrects and builds upon previous studies by carefully accounting for the gauge dependence of relevant quantities. We work in a preferred frame for the laboratory, the proper detector frame, and show how to resum short-wavelength effects to provide analytic results that are exact for GWs of arbitrary wavelength. This formalism allows us to firmly establish that, contrary to previous claims, cavity experiments designed for the detection of axion dark matter only need to reanalyze existing data to search for high-frequency GWs with strains as small as $h \sim 10^{-22} - 10^{-21}$. We also argue that directional detection is possible in principle using readout of multiple cavity modes. Further improvements in sensitivity are expected with cutting-edge advances in superconducting cavity technology.

Comments: 20 pages + appendix, 7 figures
Subjects: High Energy Physics – Phenomenology (hep-ph); High Energy Astrophysical Phenomena (astro-ph.HE); Instrumentation and Methods for Astrophysics (astro-ph.IM); High Energy Physics – Experiment (hep-ex)
Cite as: arXiv:2112.11465 [hep-ph]
(or arXiv:2112.11465v1 [hep-ph] for this version)

Submission history
From: Jan Schütte-Engel [view email]
[v1] Tue, 21 Dec 2021 19:00:01 UTC (3,548 KB)

II. GW ELECTRODYNAMICS IN THE PROPER DETECTOR FRAME

A. Analogies with Axion Dark Matter Detection

FIG. 1. A cartoon illustrating the differences between GW-EM conversion (left) and axion-EM conversion (right) in the presence of an external magnetic field $\mathbf{B}_0$. The GW effective current is proportional to $\omega_g h B_0$, with a direction dependent on the GW polarization and a typical quadrupole pattern, yielding a signal field with amplitude hB_0. The axion effective current is proportional to $\omega_a \theta_a B_0$, with a direction parallel to the external field $\mathbf{B}_0$, yielding a signal field with amplitude $\theta_a B_0$. The differing geometry of the effective current yields different selection rules for coupling the GW and axion to cavity modes.

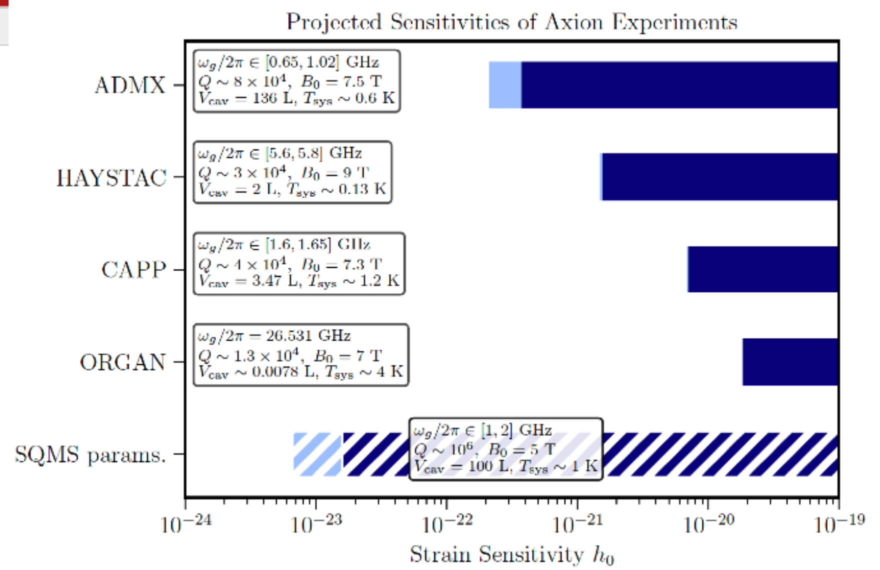


FIG. 4. Projected sensitivity of axion experiments to high-frequency GWs, assuming an integration time of $t_{\rm int} = 2$ min for ADMX, HAYSTAC and CAPP, $t_{\rm int} = 4$ day for ORGAN, and $t_{\rm int} = 1$ day for the SQMS parameters. These integration times are characteristic of data-taking runs in each experiment. The GW-cavity coupling coefficient is fixed to $\eta_n = 0.1$ for each experiment, and the signal bandwidth $\Delta\nu$ is conservatively fixed to the linewidth of the cavity. Dark (light) blue regions indicate the sensitivity at the lowest (highest) resonant frequency of the tunable signal mode. For ADMX [46, 120, 122], HAYSTAC [47], and CAPP [123], the signal mode is TM_{010}, but for ORGAN [48] the signal mode is TM_{020}. The system temperature $T_{\rm sys}$ defining the thermal noise floor of each experiment is given in the figure, along with relevant experimental parameters including the loaded cavity quality factor Q.

$$j_{\rm eff} \supset g_{a\gamma\gamma}\partial_t a \mathbf{B}_0 \simeq \omega_a \theta_a \mathbf{B}_0 \qquad \mathbf{E}_a = g_{a\gamma\gamma} a \mathbf{B}_0 = \theta_a \mathbf{B}_0$$

$$j^\mu_{\rm eff} \equiv \partial_\nu \left(\frac{1}{2} h F^{\mu\nu} + h^\nu_\alpha F^{\alpha\mu} - h^\mu_\alpha F^{\alpha\nu} \right)$$

$$j_{\rm eff} \sim \omega_g h B_0$$

identifying $\theta_a \sim h$

Comparing the Sensitivity of Dissimilar Photonic Axion Haloscopes to Axion Dark Matter and High Frequency Gravitational Waves

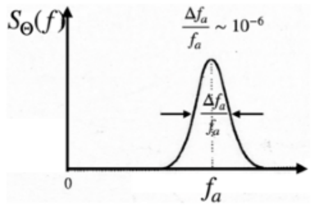
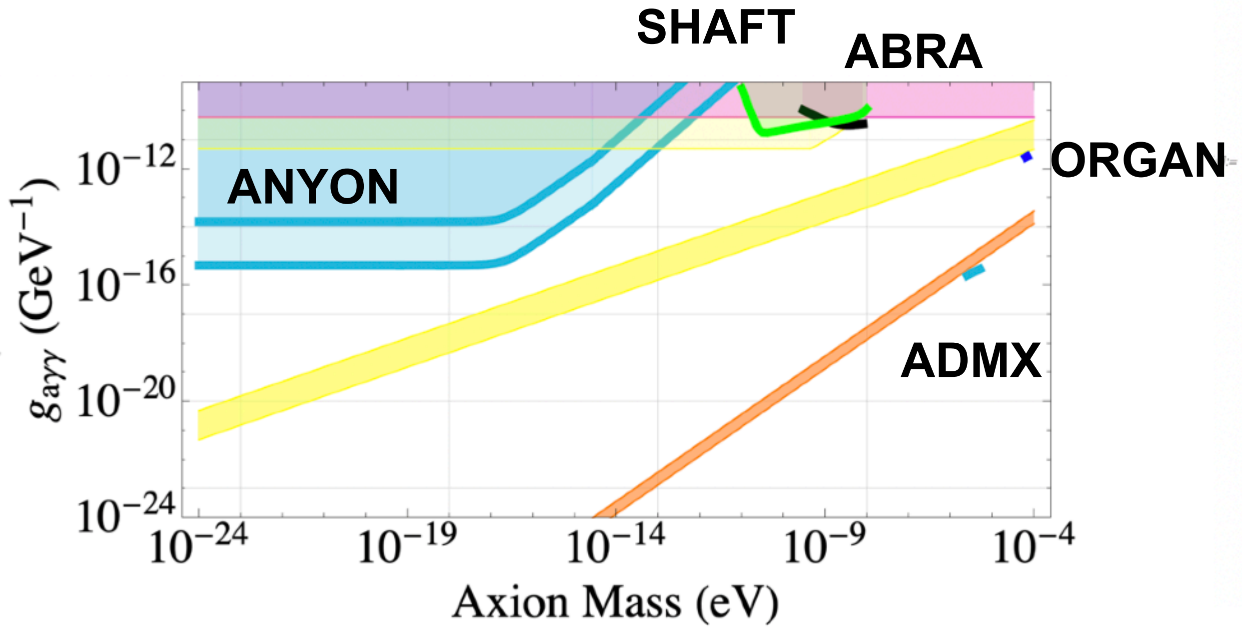
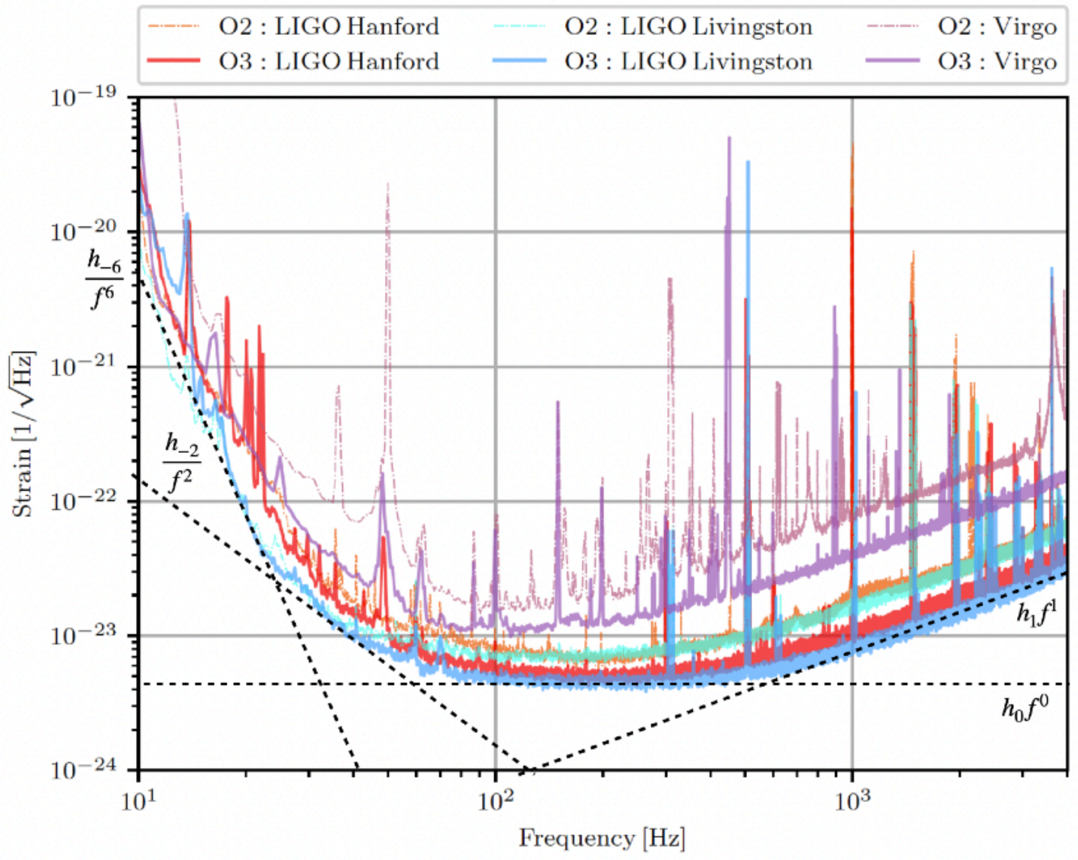
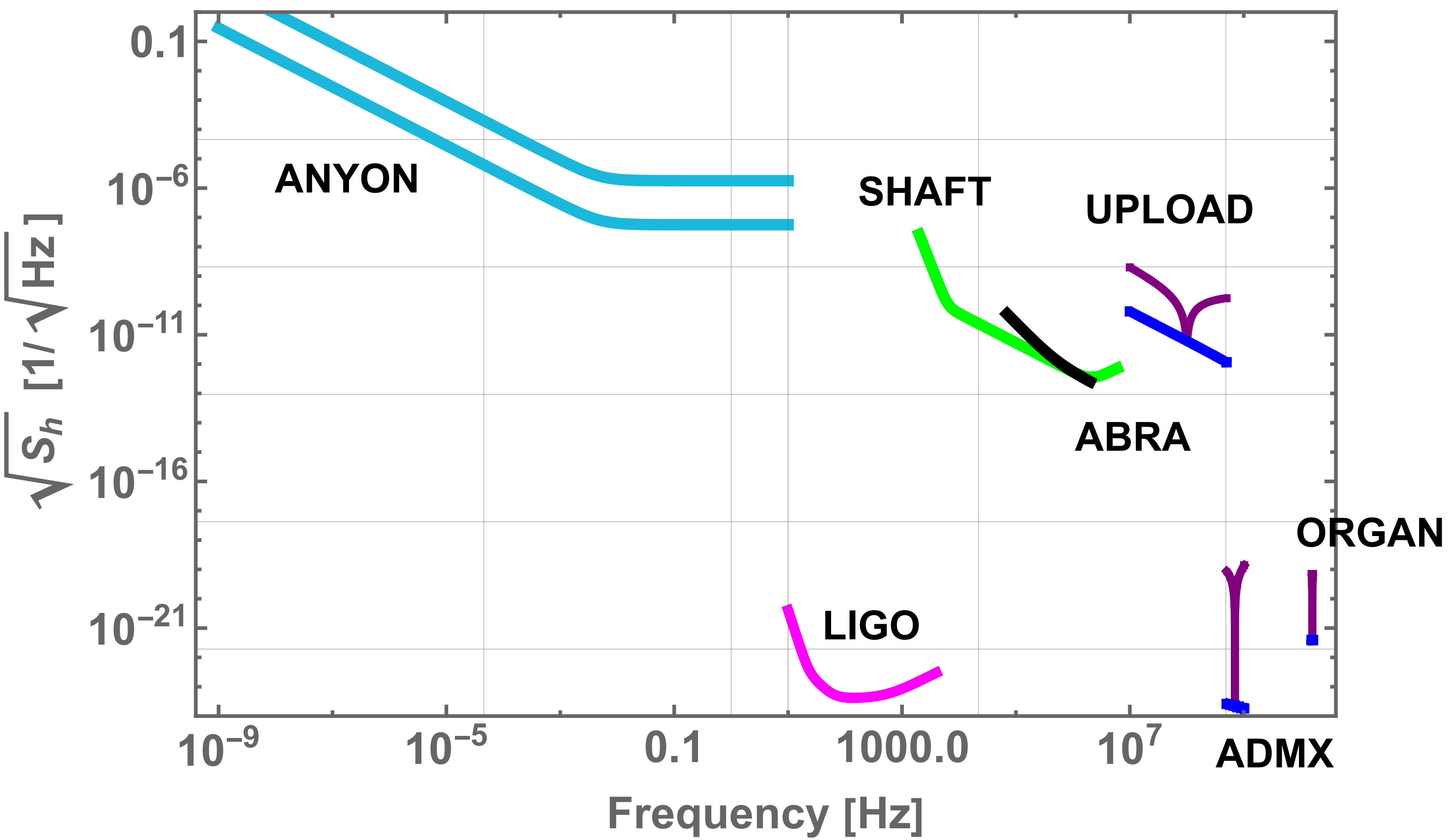


FIG. 1: Approximation of a virialized dark matter axion as a narrow band noise source with an effective Q-factor of $Q_a \sim \frac{f_a}{\Delta f_a}$. In actual fact it takes on a Maxwell–Boltzmann distribution, here it is approximated as a Lorentzian distribution.



LIGO Strain SD





ADMX and ORGAN (purple) with current tuning locus (blue); 0.6-1.2 GHz for ADMX and 15.2 to 16.2 GHz for ORGAN

Photon Haloscopes

- Axions convert into photons in presence of a background AC electromagnetic field

Axion Equation of Motion:

Klein-Gordon equation for massive spin 0 particle

$$a(t) = \frac{1}{2}\left(\tilde{a}e^{-j\omega_a t} + \tilde{a}^* e^{j\omega_a t}\right)$$
$$= \mathrm{Re}\left(\tilde{a}e^{-j\omega_a t}\right)$$

Modified Axion Electrodynamics

(Represents two photons)

$$\nabla \cdot \vec{E} = \frac{\rho_e}{\varepsilon_0} + c g_{a\gamma\gamma} \vec{B} \,.\, \nabla a \rightarrow$$

$$\nabla \times \vec{B} - \frac{1}{c^2}\partial_t \vec{E} =$$
$$\mu_0 \vec{J}_e - g_{a\gamma\gamma}\epsilon_0 c \left(\vec{B}\partial_t a + \nabla a \times \vec{E}\right)$$

$$\nabla \cdot \vec{B} = 0$$
$$\nabla \times \vec{E} + \partial_t \vec{B} = 0$$

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$$\nabla \cdot \vec{B} = 0$$
$$\nabla \times \vec{E} + \partial_t \vec{B} = 0$$

→

1) Background field (subscript zero)

2) Created Photon Field (subscript 1)

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$$\mu_0 \vec{J}_e - g_{a\gamma\gamma}\epsilon_0 c \left(\vec{B}\partial_t a + \nabla a \times \vec{E}\right)$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} + \partial_t \vec{B} = 0$$

1) Background field (subscript zero)

2) Created Photon Field (subscript 1)

$$\epsilon_0 \nabla \cdot \vec{E}_1 = \rho_{e1} + \rho_{ab}$$

$$\frac{1}{\mu_0}\nabla \times \vec{B}_1 - \epsilon_0 \partial_t \vec{E}_1 = \vec{J}_{e1} + \vec{J}_{ab} + \vec{J}_{ae}$$

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$$\mu_0 \vec{J}_e - g_{a\gamma\gamma}\epsilon_0 c \left(\vec{B}\partial_t a + \nabla a \times \vec{E}\right)$$
$$\nabla \cdot \vec{B} = 0$$
$$\nabla \times \vec{E} + \partial_t \vec{B} = 0$$

$$\epsilon_0 \nabla \cdot \vec{E}_1 = \rho_{e1} + \rho_{ab}$$
$$\frac{1}{\mu_0}\nabla \times \vec{B}_1 - \epsilon_0 \partial_t \vec{E}_1 = \vec{J}_{e1} + \vec{J}_{ab} + \vec{J}_{ae}$$

$$\rho_{ab} = g_{a\gamma\gamma}\epsilon_0 c \nabla \cdot \left(a(t)\vec{B}_0(\vec{r},t)\right)$$
$$\vec{J}_{ab} = -g_{a\gamma\gamma}\epsilon_0 c \partial_t \left(a(t)\vec{B}_0(\vec{r},t)\right)$$
$$\vec{J}_{ae} = -g_{a\gamma\gamma}\epsilon_0 c \nabla \times \left(a(t)\vec{E}_0(\vec{r},t)\right)$$

$$\nabla \cdot \vec{J}_{ab} = -\partial_t \rho_{ab}$$

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$$\nabla \times \vec{E} + \partial_t \vec{B} = 0$$

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$$\vec{J}_{ab} = -\, g_{a\gamma\gamma}\epsilon_0 c \partial_t \left(a(t)\vec{B}_0(\vec{r},t)\right)$$

$$\vec{J}_{ae} = -\, g_{a\gamma\gamma}\epsilon_0 c \nabla \times \left(a(t)\vec{E}_0(\vec{r},t)\right)$$

$$\nabla \cdot \vec{J}_{ab} = -\,\partial_t \rho_{ab}$$

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$$\vec{J}_{ae} = -\, g_{a\gamma\gamma}\epsilon_0 c \nabla \times \left(a(t)\vec{E}_0(\vec{r},t)\right)$$

$$\nabla \cdot \vec{J}_{ab} = -\,\partial_t \rho_{ab}$$

Source Terms generate Photons->
From background fields mixing with axion

Photonic Haloscope Equations in terms of Auxiliary Fields

Modified Axion Electrodynamics

(Represents two photons)

$$\nabla \cdot \vec{E} = \frac{\rho_e}{\varepsilon_0} + c g_{a\gamma\gamma} \vec{B} . \nabla a$$

$$\nabla \times \vec{B} - \frac{1}{c^2} \partial_t \vec{E} =$$

$$\mu_0 \vec{J}_e - g_{a\gamma\gamma} \epsilon_0 c \left(\vec{B} \partial_t a + \nabla a \times \vec{E} \right)$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} + \partial_t \vec{B} = 0$$

Applied Background Field

$$\nabla \times \vec{B}_0 = \mu_0 \epsilon_0 \partial_t \vec{E}_0 + \mu_0 \vec{J}_{e_0}$$

$$\nabla \times \vec{E}_0 = - \partial_t \vec{B}_0$$

$$\nabla \cdot \vec{B}_0 = 0$$

$$\nabla \cdot \vec{E}_0 = \epsilon_0^{-1} \rho_{e_0}$$

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Modified Axion Electrodynamics

(Represents two photons)

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$$\nabla \times \vec{E}_0 = -\partial_t \vec{B}_0$$

$$\nabla \cdot \vec{B}_0 = 0$$

$$\nabla \cdot \vec{E}_0 = \epsilon_0^{-1} \rho_{e_0}$$

Measure Created Photon

Photonic Haloscope Equations in terms of Auxiliary Fields

Modified Axion Electrodynamics

(Represents two photons)

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$$\nabla \times \vec{B} - \frac{1}{c^2}\partial_t \vec{E} =$$

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$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} + \partial_t \vec{B} = 0$$

Applied Background Field

$$\nabla \times \vec{B}_0 = \mu_0 \epsilon_0 \partial_t \vec{E}_0 + \mu_0 \vec{J}_{e_0}$$

$$\nabla \times \vec{E}_0 = -\partial_t \vec{B}_0$$

$$\nabla \cdot \vec{B}_0 = 0$$

$$\nabla \cdot \vec{E}_0 = \epsilon_0^{-1} \rho_{e_0}$$

Measure Created Photon

$$\nabla \cdot \left(\vec{E}_1(\vec{r},t) - g_{a\gamma\gamma} a(t) c \vec{B}_0(\vec{r},t) \right) = \frac{\rho_{e_1}}{\epsilon_0}$$

$$\nabla \times \left(\vec{B}_1(\vec{r},t) + \frac{g_{a\gamma\gamma} a(t)}{c} \vec{E}_0(\vec{r},t) \right)$$

$$-\frac{1}{c^2}\partial_t \left(\vec{E}_1(\vec{r},t) - g_{a\gamma\gamma} a(\vec{r},t) c \vec{B}_0(\vec{r},t) \right) = \mu_0 \vec{J}_{e_1}$$

$$\nabla \cdot \vec{B}_1(\vec{r},t) = 0$$

$$\nabla \times \vec{E}_1(\vec{r},t) + \partial_t \vec{B}_1(\vec{r},t) = 0.$$

Photonic Haloscope Equations in terms of Auxiliary Fields

Modified Axion Electrodynamics

(Represents two photons)

$$\nabla \cdot \vec{E} = \frac{\rho_e}{\varepsilon_0} + c g_{a\gamma\gamma} \vec{B} \cdot \nabla a$$

$$\nabla \times \vec{B} - \frac{1}{c^2}\partial_t \vec{E} =$$

$$\mu_0 \vec{J}_e - g_{a\gamma\gamma}\epsilon_0 c \left(\vec{B}\partial_t a + \nabla a \times \vec{E} \right)$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} + \partial_t \vec{B} = 0$$

Applied Background Field

$$\nabla \times \vec{B}_0 = \mu_0 \epsilon_0 \partial_t \vec{E}_0 + \mu_0 \vec{J}_{e_0}$$

$$\nabla \times \vec{E}_0 = -\partial_t \vec{B}_0$$

$$\nabla \cdot \vec{B}_0 = 0$$

$$\nabla \cdot \vec{E}_0 = \epsilon_0^{-1} \rho_{e_0}$$

Measure Created Photon

$$\nabla \cdot \left(\vec{E}_1(\vec{r},t) - g_{a\gamma\gamma} a(t) c \vec{B}_0(\vec{r},t) \right) = \frac{\rho_{e_1}}{\epsilon_0}$$

$$\nabla \times \left(\vec{B}_1(\vec{r},t) + \frac{g_{a\gamma\gamma} a(t)}{c} \vec{E}_0(\vec{r},t) \right)$$

$$-\frac{1}{c^2}\partial_t \left(\vec{E}_1(\vec{r},t) - g_{a\gamma\gamma} a(\vec{r},t) c \vec{B}_0(\vec{r},t) \right) = \mu_0 \vec{J}_{e_1}$$

$$\nabla \cdot \vec{B}_1(\vec{r},t) = 0$$

$$\nabla \times \vec{E}_1(\vec{r},t) + \partial_t \vec{B}_1(\vec{r},t) = 0.$$

Photonic Haloscope Equations in terms of Auxiliary Fields

Modified Axion Electrodynamics

(Represents two photons)

$$\nabla \cdot \vec{E} = \frac{\rho_e}{\varepsilon_0} + c g_{a\gamma\gamma} \vec{B} \,.\, \nabla a$$

$$\nabla \times \vec{B} - \frac{1}{c^2}\partial_t \vec{E} =$$

$$\mu_0 \vec{J}_e - g_{a\gamma\gamma}\epsilon_0 c \left(\vec{B}\,\partial_t a + \nabla a \times \vec{E} \right)$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} + \partial_t \vec{B} = 0$$

Applied Background Field

$$\nabla \times \vec{B}_0 = \mu_0 \epsilon_0 \partial_t \vec{E}_0 + \mu_0 \vec{J}_{e_0}$$

$$\nabla \times \vec{E}_0 = -\,\partial_t \vec{B}_0$$

$$\nabla \cdot \vec{B}_0 = 0$$

$$\nabla \cdot \vec{E}_0 = \epsilon_0^{-1} \rho_{e_0}$$

$$\vec{\nabla} \cdot \vec{D}_1 = \rho_{e1}$$

$$\vec{\nabla} \times \vec{E}_1 = -\,\partial_t \vec{B}_1$$

$$\vec{\nabla} \cdot \vec{B}_1 = 0$$

$$\frac{1}{\mu_0} \vec{\nabla} \times \vec{B}_1 = = \vec{J}_{e1} + \epsilon_0 \partial_t \vec{E}_1 - g_{a\gamma\gamma}\epsilon_0 c \vec{B}_0 \partial_t a$$

Measure Created Photon

$$\nabla \cdot \left(\vec{E}_1(\vec{r}, t) - g_{a\gamma\gamma} a(t) c \vec{B}_0(\vec{r}, t) \right) = \frac{\rho_{e_1}}{\epsilon_0}$$

$$\nabla \times \left(\vec{B}_1(\vec{r}, t) + \frac{g_{a\gamma\gamma} a(t)}{c} \vec{E}_0(\vec{r}, t) \right)$$

$$-\frac{1}{c^2}\partial_t \left(\vec{E}_1(\vec{r}, t) - g_{a\gamma\gamma} a(\vec{r}, t) c \vec{B}_0(\vec{r}, t) \right) = \mu_0 \vec{J}_{e_1}$$

$$\nabla \cdot \vec{B}_1(\vec{r}, t) = 0$$

$$\nabla \times \vec{E}_1(\vec{r}, t) + \partial_t \vec{B}_1(\vec{r}, t) = 0.$$

Photonic Haloscope Equations in terms of Auxiliary Fields

Modified Axion Electrodynamics

(Represents two photons)

$$\nabla \cdot \vec{E} = \frac{\rho_e}{\varepsilon_0} + c g_{a\gamma\gamma} \vec{B} . \nabla a$$

$$\nabla \times \vec{B} - \frac{1}{c^2}\partial_t \vec{E} =$$

$$\mu_0 \vec{J}_e - g_{a\gamma\gamma}\epsilon_0 c \left(\vec{B}\partial_t a + \nabla a \times \vec{E} \right)$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} + \partial_t \vec{B} = 0$$

Applied Background Field

$$\nabla \times \vec{B}_0 = \mu_0 \epsilon_0 \partial_t \vec{E}_0 + \mu_0 \vec{J}_{e_0}$$

$$\nabla \times \vec{E}_0 = -\partial_t \vec{B}_0$$

$$\nabla \cdot \vec{B}_0 = 0$$

$$\nabla \cdot \vec{E}_0 = \epsilon_0^{-1} \rho_{e_0}$$

Constitutive Relations(in vacuum)

$$\vec{\nabla} \cdot \vec{D}_1 = \rho_{e1}$$

$$\vec{\nabla} \times \vec{E}_1 = -\partial_t \vec{B}_1$$

$$\vec{\nabla} \cdot \vec{B}_1 = 0$$

$$\frac{1}{\mu_0}\vec{\nabla} \times \vec{B}_1 = = \vec{J}_{e1} + \epsilon_0 \partial_t \vec{E}_1 - g_{a\gamma\gamma}\epsilon_0 c \vec{B}_0 \partial_t a$$

Measure Created Photon

$$\nabla \cdot \left(\vec{E}_1(\vec{r},t) - g_{a\gamma\gamma} a(t) c \vec{B}_0(\vec{r},t) \right) = \frac{\rho_{e_1}}{\epsilon_0}$$

$$\nabla \times \left(\vec{B}_1(\vec{r},t) + \frac{g_{a\gamma\gamma} a(t)}{c} \vec{E}_0(\vec{r},t) \right)$$

$$-\frac{1}{c^2}\partial_t \left(\vec{E}_1(\vec{r},t) - g_{a\gamma\gamma} a(\vec{r},t) c \vec{B}_0(\vec{r},t) \right) = \mu_0 \vec{J}_{e_1}$$

$$\nabla \cdot \vec{B}_1(\vec{r},t) = 0$$

$$\nabla \times \vec{E}_1(\vec{r},t) + \partial_t \vec{B}_1(\vec{r},t) = 0.$$

Photonic Haloscope Equations in terms of Auxiliary Fields

Modified Axion Electrodynamics

(Represents two photons)

$$\nabla \cdot \vec{E} = \frac{\rho_e}{\varepsilon_0} + c g_{a\gamma\gamma} \vec{B} . \nabla a$$

$$\nabla \times \vec{B} - \frac{1}{c^2} \partial_t \vec{E} =$$

$$\mu_0 \vec{J}_e - g_{a\gamma\gamma} \epsilon_0 c \left(\vec{B} \partial_t a + \nabla a \times \vec{E} \right)$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} + \partial_t \vec{B} = 0$$

Applied Background Field

$$\nabla \times \vec{B}_0 = \mu_0 \epsilon_0 \partial_t \vec{E}_0 + \mu_0 \vec{J}_{e_0}$$

$$\nabla \times \vec{E}_0 = -\partial_t \vec{B}_0$$

$$\nabla \cdot \vec{B}_0 = 0$$

$$\nabla \cdot \vec{E}_0 = \epsilon_0^{-1} \rho_{e_0}$$

$$\vec{\nabla} \cdot \vec{D}_1 = \rho_{e1}$$

$$\vec{\nabla} \times \vec{E}_1 = -\partial_t \vec{B}_1$$

$$\vec{\nabla} \cdot \vec{B}_1 = 0$$

$$\frac{1}{\mu_0} \vec{\nabla} \times \vec{B}_1 = = \vec{J}_{e1} + \epsilon_0 \partial_t \vec{E}_1 - g_{a\gamma\gamma} \epsilon_0 c \vec{B}_0 \partial_t a$$

Constitutive Relations(in vacuum)

$$\frac{1}{\epsilon_0} \vec{D}_1 = \vec{E}_1 - g_{a\gamma\gamma} c a \vec{B}_0$$

Measure Created Photon

$$\nabla \cdot \left(\vec{E}_1(\vec{r}, t) - g_{a\gamma\gamma} a(t) c \vec{B}_0(\vec{r}, t) \right) = \frac{\rho_{e_1}}{\epsilon_0}$$

$$\nabla \times \left(\vec{B}_1(\vec{r}, t) + \frac{g_{a\gamma\gamma} a(t)}{c} \vec{E}_0(\vec{r}, t) \right)$$

$$-\frac{1}{c^2} \partial_t \left(\vec{E}_1(\vec{r}, t) - g_{a\gamma\gamma} a(\vec{r}, t) c \vec{B}_0(\vec{r}, t) \right) = \mu_0 \vec{J}_{e_1}$$

$$\nabla \cdot \vec{B}_1(\vec{r}, t) = 0$$

$$\nabla \times \vec{E}_1(\vec{r}, t) + \partial_t \vec{B}_1(\vec{r}, t) = 0.$$

Photonic Haloscope Equations in terms of Auxiliary Fields

Modified Axion Electrodynamics

(Represents two photons)

$$\nabla \cdot \vec{E} = \frac{\rho_e}{\varepsilon_0} + c g_{a\gamma\gamma} \vec{B} \,.\, \nabla a$$

$$\nabla \times \vec{B} - \frac{1}{c^2}\partial_t \vec{E} =$$

$$\mu_0 \vec{J}_e - g_{a\gamma\gamma}\epsilon_0 c \left(\vec{B} \partial_t a + \nabla a \times \vec{E} \right)$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} + \partial_t \vec{B} = 0$$

Applied Background Field

$$\nabla \times \vec{B}_0 = \mu_0 \epsilon_0 \partial_t \vec{E}_0 + \mu_0 \vec{J}_{e_0}$$

$$\nabla \times \vec{E}_0 = -\partial_t \vec{B}_0$$

$$\nabla \cdot \vec{B}_0 = 0$$

$$\nabla \cdot \vec{E}_0 = \epsilon_0^{-1} \rho_{e_0}$$

$$\vec{\nabla} \cdot \vec{D}_1 = \rho_{e1}$$

$$\vec{\nabla} \times \vec{E}_1 = -\partial_t \vec{B}_1$$

$$\vec{\nabla} \cdot \vec{B}_1 = 0$$

$$\frac{1}{\mu_0} \vec{\nabla} \times \vec{B}_1 = = \vec{J}_{e1} + \epsilon_0 \partial_t \vec{E}_1 - g_{a\gamma\gamma}\epsilon_0 c \vec{B}_0 \partial_t a$$

$$\frac{1}{\epsilon_0} \nabla \times \vec{D}_1 = -\partial_t \vec{B}_1 - g_{a\gamma\gamma} c \nabla \times (a \vec{B}_0)$$

Constitutive Relations(in vacuum)

$$\frac{1}{\epsilon_0} \vec{D}_1 = \vec{E}_1 - g_{a\gamma\gamma} c a \vec{B}_0$$

Measure Created Photon

$$\nabla \cdot \left(\vec{E}_1(\vec{r}, t) - g_{a\gamma\gamma} a(t) c \vec{B}_0(\vec{r}, t) \right) = \frac{\rho_{e_1}}{\epsilon_0}$$

$$\nabla \times \left(\vec{B}_1(\vec{r}, t) + \frac{g_{a\gamma\gamma} a(t)}{c} \vec{E}_0(\vec{r}, t) \right)$$

$$-\frac{1}{c^2}\partial_t \left(\vec{E}_1(\vec{r}, t) - g_{a\gamma\gamma} a(\vec{r}, t) c \vec{B}_0(\vec{r}, t) \right) = \mu_0 \vec{J}_{e_1}$$

$$\nabla \cdot \vec{B}_1(\vec{r}, t) = 0$$

$$\nabla \times \vec{E}_1(\vec{r}, t) + \partial_t \vec{B}_1(\vec{r}, t) = 0.$$

Photonic Haloscope Equations in terms of Auxiliary Fields

Modified Axion Electrodynamics

(Represents two photons)

$$\nabla \cdot \vec{E} = \frac{\rho_e}{\epsilon_0} + c g_{a\gamma\gamma} \vec{B} \,.\, \nabla a$$

$$\nabla \times \vec{B} - \frac{1}{c^2}\partial_t \vec{E} =$$

$$\mu_0 \vec{J}_e - g_{a\gamma\gamma}\epsilon_0 c \left(\vec{B}\partial_t a + \nabla a \times \vec{E} \right)$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} + \partial_t \vec{B} = 0$$

Applied Background Field

$$\nabla \times \vec{B}_0 = \mu_0 \epsilon_0 \partial_t \vec{E}_0 + \mu_0 \vec{J}_{e_0}$$

$$\nabla \times \vec{E}_0 = -\partial_t \vec{B}_0$$

$$\nabla \cdot \vec{B}_0 = 0$$

$$\nabla \cdot \vec{E}_0 = \epsilon_0^{-1} \rho_{e_0}$$

Measure Created Photon

$$\nabla \cdot \left(\vec{E}_1(\vec{r},t) - g_{a\gamma\gamma} a(t) c \vec{B}_0(\vec{r},t) \right) = \frac{\rho_{e_1}}{\epsilon_0}$$

$$\nabla \times \left(\vec{B}_1(\vec{r},t) + \frac{g_{a\gamma\gamma} a(t)}{c} \vec{E}_0(\vec{r},t) \right)$$

$$-\frac{1}{c^2}\partial_t \left(\vec{E}_1(\vec{r},t) - g_{a\gamma\gamma} a(\vec{r},t) c \vec{B}_0(\vec{r},t) \right) = \mu_0 \vec{J}_{e_1}$$

$$\nabla \cdot \vec{B}_1(\vec{r},t) = 0$$

$$\nabla \times \vec{E}_1(\vec{r},t) + \partial_t \vec{B}_1(\vec{r},t) = 0.$$

$$\vec{\nabla} \cdot \vec{D}_1 = \rho_{e1}$$

$$\vec{\nabla} \times \vec{E}_1 = -\partial_t \vec{B}_1$$

$$\vec{\nabla} \cdot \vec{B}_1 = 0$$

$$\frac{1}{\mu_0}\vec{\nabla} \times \vec{B}_1 = = \vec{J}_{e1} + \epsilon_0 \partial_t \vec{E}_1 - g_{a\gamma\gamma}\epsilon_0 c \vec{B}_0 \partial_t a$$

$$\frac{1}{\epsilon_0}\nabla \times \vec{D}_1 = -\partial_t \vec{B}_1 - g_{a\gamma\gamma} c \nabla \times (a \vec{B}_0)$$

$$\nabla a = 0$$

Constitutive Relations(in vacuum)

$$\frac{1}{\epsilon_0}\vec{D}_1 = \vec{E}_1 - g_{a\gamma\gamma} c a \vec{B}_0$$

Photonic Haloscope Equations in terms of Auxiliary Fields

Modified Axion Electrodynamics

(Represents two photons)

$$\nabla\cdot\vec{E}=\frac{\rho_e}{\varepsilon_0}+cg_{a\gamma\gamma}\vec{B}\,.\nabla a$$

$$\nabla\times\vec{B}-\frac{1}{c^2}\partial_t\vec{E}=$$

$$\mu_0\vec{J}_e-g_{a\gamma\gamma}\epsilon_0 c\left(\vec{B}\partial_t a+\nabla a\times\vec{E}\right)$$

$$\nabla\cdot\vec{B}=0$$

$$\nabla\times\vec{E}+\partial_t\vec{B}=0$$

Applied Background Field

$$\nabla\times\vec{B}_0=\mu_0\epsilon_0\partial_t\vec{E}_0+\mu_0\vec{J}_{e_0}$$

$$\nabla\times\vec{E}_0=-\partial_t\vec{B}_0$$

$$\nabla\cdot\vec{B}_0=0$$

$$\nabla\cdot\vec{E}_0=\epsilon_0^{-1}\rho_{e_0}$$

Measure Created Photon

$$\nabla\cdot\left(\vec{E}_1(\vec{r},t)-g_{a\gamma\gamma}a(t)c\vec{B}_0(\vec{r},t)\right)=\frac{\rho_{e_1}}{\epsilon_0}$$

$$\nabla\times\left(\vec{B}_1(\vec{r},t)+\frac{g_{a\gamma\gamma}a(t)}{c}\vec{E}_0(\vec{r},t)\right)$$

$$-\frac{1}{c^2}\partial_t\left(\vec{E}_1(\vec{r},t)-g_{a\gamma\gamma}a(\vec{r},t)c\vec{B}_0(\vec{r},t)\right)=\mu_0\vec{J}_{e_1}$$

$$\nabla\cdot\vec{B}_1(\vec{r},t)=0$$

$$\nabla\times\vec{E}_1(\vec{r},t)+\partial_t\vec{B}_1(\vec{r},t)=0.$$

$$\vec{\nabla}\cdot\vec{D}_1=\rho_{e1}$$

$$\vec{\nabla}\times\vec{E}_1=-\partial_t\vec{B}_1$$

$$\vec{\nabla}\cdot\vec{B}_1=0$$

$$\frac{1}{\mu_0}\vec{\nabla}\times\vec{B}_1==\vec{J}_{e1}+\epsilon_0\partial_t\vec{E}_1-g_{a\gamma\gamma}\epsilon_0 c\vec{B}_0\partial_t a$$

$$\frac{1}{\epsilon_0}\nabla\times\vec{D}_1=-\partial_t\vec{B}_1-g_{a\gamma\gamma}c\nabla\times(a\vec{B}_0)$$

$$\nabla a=0$$

$$\frac{1}{\epsilon_0}\nabla\times\vec{D}_1=-\partial_t\vec{B}_1-\frac{g_{a\gamma\gamma}a}{c}\partial_t\vec{E}_0-g_{a\gamma\gamma}ac\mu_0\vec{J}_{e0}$$

Constitutive Relations(in vacuum)

$$\frac{1}{\epsilon_0}\vec{D}_1=\vec{E}_1-g_{a\gamma\gamma}ca\vec{B}_0$$

Photonic Haloscope Equations in terms of Auxiliary Fields

Modified Axion Electrodynamics

(Represents two photons)

$$\nabla \cdot \vec{E} = \frac{\rho_e}{\epsilon_0} + c g_{a\gamma\gamma} \vec{B} . \nabla a$$

$$\nabla \times \vec{B} - \frac{1}{c^2}\partial_t \vec{E} =$$

$$\mu_0 \vec{J}_e - g_{a\gamma\gamma}\epsilon_0 c \left(\vec{B}\partial_t a + \nabla a \times \vec{E} \right)$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} + \partial_t \vec{B} = 0$$

Applied Background Field

$$\nabla \times \vec{B}_0 = \mu_0 \epsilon_0 \partial_t \vec{E}_0 + \mu_0 \vec{J}_{e_0}$$

$$\nabla \times \vec{E}_0 = -\partial_t \vec{B}_0$$

$$\nabla \cdot \vec{B}_0 = 0$$

$$\nabla \cdot \vec{E}_0 = \epsilon_0^{-1} \rho_{e_0}$$

Measure Created Photon

$$\nabla \cdot \left(\vec{E}_1(\vec{r},t) - g_{a\gamma\gamma} a(t) c \vec{B}_0(\vec{r},t) \right) = \frac{\rho_{e_1}}{\epsilon_0}$$

$$\nabla \times \left(\vec{B}_1(\vec{r},t) + \frac{g_{a\gamma\gamma} a(t)}{c} \vec{E}_0(\vec{r},t) \right)$$

$$-\frac{1}{c^2}\partial_t \left(\vec{E}_1(\vec{r},t) - g_{a\gamma\gamma} a(\vec{r},t) c \vec{B}_0(\vec{r},t) \right) = \mu_0 \vec{J}_{e_1}$$

$$\nabla \cdot \vec{B}_1(\vec{r},t) = 0$$

$$\nabla \times \vec{E}_1(\vec{r},t) + \partial_t \vec{B}_1(\vec{r},t) = 0.$$

$$\vec{\nabla} \cdot \vec{D}_1 = \rho_{e1}$$

$$\vec{\nabla} \times \vec{E}_1 = -\partial_t \vec{B}_1$$

$$\vec{\nabla} \cdot \vec{B}_1 = 0$$

$$\frac{1}{\mu_0}\vec{\nabla} \times \vec{B}_1 = = \vec{J}_{e1} + \epsilon_0 \partial_t \vec{E}_1 - g_{a\gamma\gamma}\epsilon_0 c \vec{B}_0 \partial_t a$$

$$\frac{1}{\epsilon_0}\nabla \times \vec{D}_1 = -\partial_t \vec{B}_1 - g_{a\gamma\gamma} c \nabla \times (a \vec{B}_0)$$

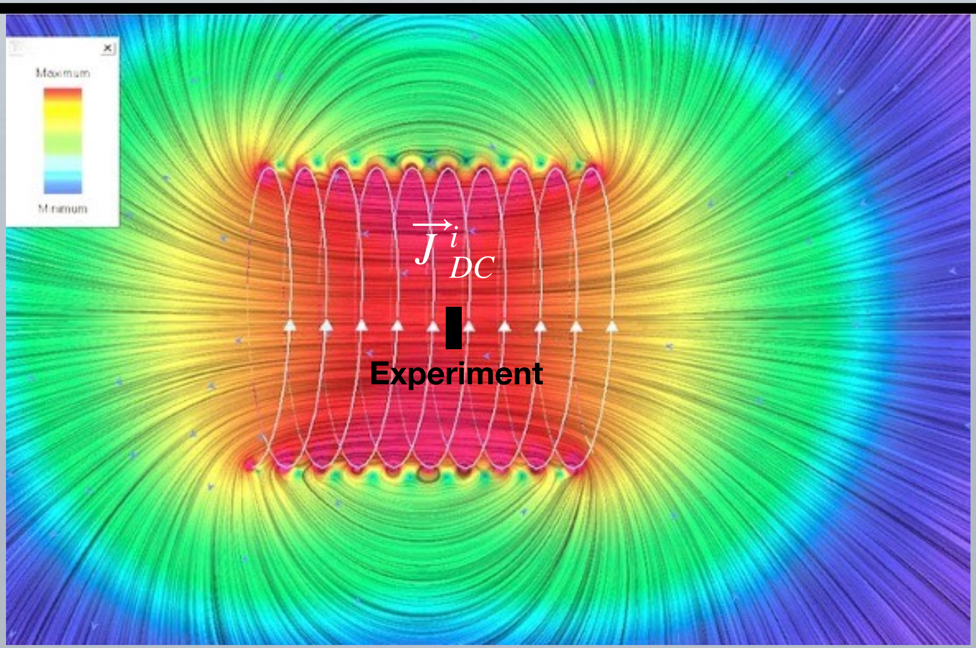
$$\nabla a = 0$$

$$\frac{1}{\epsilon_0}\nabla \times \vec{D}_1 = -\partial_t \vec{B}_1 - \frac{g_{a\gamma\gamma} a}{c}\partial_t \vec{E}_0 - g_{a\gamma\gamma} a c \mu_0 \vec{J}_{e0}$$

Constitutive Relations(in vacuum)

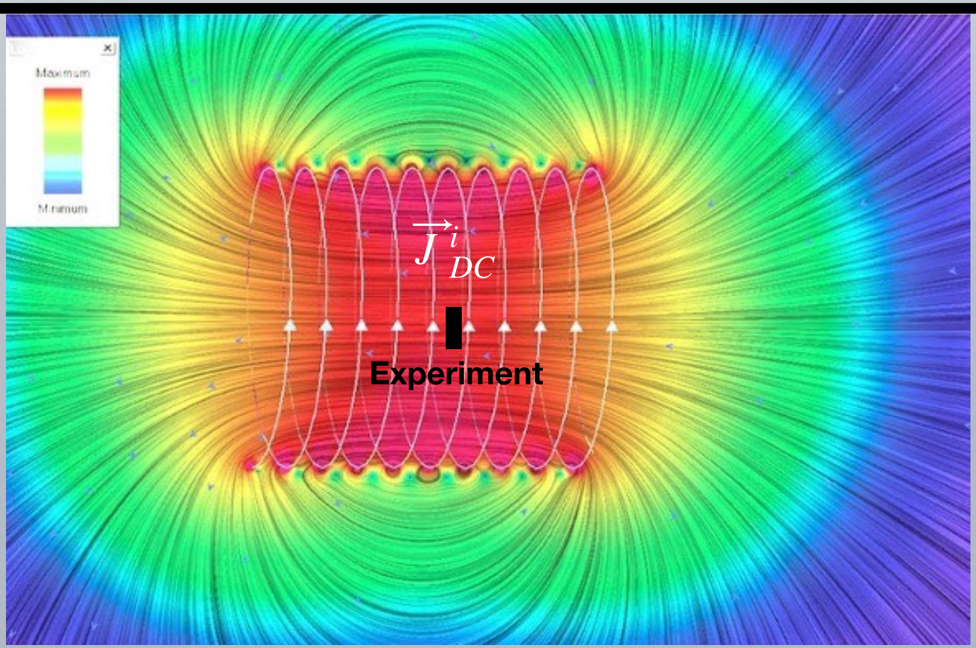
$$\frac{1}{\epsilon_0}\vec{D}_1 = \vec{E}_1 - g_{a\gamma\gamma} c a \vec{B}_0$$

Eg. Solenoidal DC Magnetic field: Defined by a surface when $\lambda_a >$ Experiment



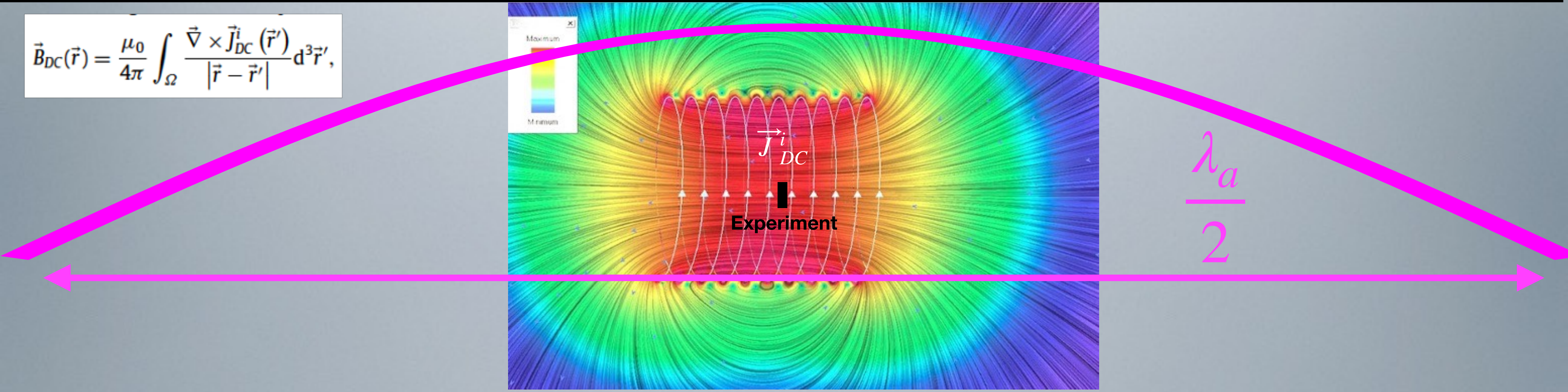
Eg. Solenoidal DC Magnetic field: Defined by a surface when $\lambda_a >$ Experiment

$$\vec{B}_{DC}(\vec{r}) = \frac{\mu_0}{4\pi} \int_{\Omega} \frac{\vec{\nabla} \times \vec{J}^{i}_{DC}(\vec{r}')}{|\vec{r} - \vec{r}'|} d^3\vec{r}',$$



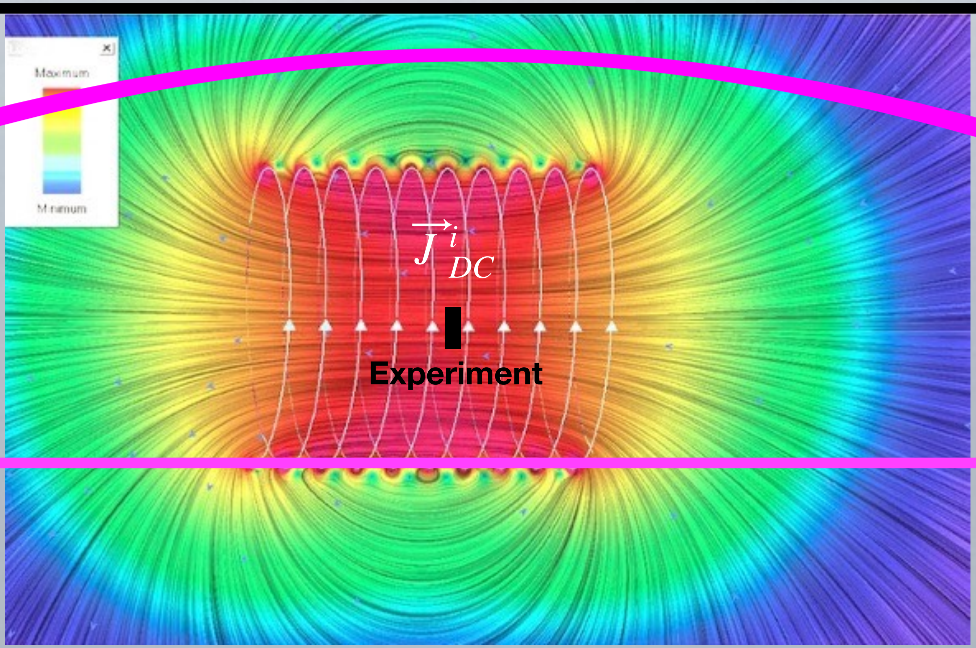
Eg. Solenoidal DC Magnetic field: Defined by a surface when $\lambda_a >$ Experiment

$$\vec{B}_{DC}(\vec{r}) = \frac{\mu_0}{4\pi}\int_{\Omega}\frac{\vec{\nabla}\times\vec{J}^{i}_{DC}(\vec{r}')}{|\vec{r}-\vec{r}'|}d^3\vec{r}',$$



Eg. Solenoidal DC Magnetic field: Defined by a surface when $\lambda_a >$ Experiment

$$\vec{B}_{DC}(\vec{r}) = \frac{\mu_0}{4\pi}\int_\Omega \frac{\vec{\nabla}\times\vec{J}^i_{DC}(\vec{r}')}{|\vec{r}-\vec{r}'|}d^3\vec{r}',$$

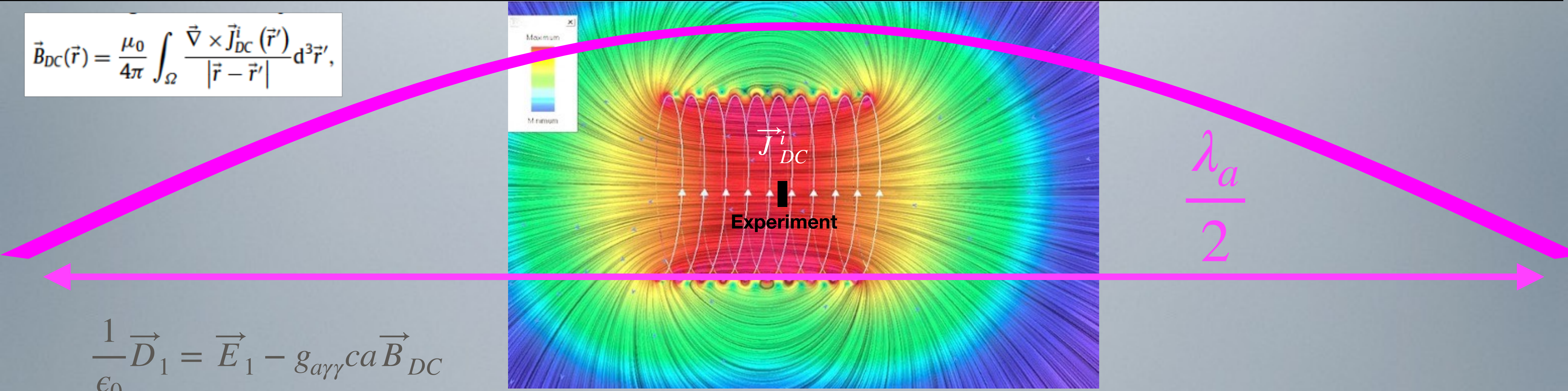


$$\frac{\lambda_a}{2}$$

$$\frac{1}{\epsilon_0}\vec{D}_1 = \vec{E}_1 - g_{a\gamma\gamma}ca\vec{B}_{DC}$$

Eg. Solenoidal DC Magnetic field: Defined by a surface when $\lambda_a >$ Experiment

$$\vec{B}_{DC}(\vec{r}) = \frac{\mu_0}{4\pi} \int_\Omega \frac{\vec{\nabla} \times \vec{J}^i_{DC}(\vec{r}')}{|\vec{r} - \vec{r}'|} d^3\vec{r}',$$

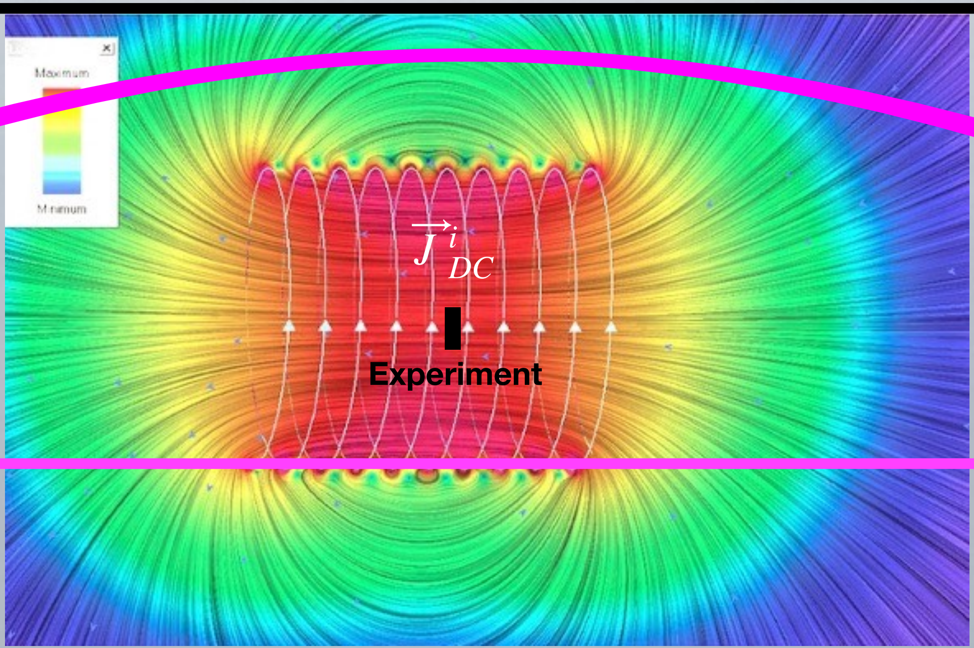


$$\frac{1}{\epsilon_0}\vec{D}_1 = \vec{E}_1 - g_{a\gamma\gamma} c a \vec{B}_{DC}$$

$$\frac{1}{\epsilon_0}\vec{P}_{1a} = -\, g_{a\gamma\gamma} a(t) c \vec{B}_{DC}(\vec{r}, t)$$

Eg. Solenoidal DC Magnetic field: Defined by a surface when $\lambda_a >$ Experiment

$$\vec{B}_{DC}(\vec{r}) = \frac{\mu_0}{4\pi}\int_{\Omega}\frac{\vec{\nabla}\times\vec{J}^{i}_{DC}(\vec{r}')}{|\vec{r}-\vec{r}'|}d^3\vec{r}',$$



$$\frac{\lambda_a}{2}$$

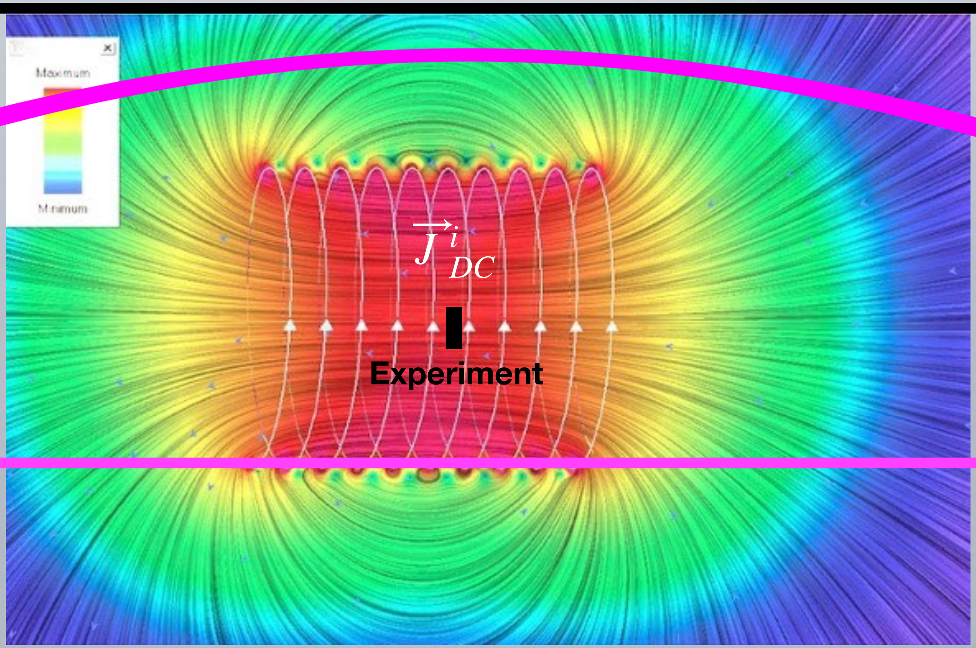
$$\frac{1}{\epsilon_0}\vec{D}_1 = \vec{E}_1 - g_{a\gamma\gamma}ca\vec{B}_{DC}$$

$$\frac{1}{\epsilon_0}\vec{P}_{1a} = -g_{a\gamma\gamma}a(t)c\vec{B}_{DC}(\vec{r},t)$$

$$= -\frac{1}{4\pi}\int_{\Omega}\frac{\vec{\nabla}\times\vec{J}^{i}_{ma}(\vec{r}',t)}{|\vec{r}-\vec{r}'|}d^3\vec{r}'$$

Eg. Solenoidal DC Magnetic field: Defined by a surface when $\lambda_a >$ Experiment

$$\vec{B}_{DC}(\vec{r}) = \frac{\mu_0}{4\pi}\int_{\Omega}\frac{\vec{\nabla}\times\vec{J}^{i}_{DC}(\vec{r}')}{|\vec{r}-\vec{r}'|}\mathrm{d}^3\vec{r}',$$



$$\frac{\lambda_a}{2}$$

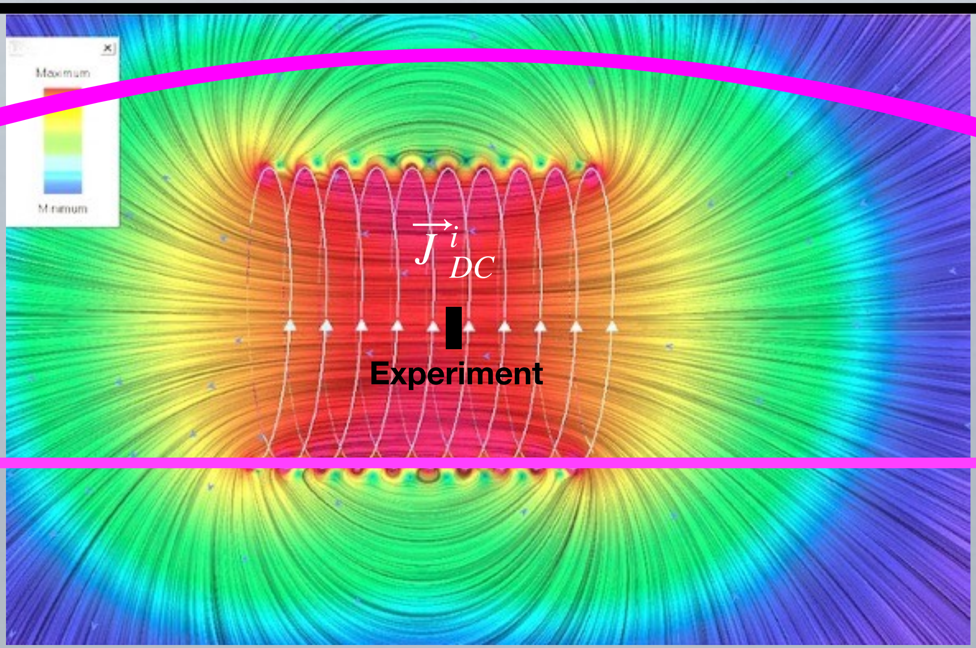
$$\frac{1}{\epsilon_0}\vec{D}_1 = \vec{E}_1 - g_{a\gamma\gamma}ca\vec{B}_{DC}$$

$$\frac{1}{\epsilon_0}\vec{P}_{1a} = -g_{a\gamma\gamma}a(t)c\vec{B}_{DC}(\vec{r},t) = -\frac{1}{4\pi}\int_{\Omega}\frac{\vec{\nabla}\times\vec{J}^{i}_{ma}(\vec{r}',t)}{|\vec{r}-\vec{r}'|}\mathrm{d}^3\vec{r}'$$

$$\frac{1}{\epsilon_0}\nabla\times\vec{P}_{a1} = -g_{a\gamma\gamma}a(t)c\mu_0\vec{J}_{DC}$$

Eg. Solenoidal DC Magnetic field: Defined by a surface when $\lambda_a >$ Experiment

$$\vec{B}_{DC}(\vec{r}) = \frac{\mu_0}{4\pi}\int_\Omega \frac{\vec{\nabla}\times\vec{J}^i_{DC}(\vec{r}\,')}{|\vec{r}-\vec{r}\,'|}d^3\vec{r}\,',$$



$$\frac{\lambda_a}{2}$$

$$\frac{1}{\epsilon_0}\vec{D}_1 = \vec{E}_1 - g_{a\gamma\gamma}ca\vec{B}_{DC}$$

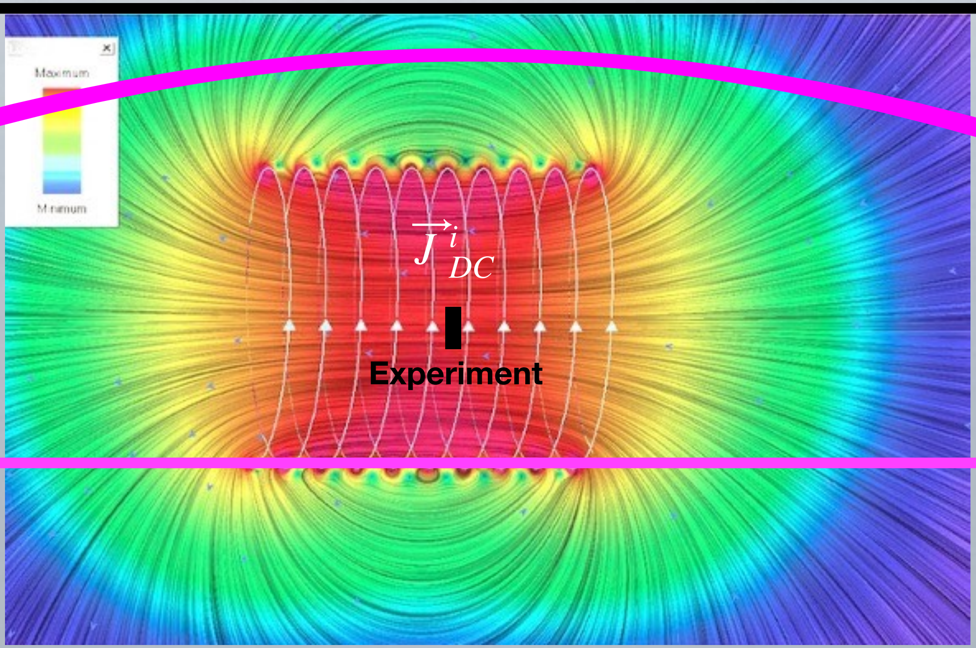
$$\frac{1}{\epsilon_0}\vec{P}_{1a} = -g_{a\gamma\gamma}a(t)c\vec{B}_{DC}(\vec{r},t) = -\frac{1}{4\pi}\int_\Omega \frac{\vec{\nabla}\times\vec{J}^i_{ma}(\vec{r}\,',t)}{|\vec{r}-\vec{r}\,'|}d^3\vec{r}\,'$$

$$\frac{1}{\epsilon_0}\nabla\times\vec{P}_{a1} = -g_{a\gamma\gamma}a(t)c\mu_0\vec{J}_{DC}$$

$$\vec{J}^i_{ma}(\vec{r},t) = g_{a\gamma\gamma}a(t)c\mu_0\vec{J}^i_{DC}(\vec{r})$$

Eg. Solenoidal DC Magnetic field: Defined by a surface when $\lambda_a >$ Experiment

$$\vec{B}_{DC}(\vec{r}) = \frac{\mu_0}{4\pi}\int_{\Omega}\frac{\vec{\nabla}\times\vec{J}^{i}_{DC}(\vec{r}')}{|\vec{r}-\vec{r}'|}d^3\vec{r}',$$



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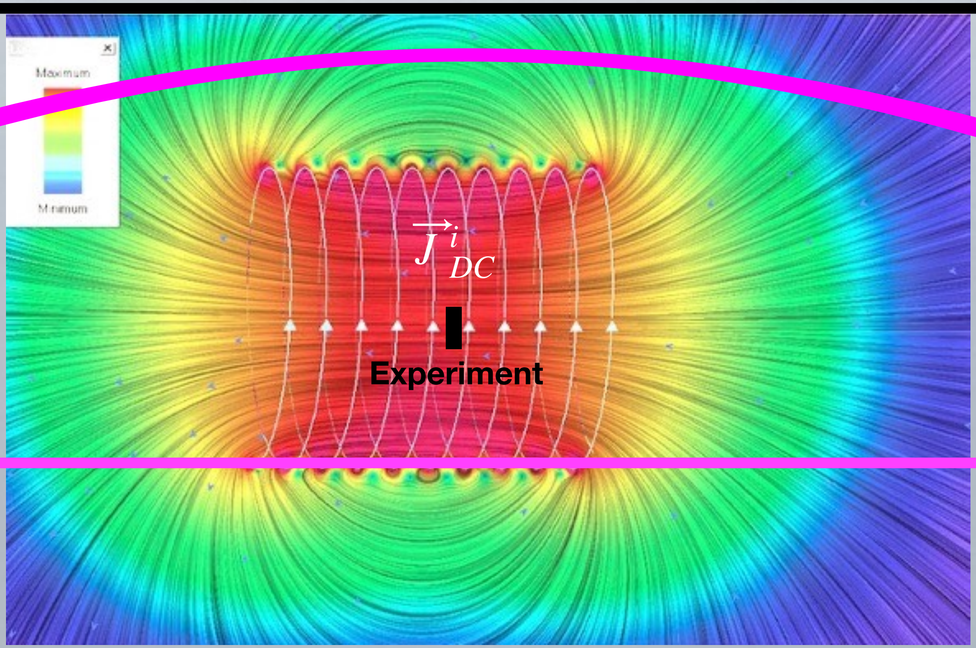

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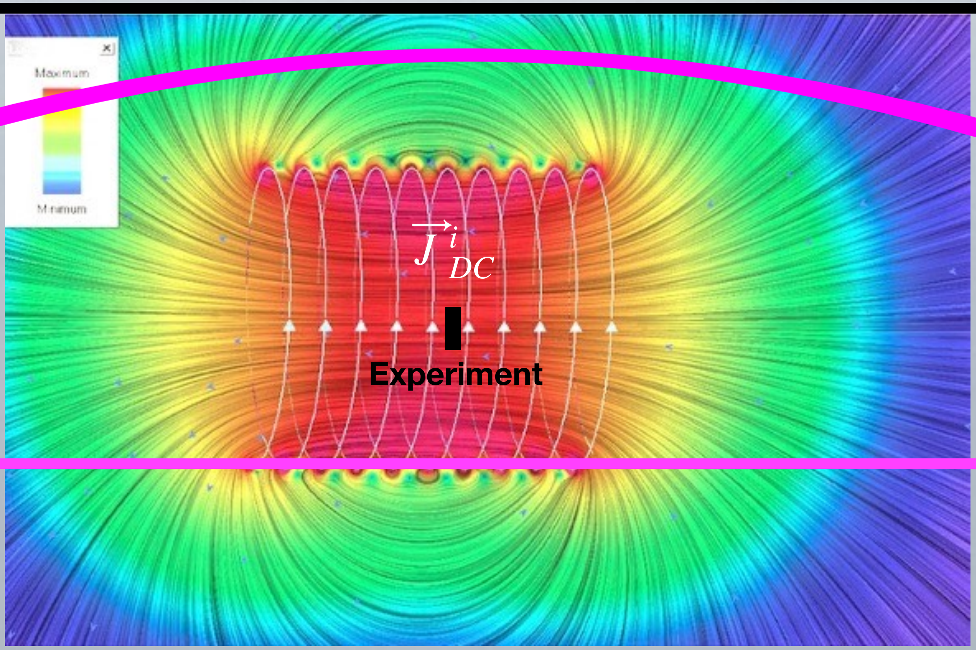

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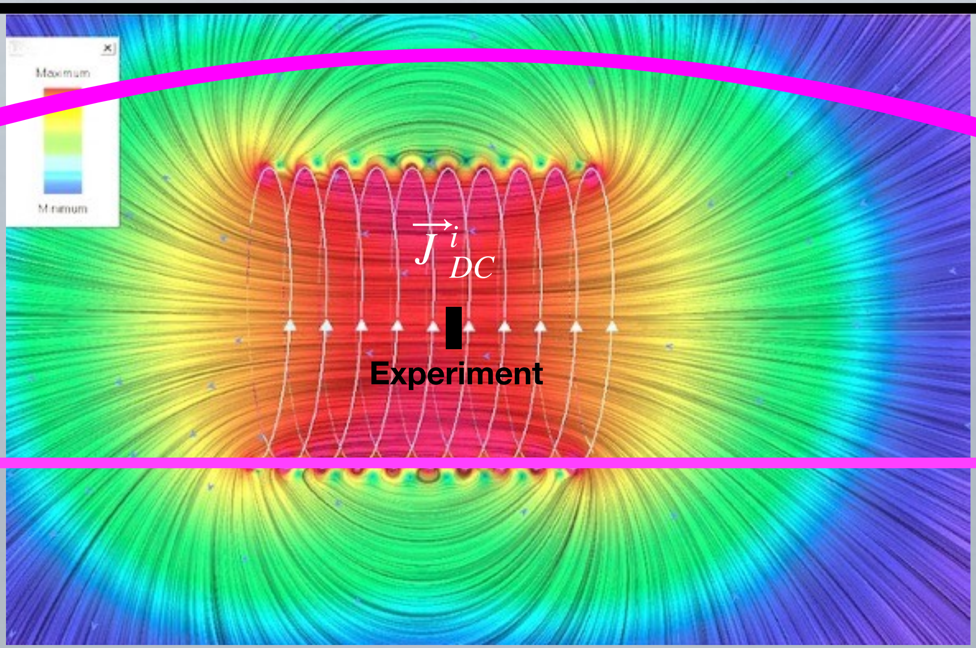

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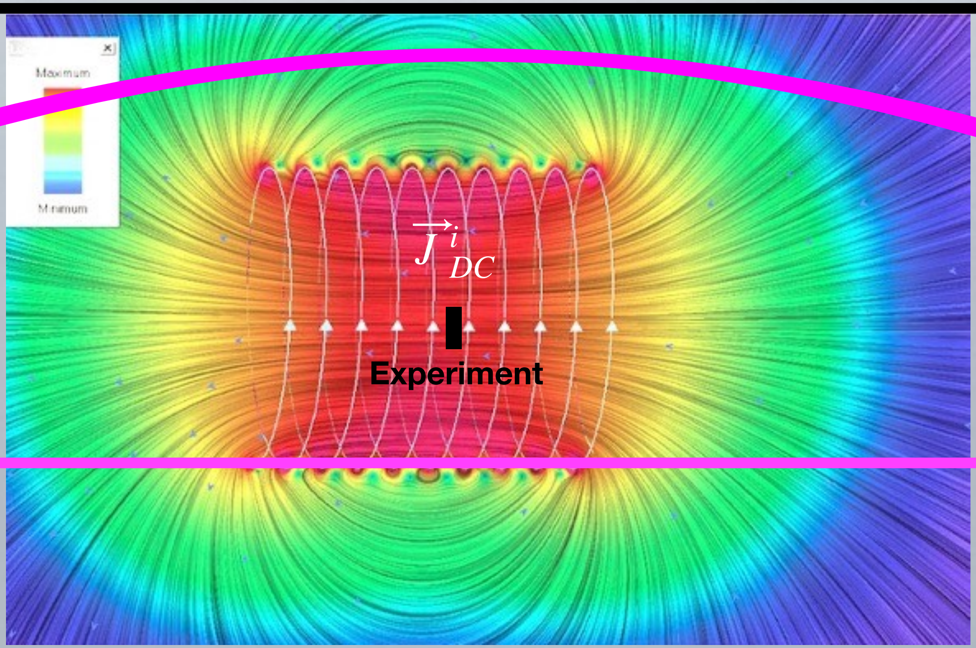

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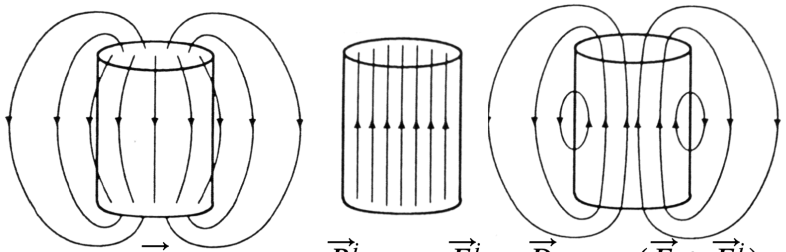
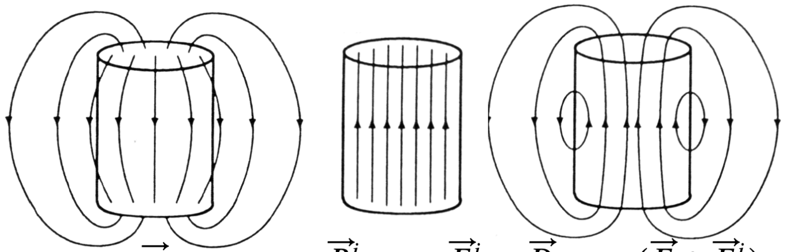
Polarization generated by axion induced fictitious magnetic current boundary -> similar to an electret or voltage source : Has an Electric Vector Potential!

ELECTRET VOLTAGE SOURCE



$\vec{E}$ $\quad$ $\vec{P}^{i}_{b} = \epsilon_0 \epsilon_r \vec{E}^{i}_{b}$ $\quad$ $\vec{D} = \epsilon_0 \epsilon_r (\vec{E} + \vec{E}^{i}_{b})$

ELECTRET VOLTAGE SOURCE



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Bulk polarization modifies the boundary Luttinger theorem

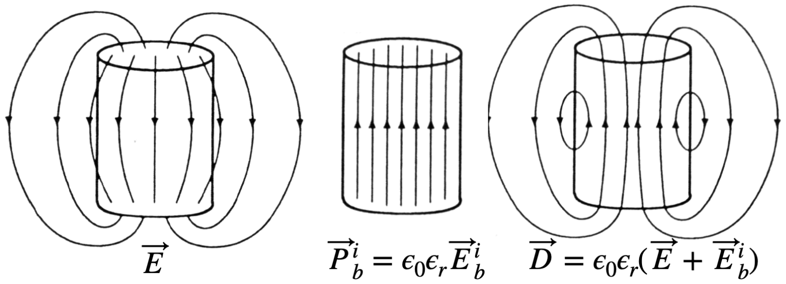
PHYSICAL REVIEW RESEARCH **3**, 023011 (2021)

Electric polarization as a nonquantized topological response and boundary Luttinger theorem

Xue-Yang Song,[1,2] Yin-Chen He,[2] Ashvin Vishwanath,[1] and Chong Wang[2]
[1]*Department of Physics, Harvard University, Cambridge, Massachusetts 02138, USA*
[2]*Perimeter Institute for Theoretical Physics, Waterloo, Ontario N2L 2Y5, Canada*

(Received 22 February 2021; accepted 5 March 2021; published 2 April 2021)

ELECTRET VOLTAGE SOURCE



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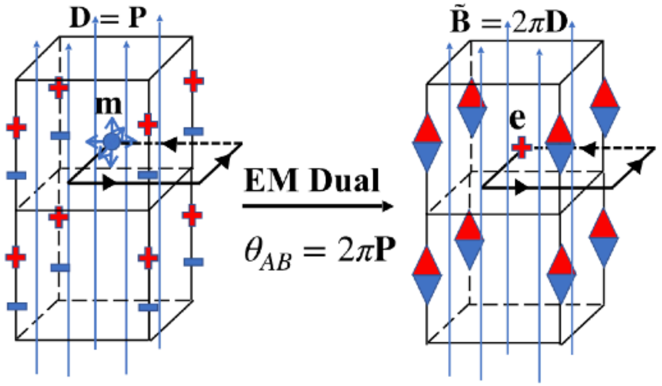

FIG. 2. The semiclassical picture relates projective representation of monopoles to polarization by electromagnetic duality that exchanges magnetic monopole and electric charge. The electric displacement field (left) $\mathbf{D} = \mathbf{P}$ is mapped to magnetic field (right) $\tilde{\mathbf{B}} = 2\pi\mathbf{D}$ and the monopole to an electric charge. The Aharonom-Bohm (AB) phase θ_{AB} seen by the electric charge is proportional to magnetic field (right), hence the monopole Berry phase proportional to displacement field (left).

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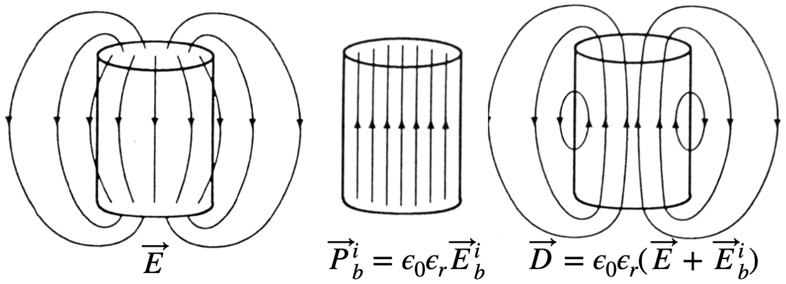

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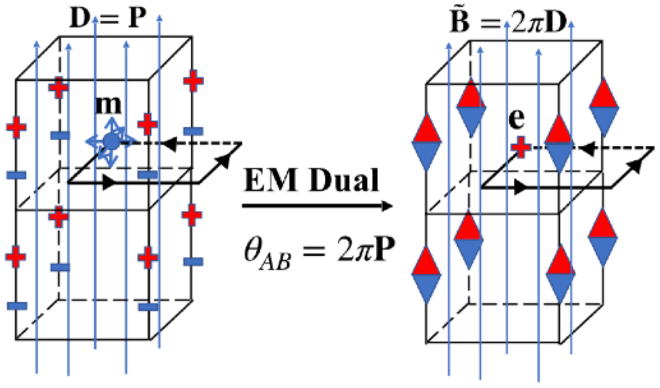
TABLE I. Polarization density **P** is related to the properties of the monopoles in dimensions $d = 1, 2, 3$.

	Monopole property	Polarization
1D	Berry phase	$\Phi = 2\pi P$
2D	Momentum	$\mathbf{k}_{\mathcal{M}} = 2\pi\hat{z} \times \mathbf{P}$
3D	Projective momentum	$T_j^{-1}T_i^{-1}T_jT_i = \exp(i2\pi\epsilon^{ijk}P_k)$

We summarize the connection between bulk polarization and monopole (instanton) properties in $d = 1, 2, 3$ in Table I.



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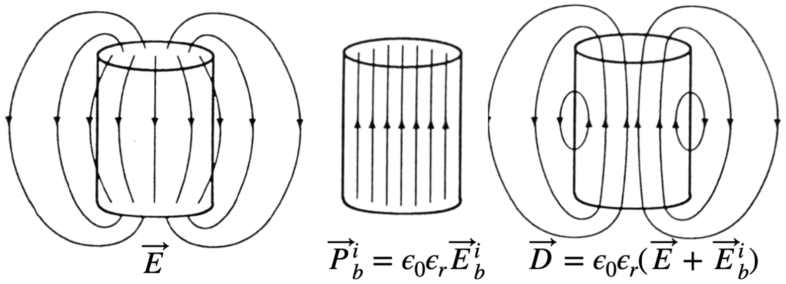

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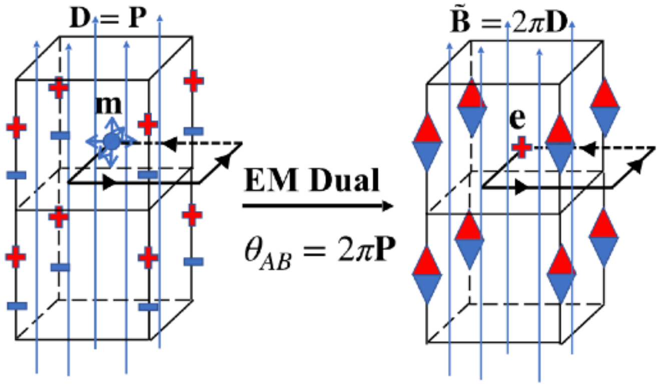

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APPENDIX C: POLARIZATION AND OTHER TOPOLOGICAL QUANTITIES

$$\frac{\Theta}{4\pi^2}\mathbf{E}\cdot\mathbf{B}, \quad \text{(C1)}$$

$$\Delta\mathbf{P} = \frac{\Theta}{4\pi^2}\mathbf{B}. \quad \text{(C2)}$$

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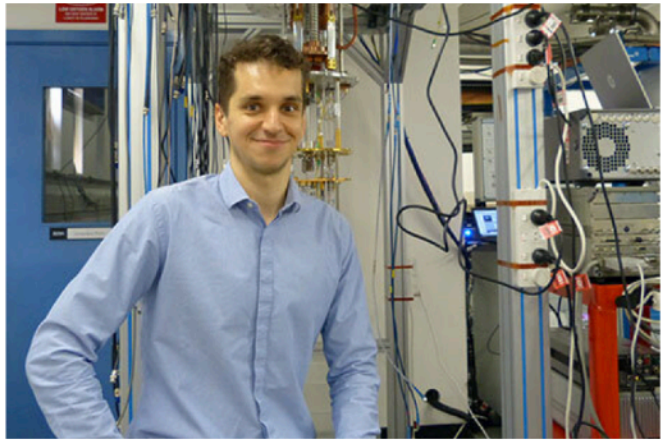
Electrodynamics of Free- and Bound-Charge Electricity Generators Using Impressed Sources

Michael E. Tobar, Ben T. McAllister, and Maxim Goryachev

Phys. Rev. Applied **15**, 014007 – Published 6 January 2021

Professor Michael Tobar

Director



Dr Maxim Goryachev

Lecturer, Research Intensive

Dr Ben McAllister

Forrest Prospect Fellow

Sensitivity of Low-Mass and Resonant Axion Haloscopes

PHYSICAL REVIEW D **105**, 045009 (2022)

Poynting vector controversy in axion modified electrodynamics

Michael E. Tobar,* Ben T. McAllister, and Maxim Goryachev

ARC Centre of Excellence for Engineered Quantum Systems and ARC Centre of Excellence for Dark Matter Particle Physics, Department of Physics, University of Western Australia, 35 Stirling Highway, Crawley, Western Australia 6009, Australia

(Received 9 September 2021; accepted 28 January 2022; published 15 February 2022)

$$E = mc^2$$

Physics of the Dark Universe 26 (2019) 100339

Contents lists available at ScienceDirect

Physics of the Dark Universe

journal homepage: www.elsevier.com/locate/dark

Modified axion electrodynamics as impressed electromagnetic sources through oscillating background polarization and magnetization

Michael E. Tobar*, Ben T. McAllister, Maxim Goryachev

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Physics of the Dark Universe 30 (2020) 100624

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Broadband electrical action sensing techniques with conducting wires for low-mass dark matter axion detection

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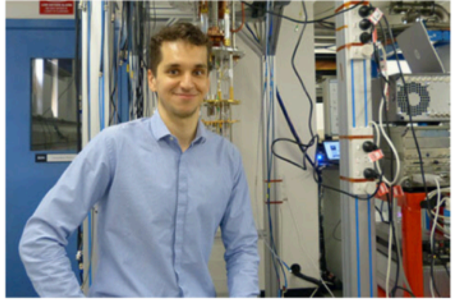

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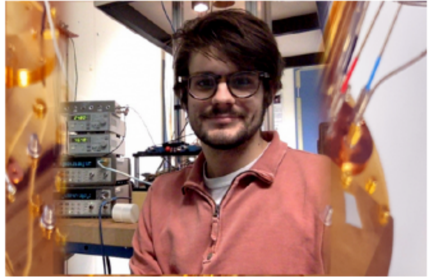
Dr Ben McAllister
Research Associate

Professor Eugene Ivanov
Winthrop Research Professor—Dept of Physics



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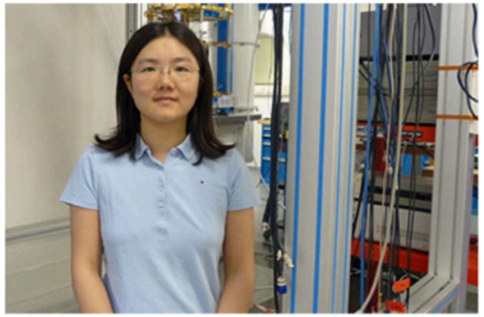
Postdoctoral Research Associate



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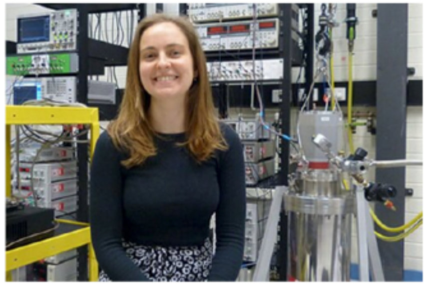

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Elrina Hartman
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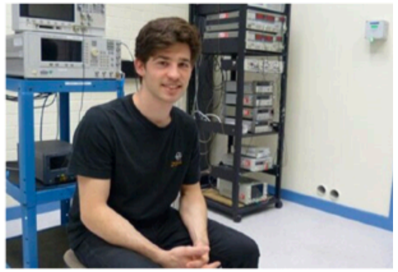

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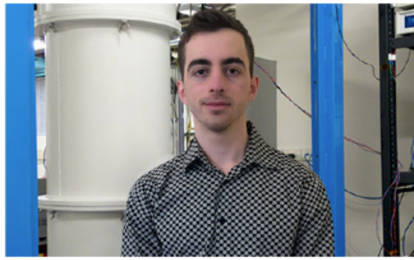
Masters

Robert Crew
BPhil (Hons) Placement

Daniel Tobar
BPhil (Hons) Placement



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BPhil (Hons)Placement

Steve Osborne
Technician

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