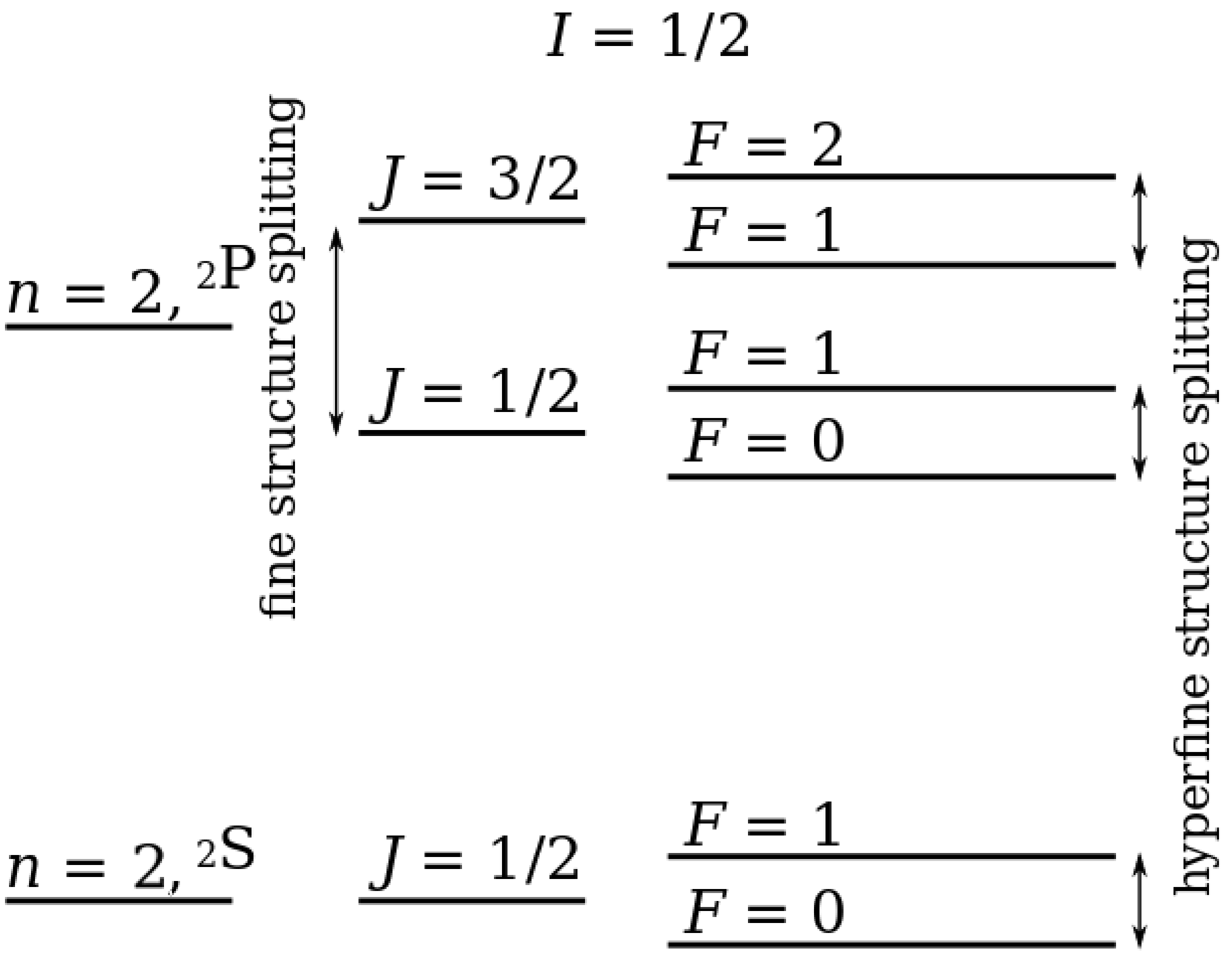
HYPERFINE-SPECTROSCOPY MEASUREMENT OF METASTABLE HYDROGEN ATOMS WITH A SONA-TRANSITION UNIT

26TH SEPTEMBER 2022 I NICOLAS FAATZ

Member of the Helmholtz Association

INTRODUCTION

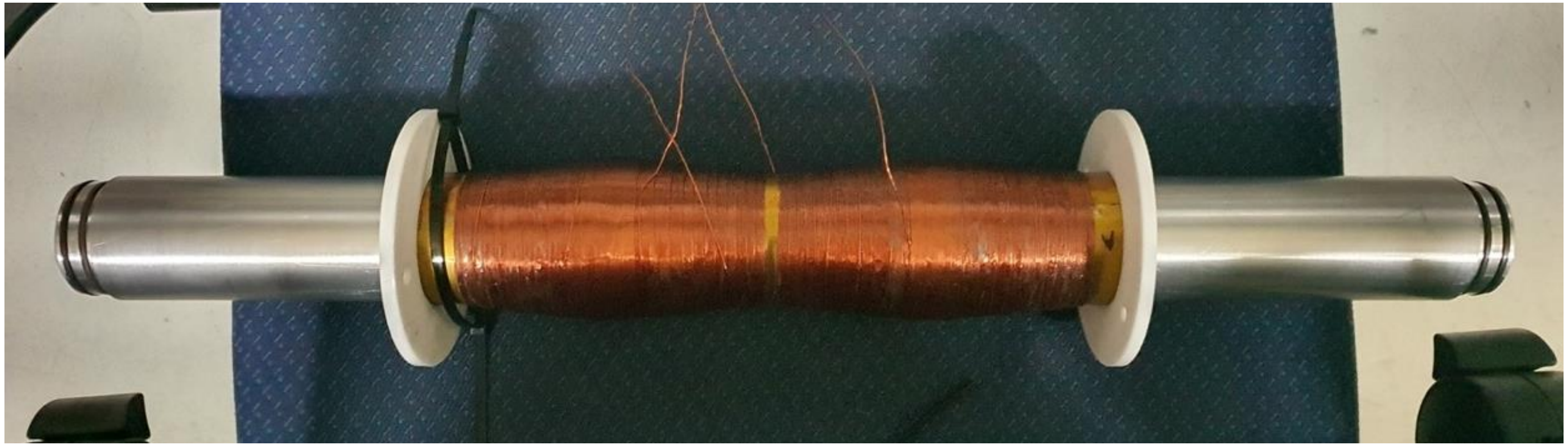


$$\vec{J} = \vec{L} \otimes 1 + 1 \otimes \vec{S}$$

$$\vec{F} = \vec{J} \otimes 1 + 1 \otimes \vec{I}$$

https://en.wikipedia.org/wiki/Hyperfine_structure

SONA TRANSITION UNIT



S. Aswani, Bachelor thesis, (2022)

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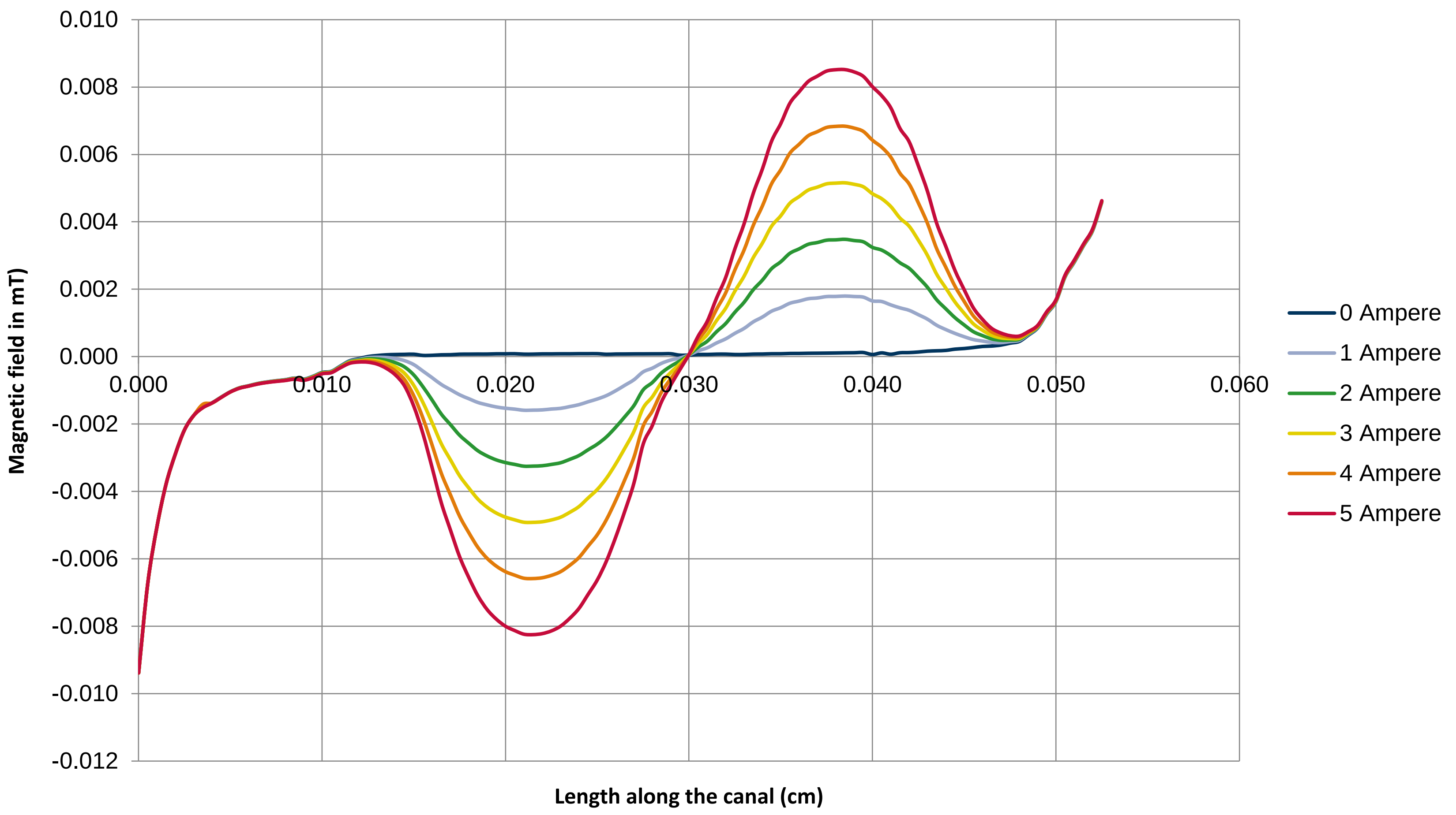
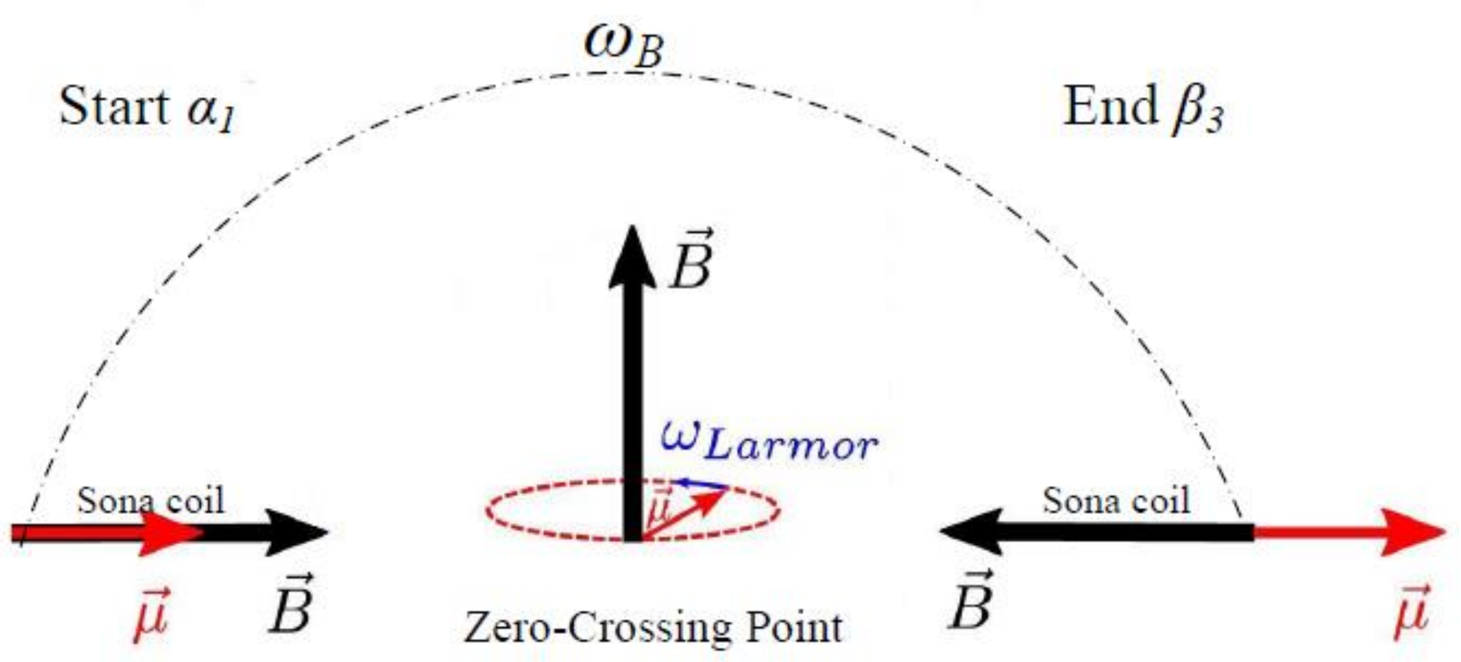


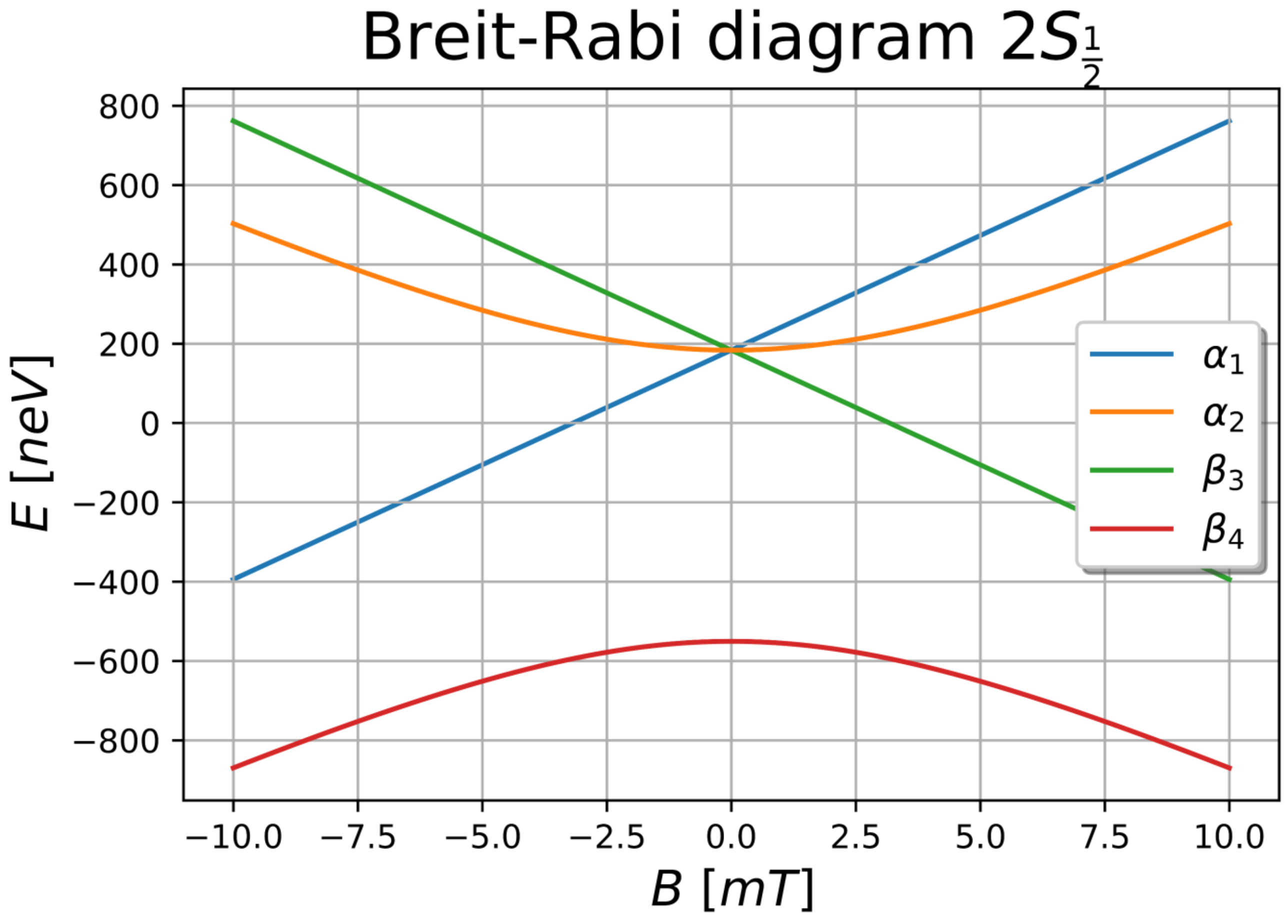
S. Aswani, Bachelor thesis, (2022)

ZERO CROSSING CONSTRAINMENT



$$\omega_B \gg \omega_{Larmor}$$

ORIGINAL IDEA OF A SONA UNIT

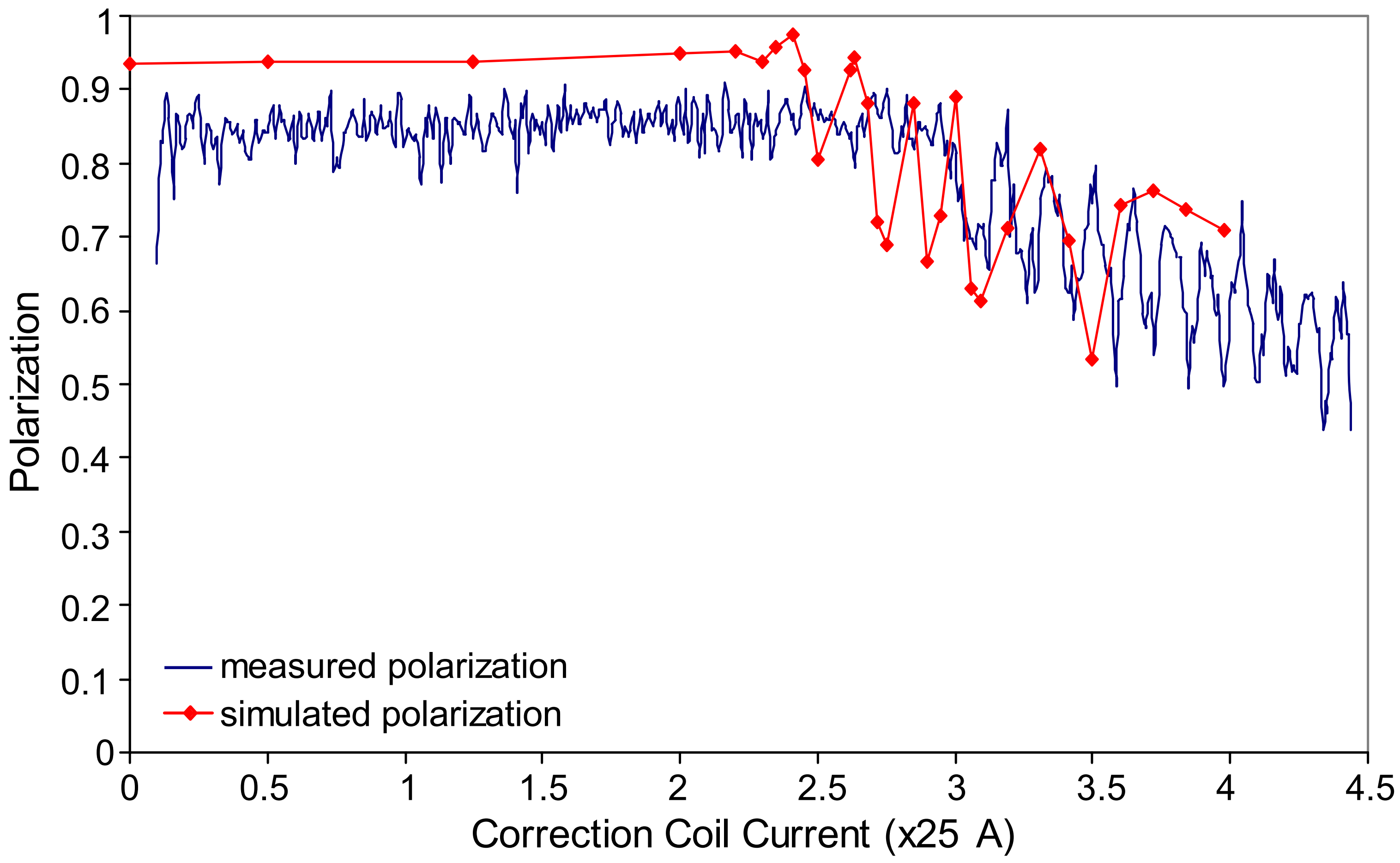


For positive quantization axis

$|\alpha_1\rangle = |\Uparrow, \uparrow\rangle$ $|\beta_3\rangle = |\Downarrow, \downarrow\rangle$

$B \Rightarrow -B$ $|\alpha_1\rangle \Leftrightarrow |\beta_3\rangle$

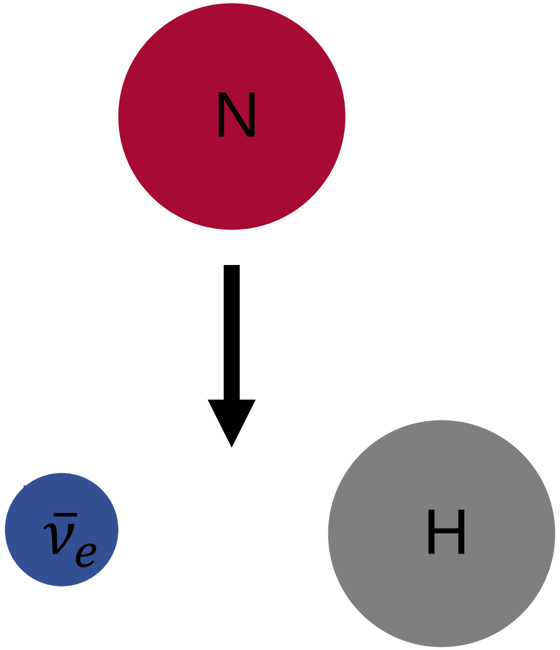
HIGH POLARIZATION MEASUREMENT AT BNL



A. Zelenski, Proceedings of EPAC08, Genoa, Italy

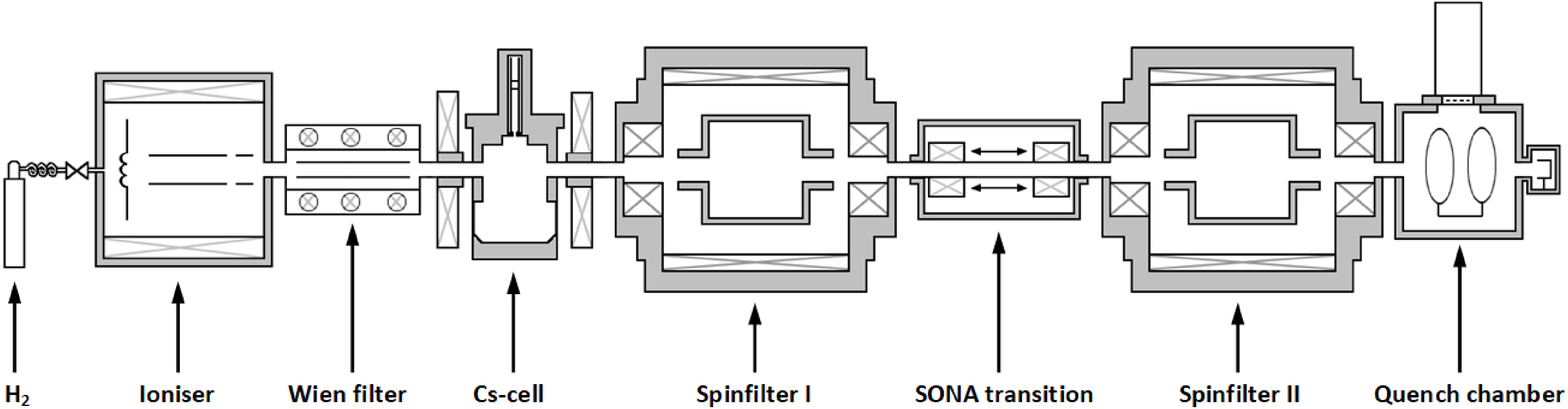
BOUND BETA DECAY

$$\Lambda = \frac{\vec{S} \cdot \vec{P}}{|\vec{P}|} \qquad n \rightarrow {H^*}_{2S_{1/2}} + \bar{\nu}_e$$



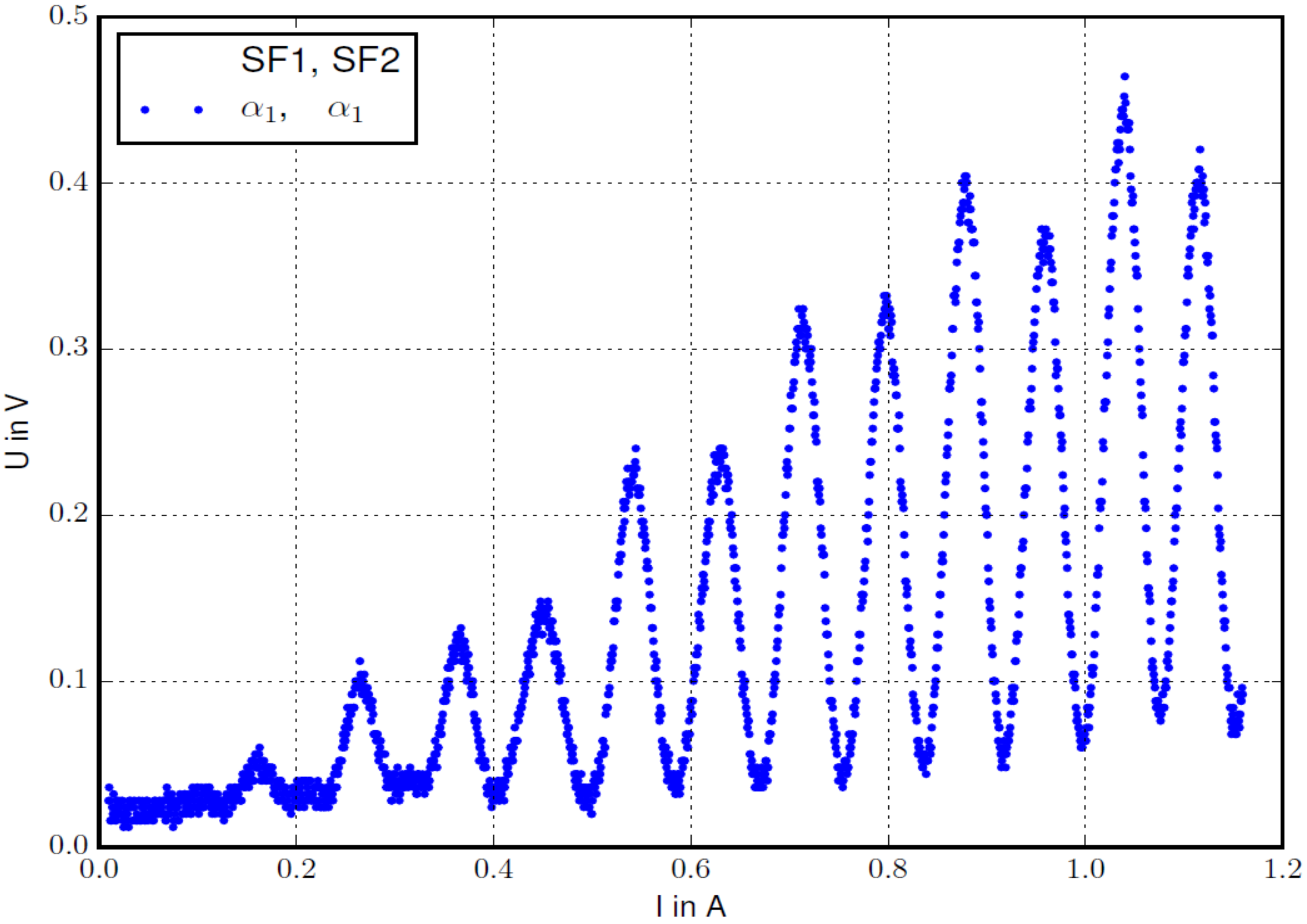
- Two body decay ➔ fixed trajectories and energies
- Antineutrinos $\bar{\nu}_e$ are righthanded
- LSP positioned such that $\bar{\nu}_e$ has spin up
- β_3 state is then forbidden
- Energy is in the range that the Lamb-shift polarimeter (LSP) can separate the single Hyperfine states

EXPERIMENTAL SET-UP



R. Engels, et al., *Eur. Phys. J. D,* vol. **75**, no. 9, p. 257, (2021)

FIRST MEASUREMENT



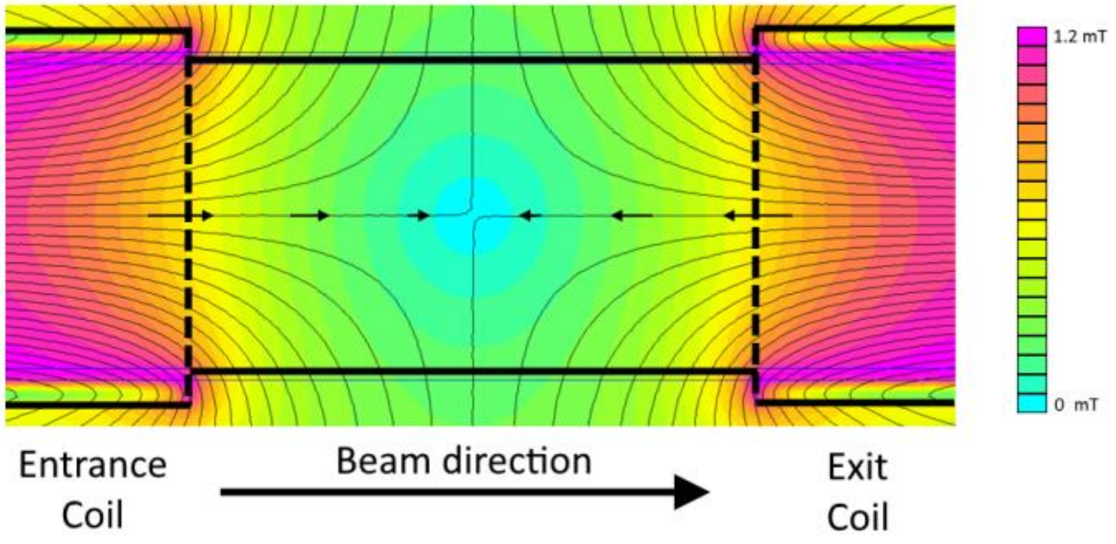
P. Buske, Bachelor thesis, (2016)

PHOTON MODEL

- Static magnetic field

$$\vec{B} \cdot \vec{\nabla} = 0 \quad \Rightarrow \quad B_r = -\frac{r}{2}\frac{\partial B_z}{\partial z}$$

$$B'_z = B_0 \sin(k'z') \qquad B'_r = -\tfrac{r}{2} B_0 k \cos(k'z')$$



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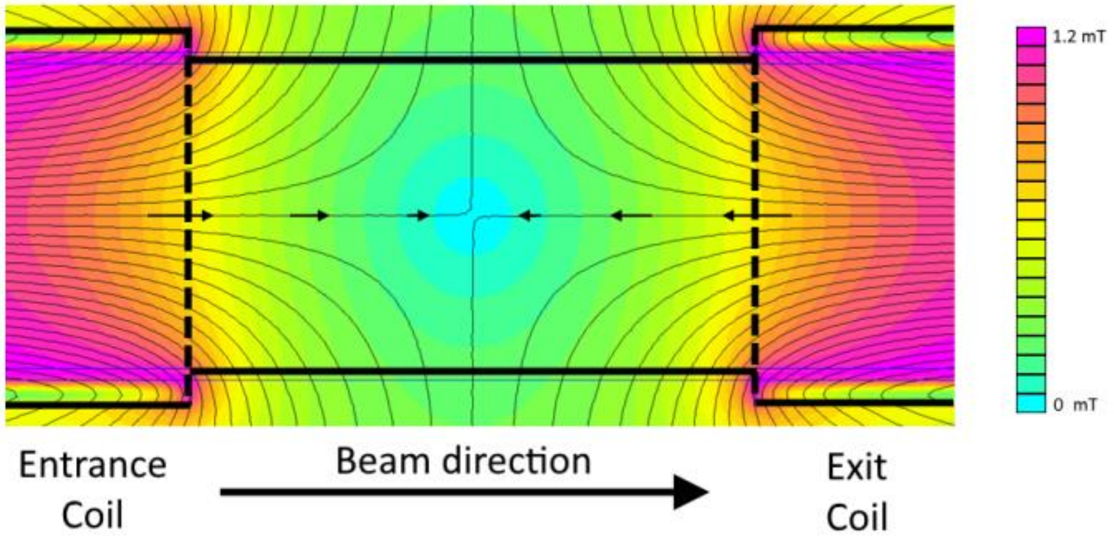
- Metastable $2S_{1/2}$ hydrogen atoms enter



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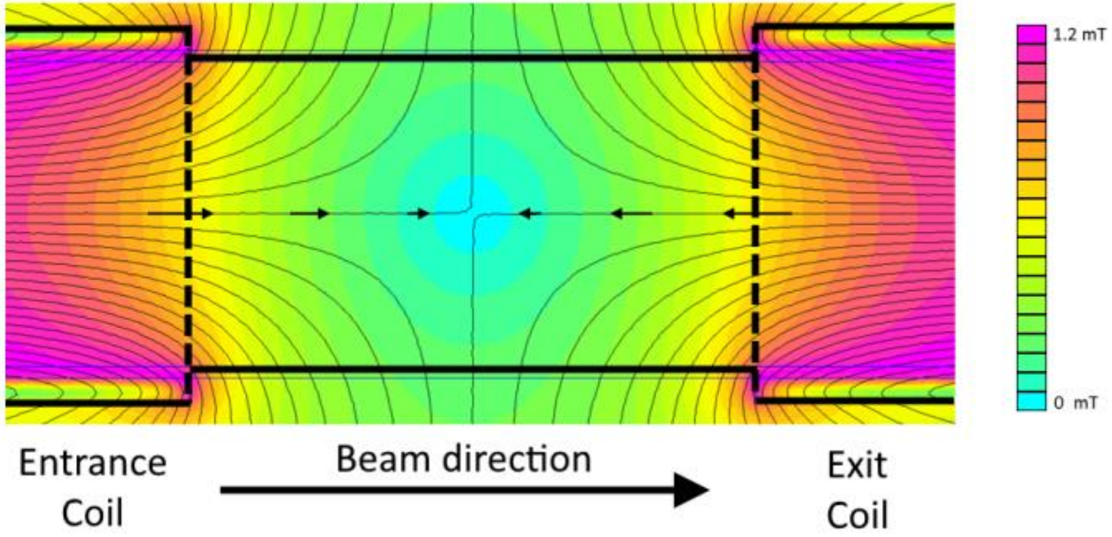

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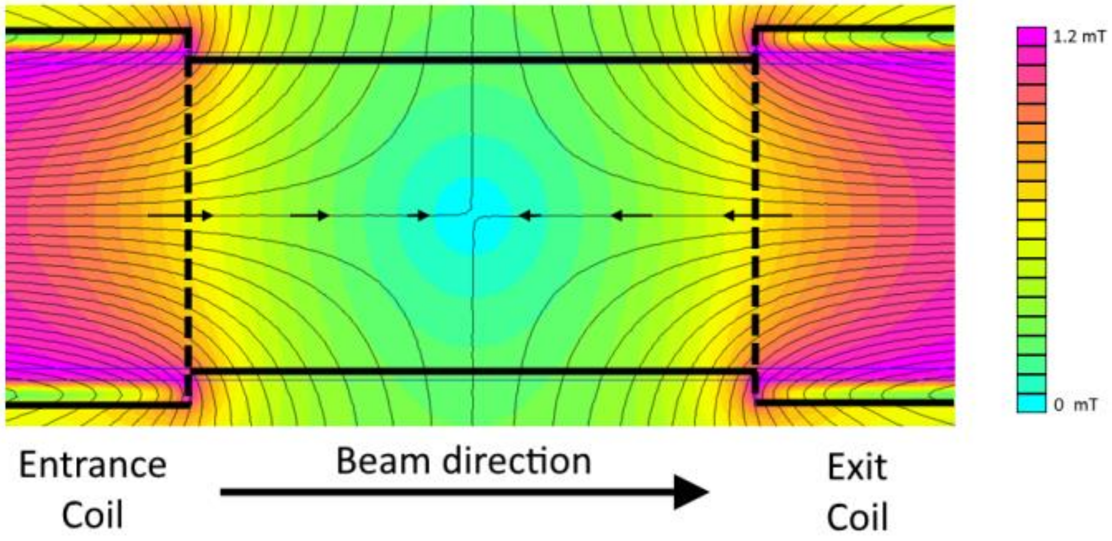
- Metastable $2S_{1/2}$ hydrogen atoms enter
- Lorentztransformation into the rest frame

$$F^{\mu\nu} = \Lambda^{\mu}{}_{\mu\prime}\Lambda^{\nu}{}_{\nu\prime}F^{\mu\prime\nu\prime}$$

$$\Rightarrow \quad B_z = B_0 \sin(kz) \qquad B_r = -\tfrac{r}{2\gamma} B_0 k \cos(kz) \qquad E_\varphi = -\tfrac{vr}{2\gamma} B_0 k \cos(kz)$$

PHOTON MODEL



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- Lorentztransformation into the rest frame

$$F^{\mu\nu} = \Lambda^{\mu}{}_{\mu'} \Lambda^{\nu}{}_{\nu'} F^{\mu'\nu'}$$

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- Creation of photons leads to transitions

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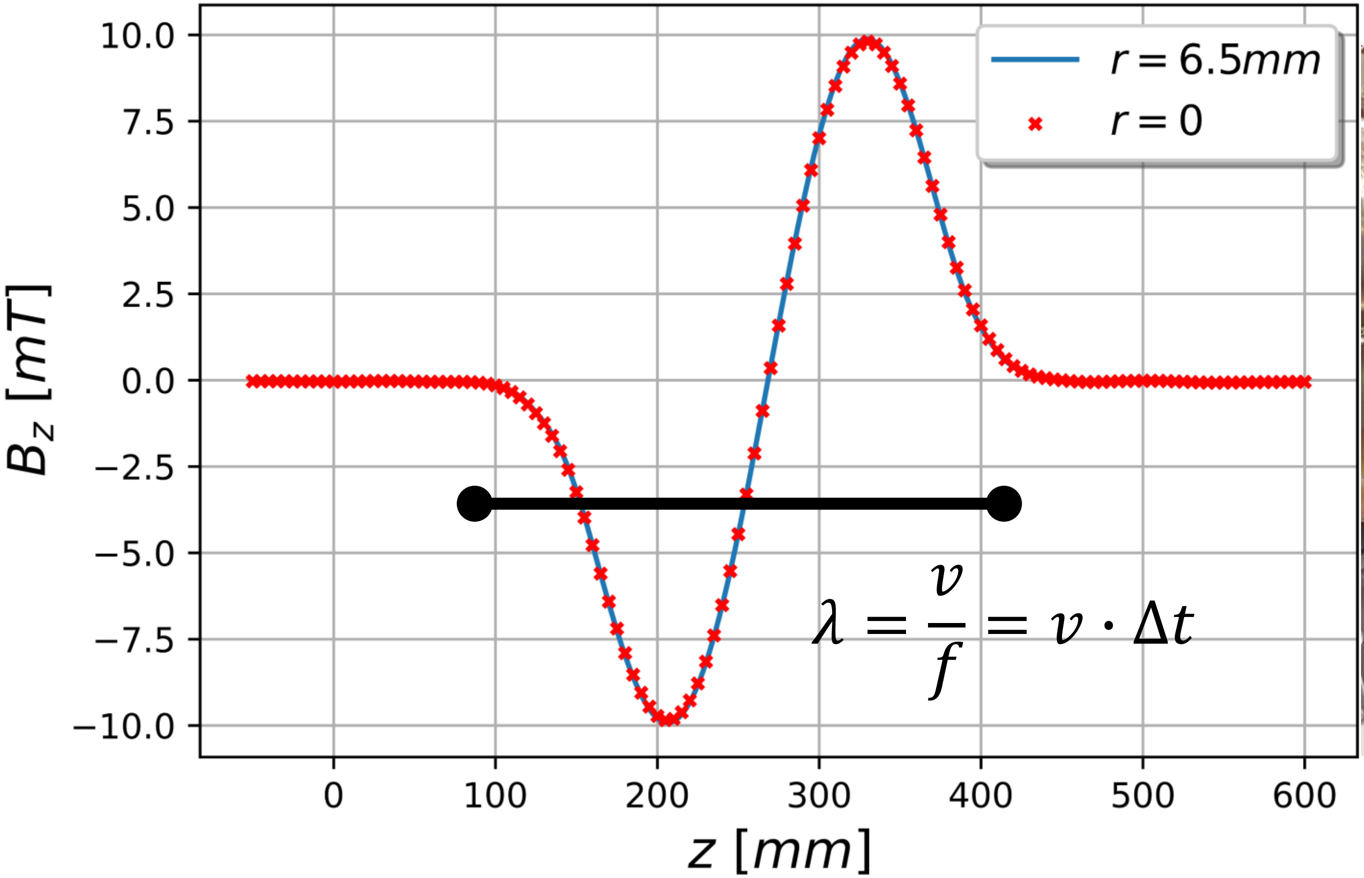
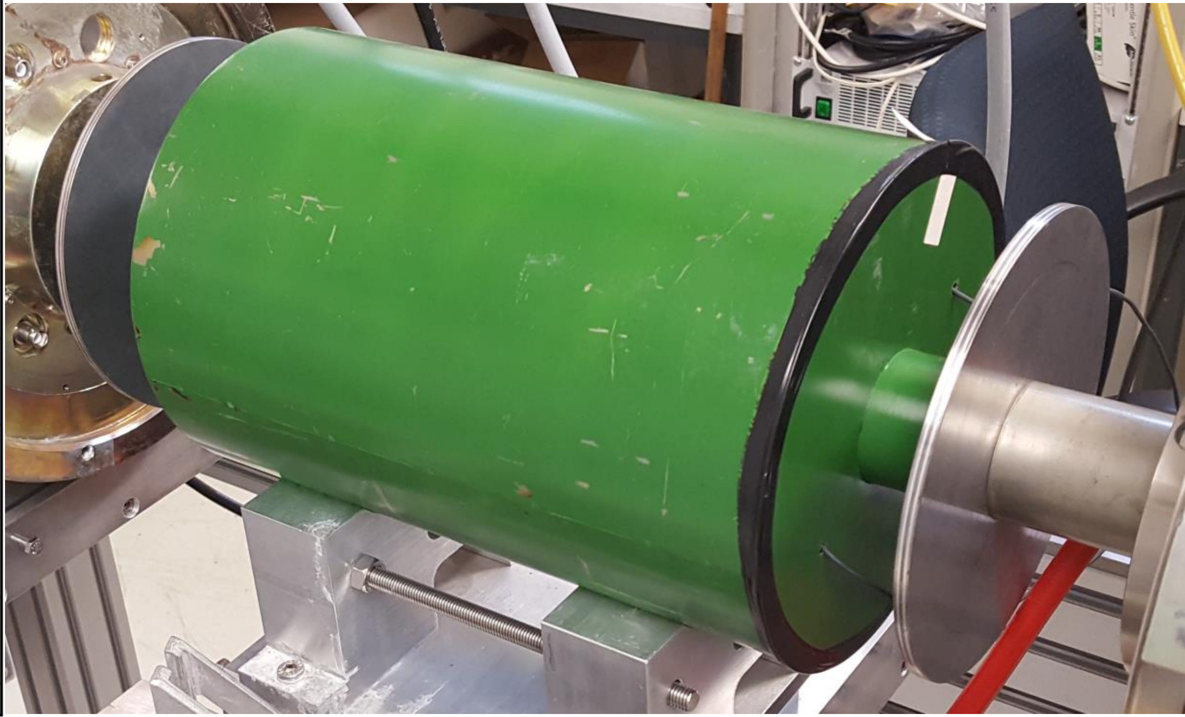
PHOTON MODEL





$$E_n = (2n - 1)hf \qquad n \in \mathbb{N}$$

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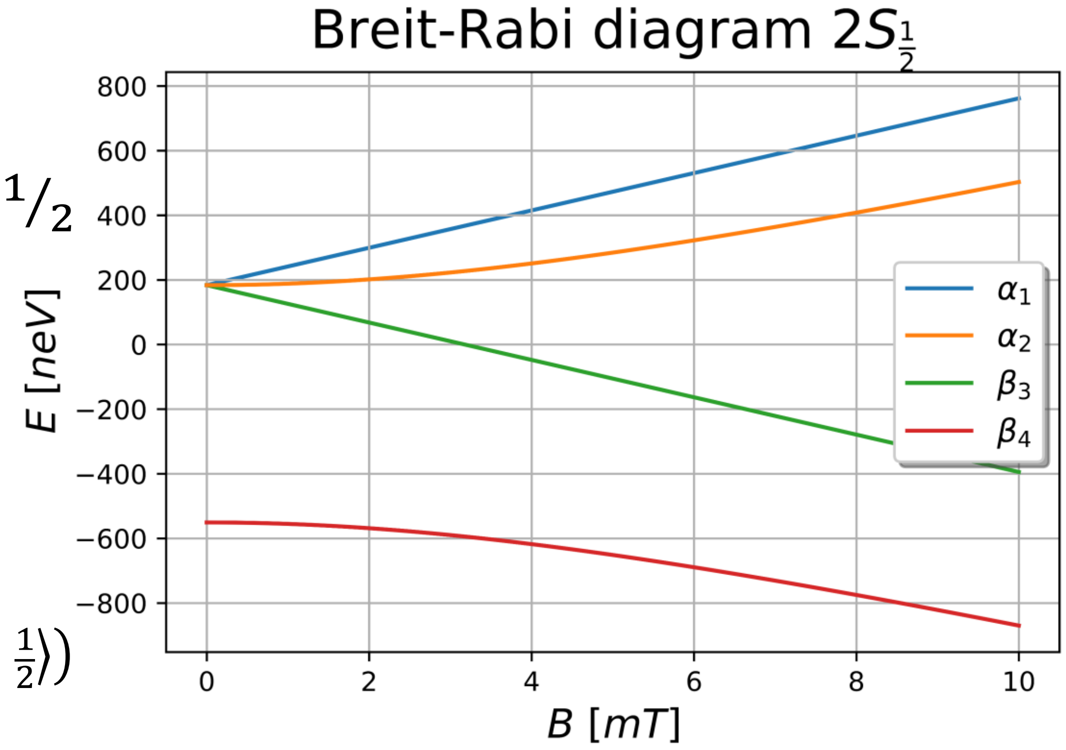
HYPERFINE STATES

- Eigenstates of the hyperfine splitting $2S_{1/2}$

$$S = \frac{1}{2} \qquad I = \frac{1}{2} \qquad F = 0,1$$



$$|\alpha_1\rangle = |m_F = 1, F = 1\rangle = |m_J = \tfrac{1}{2}, m_I = \tfrac{1}{2}\rangle$$

$$|\alpha_2\rangle = |m_F = 0, F = 1\rangle = \tfrac{1}{\sqrt{2}}\left(|m_J = \tfrac{1}{2}, m_I = -\tfrac{1}{2}\rangle + |m_J = -\tfrac{1}{2}, m_I = \tfrac{1}{2}\rangle\right)$$

$$|\beta_3\rangle = |m_F = -1, F = 1\rangle = |m_J = -\tfrac{1}{2}, m_I = -\tfrac{1}{2}\rangle$$

$$|\beta_4\rangle = |m_F = 0, F = 0\rangle = \tfrac{1}{\sqrt{2}}\left(|m_J = \tfrac{1}{2}, m_I = -\tfrac{1}{2}\rangle - |m_J = -\tfrac{1}{2}, m_I = \tfrac{1}{2}\rangle\right)$$

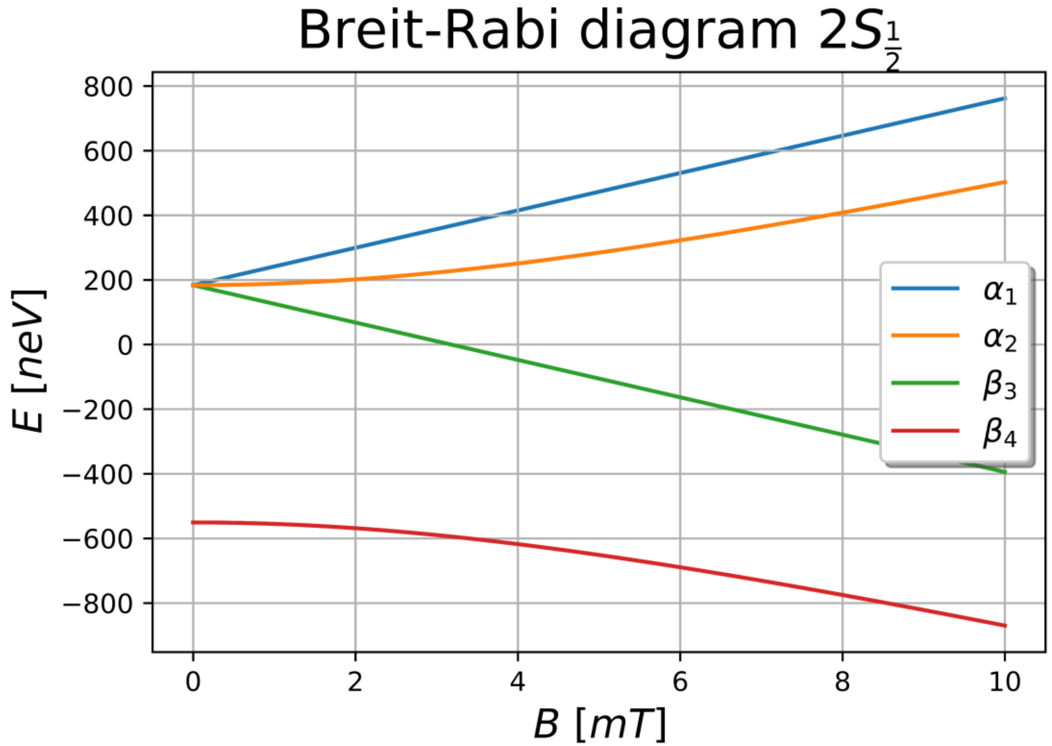
BREIT-RABI EIGENSTATES FOR $\vec{B} = B_z\vec{e}_z$

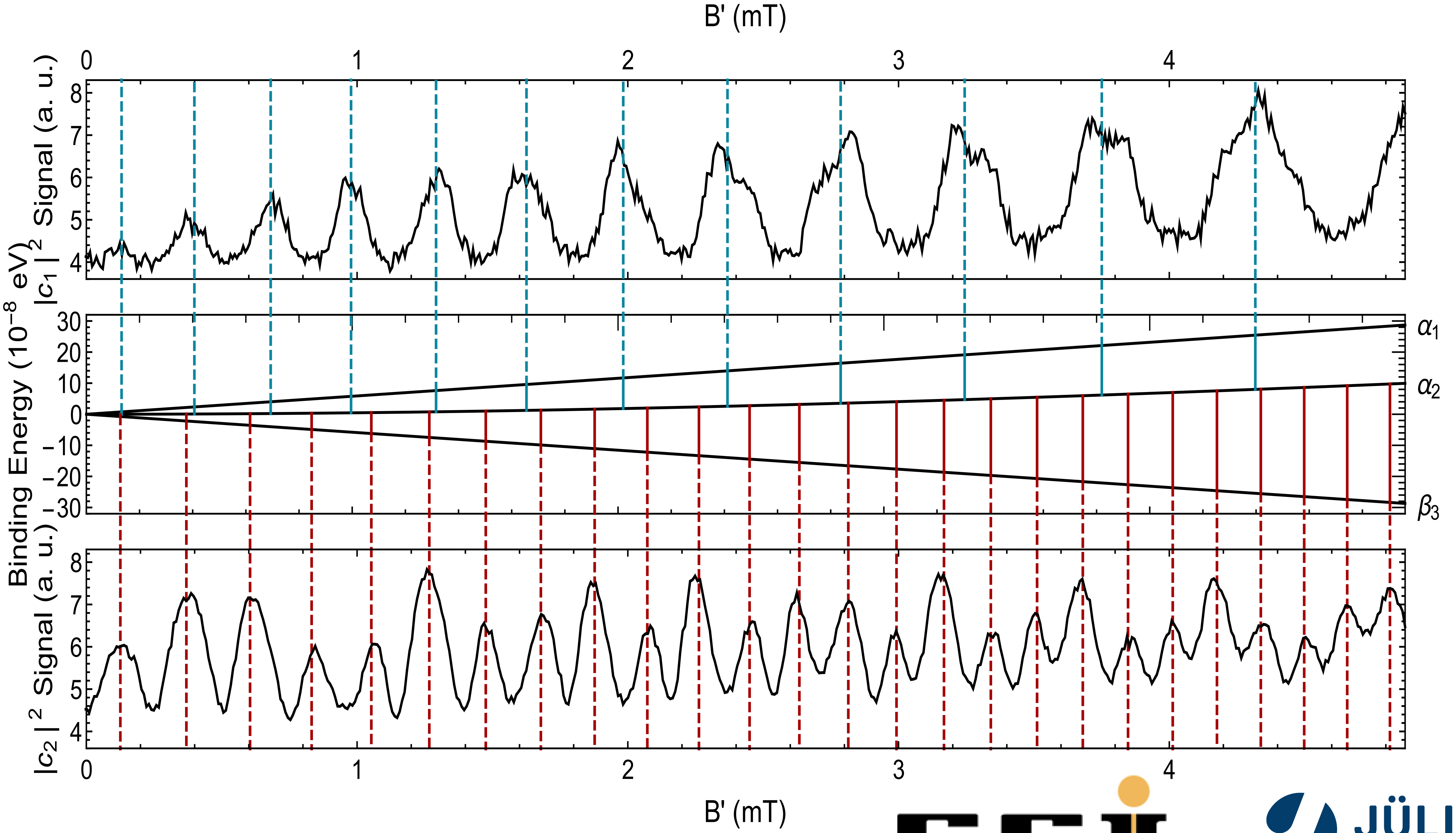
$$|v_1\rangle = |\alpha_1\rangle \qquad |v_2\rangle = \frac{1}{\sqrt{1+x^2}}\left(x|\beta_4\rangle + |\alpha_2\rangle\right)$$

$$|v_3\rangle = |\beta_3\rangle \qquad |v_4\rangle = \frac{1}{\sqrt{1+y^2}}\left(|\beta_4\rangle + y|\alpha_2\rangle\right)$$

$$E_{1,3} = \frac{A}{4} \pm \frac{1}{2}\left(g_J\mu_B - g_I\mu_K\right)B \qquad E_{2,4} = -\frac{A}{4} \pm \frac{1}{2}\sqrt{A^2 + \left(g_J\mu_B + g_I\mu_K\right)^2 B^2}$$

$$y(B) = \frac{A - \sqrt{A^2 + \left(g_J\mu_B + g_I\mu_K\right)^2 B^2}}{\left(g_J\mu_B + g_I\mu_K\right)B} \qquad x(B) = \frac{\left(g_J\mu_B + g_I\mu_K\right)B}{A + \sqrt{A^2 + \left(g_J\mu_B + g_I\mu_K\right)^2 B^2}}$$




B' (mT)
0
1
2
3
4
$|c_1|^2$ Signal (a. u.)
8
7
6
5
4
Binding Energy (10^{-8} eV)
30
20
10
0
−10
−20
−30
α_1
α_2
β_3
$|c_2|^2$ Signal (a. u.)
8
7
6
5
4
0
1
2
3
4
B' (mT)

THEORETICAL APPROACH

- Hamiltonian $$H = H_{Hyp} + V_B(t) + V_{Stark}(t)$$

$$H_{Hyp} = A\frac{\vec{I}\cdot\vec{J}}{\hbar^2}$$

$$V_B(t) = \left(g_j\mu_B\frac{\vec{J}}{\hbar} - g_I\mu_K\frac{\vec{I}}{\hbar}\right)\cdot\vec{B}(t)$$

$$V_{Stark}(t) = e\vec{E}(t)\cdot\vec{r}$$

THEORETICAL APPROACH

- Hamiltonian

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$$H_{Hyp} = A\frac{\vec{I}\cdot\vec{J}}{\hbar^2}$$

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$$V_{Stark}(t) = e\vec{E}(t)\cdot\vec{r}$$

- Timedepending perturbation

$$H_{Hyp}|m_F, F\rangle = E_{m_F,F}|m_F,F\rangle$$

$$|\psi\rangle = \sum_{F=|J-I|}^{J+I}\sum_{m_F=-F}^{F} c_{m_F,F}(t)\cdot e^{-\,iE_{m_F,F}\cdot t/\hbar}\,|m_F, F\rangle$$

$$\dot{c}_{\tilde{F},m_{\tilde{F}}} = -\frac{i}{\hbar}\sum_{F=|J-I|}^{J+I}\sum_{m_F=-F}^{F} c_{F,m_F}\, e^{-i\left(E_{F,m_F}-E_{\tilde{F},m_{\tilde{F}}}\right)t/\hbar}\left\langle\tilde{F}, m_{\tilde{F}}\middle|V(t)\middle|F, m_F\right\rangle$$

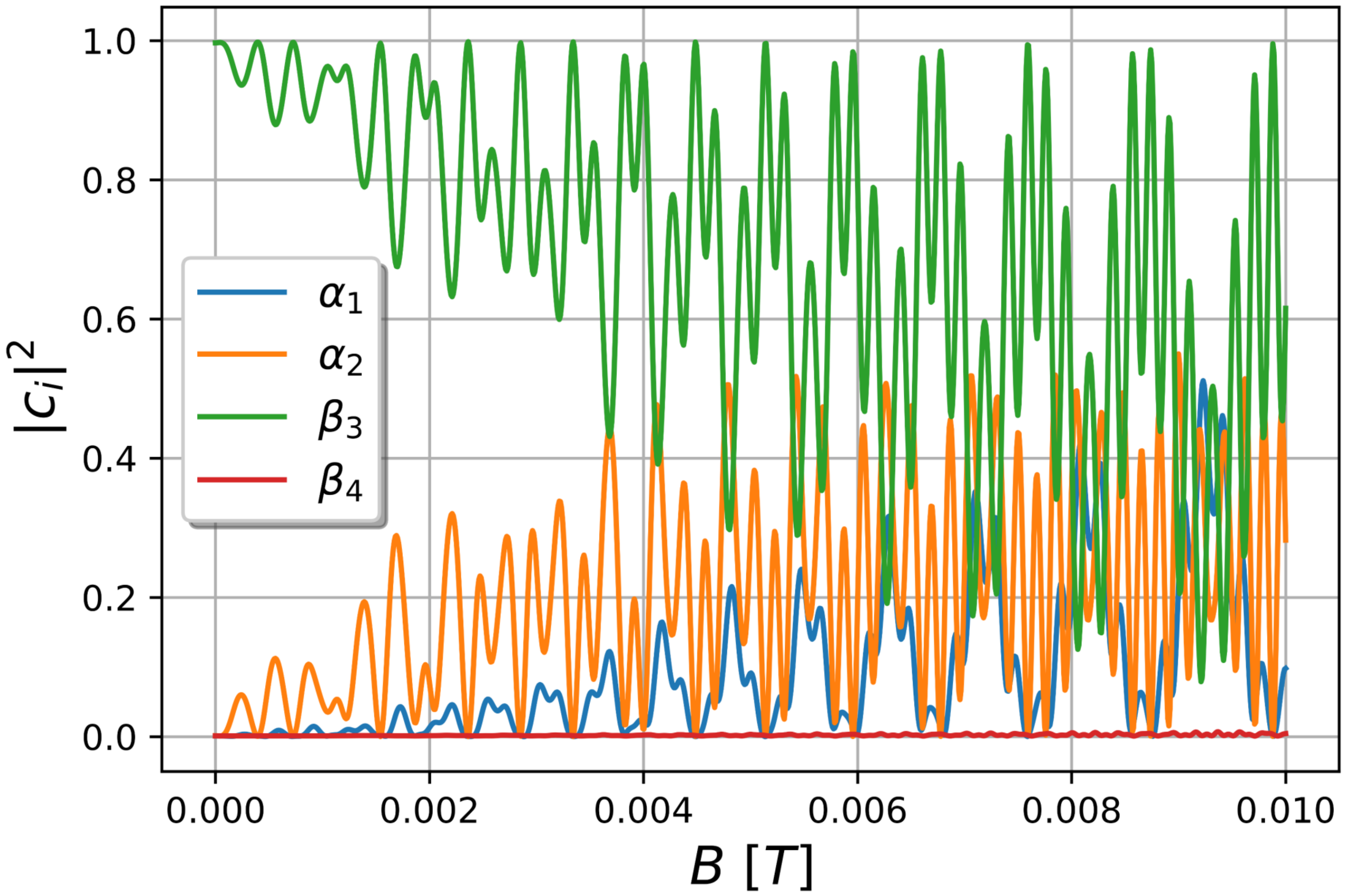
THEORETICAL APPROACH

$$\dot{c}_{\beta_4}(t) = -\frac{i}{\hbar}\left(\frac{1}{2\sqrt{2}}e^{-i\left(At/\hbar+\varphi\right)}B_r\left(g_J\mu_B + g_I\mu_K\right)c_{\beta_3} + \frac{1}{2}e^{-iAt/\hbar}B_z\left(g_J\mu_B + g_I\mu_K\right)c_{\alpha_2} - \frac{1}{2\sqrt{2}}e^{-i\left(At/\hbar-\varphi\right)}B_r\left(g_J\mu_B + g_I\mu_K\right)c_{\alpha_1}\right)$$

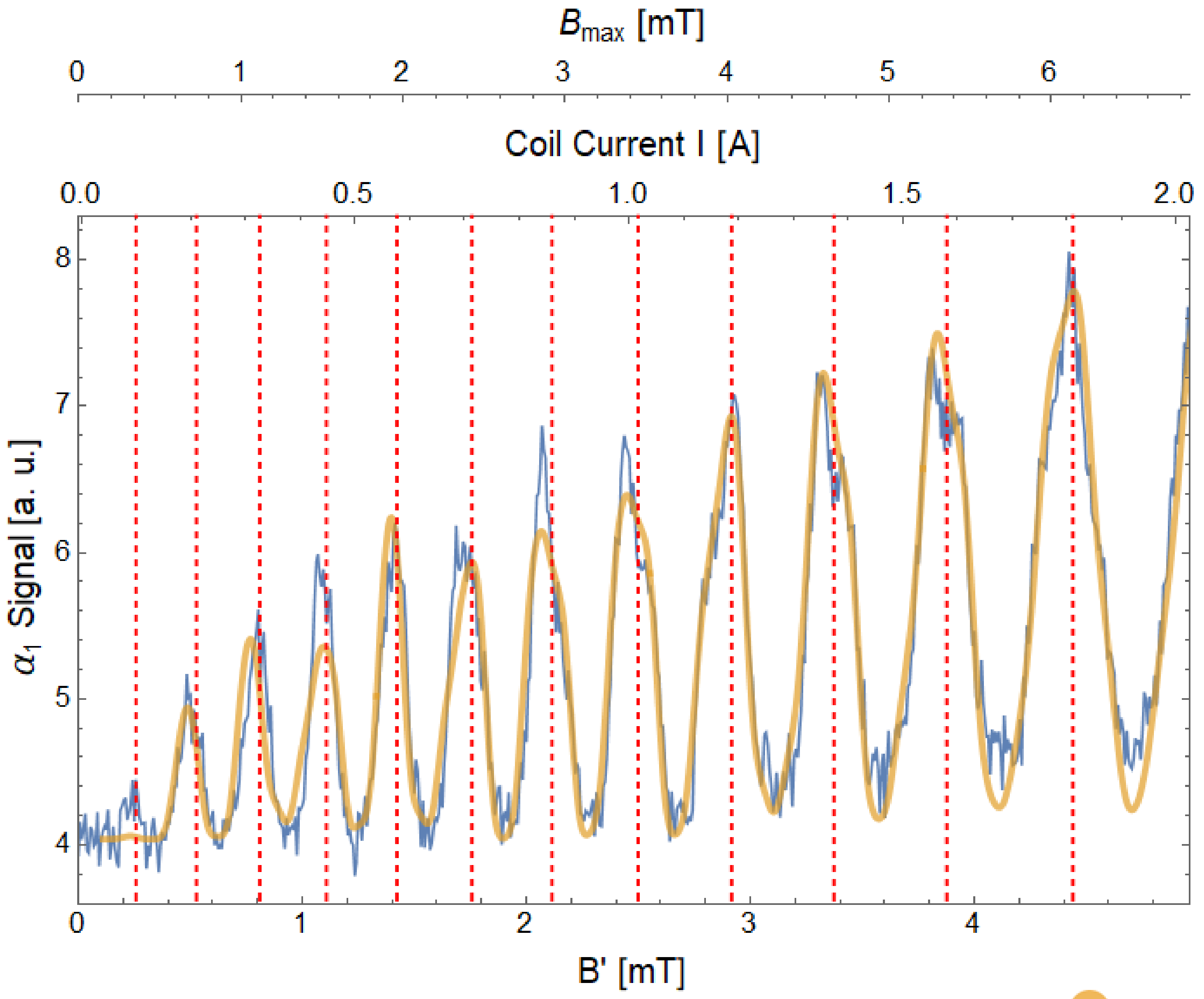
$$\dot{c}_{\beta_3}(t) = -\frac{i}{\hbar}\left(\frac{1}{2\sqrt{2}}e^{i\left(At/\hbar+\varphi\right)}B_r\left(g_J\mu_B + g_I\mu_K\right)c_{\beta_4} - \frac{1}{2}B_z\left(g_J\mu_B - g_I\mu_K\right)c_{\beta_3} + \frac{1}{2\sqrt{2}}e^{i\varphi}B_r\left(g_J\mu_B - g_I\mu_K\right)c_{\alpha_2}\right)$$

$$\dot{c}_{\alpha_2}(t) = -\frac{i}{\hbar}\left(\frac{1}{2}e^{iAt/\hbar}B_z\left(g_J\mu_B + g_I\mu_K\right)c_{\beta_4} + \frac{1}{2\sqrt{2}}B_r e^{-i\varphi}\left(g_J\mu_B - g_I\mu_K\right)c_{\beta_3} + \frac{1}{2\sqrt{2}}e^{i\varphi}B_r\left(g_J\mu_B - g_I\mu_K\right)c_{\alpha_1}\right)$$

$$\dot{c}_{\alpha_1}(t) = -\frac{i}{\hbar}\left(-\frac{1}{2\sqrt{2}}e^{i\left(At/\hbar-\varphi\right)}B_r\left(g_J\mu_B + g_I\mu_K\right)c_{\beta_4} + \frac{1}{2\sqrt{2}}B_r e^{-i\varphi}\left(g_J\mu_B - g_I\mu_K\right)c_{\alpha_2} + \frac{1}{2}B_z\left(g_J\mu_B - g_I\mu_K\right)c_{\alpha_1}\right)$$



1.0
0.8
0.6
0.4
0.2
0.0
$|c_i|^2$
α_1
α_2
β_3
β_4
0.000
0.002
0.004
0.006
0.008
0.010
B [T]



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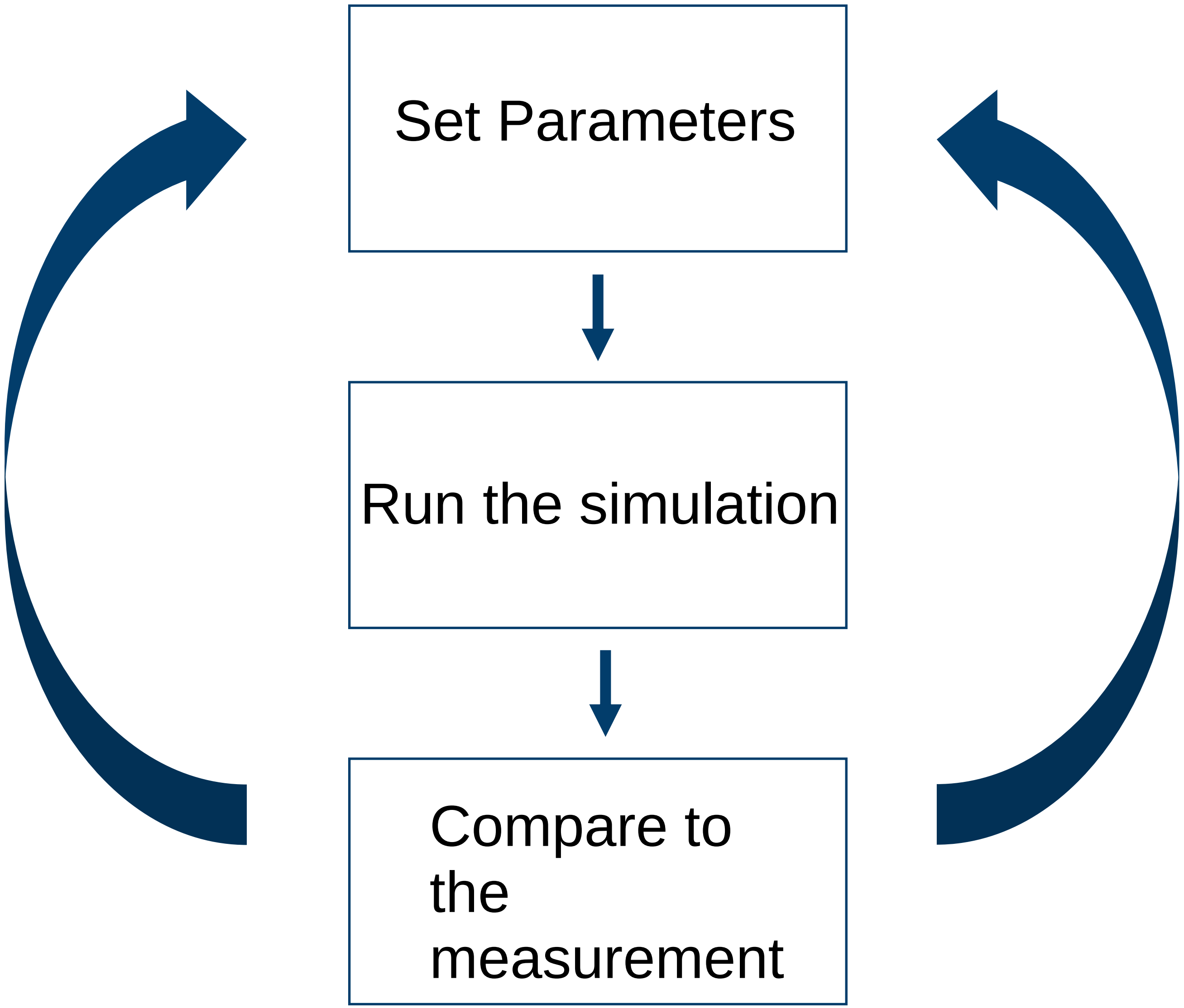
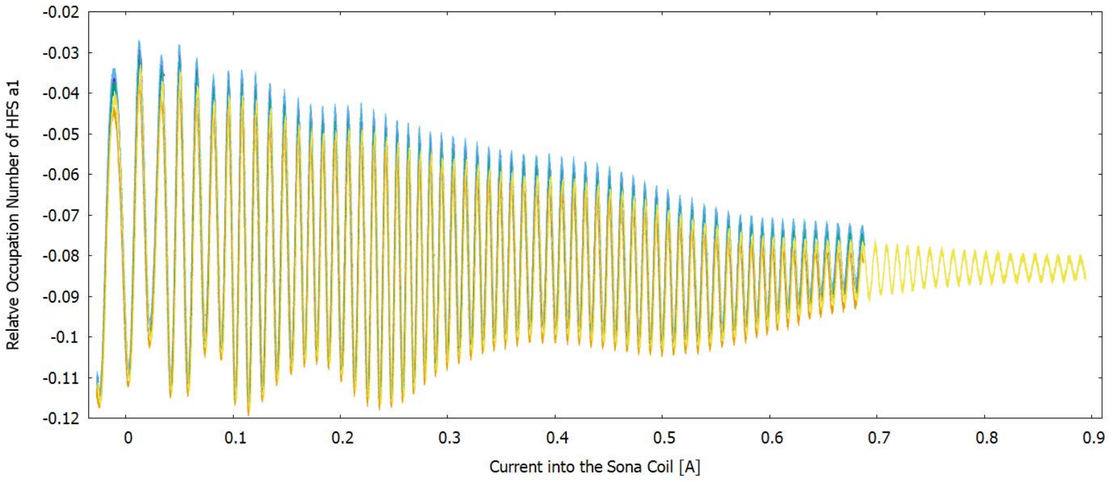
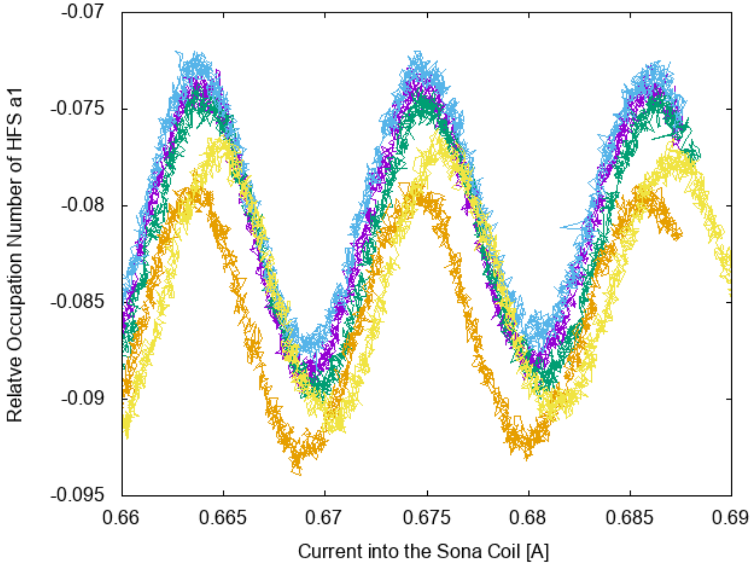

Set Parameters
Run the simulation
Compare to the measurement


Relatve Occupation Number of HFS a1
Current into the Sona Coil [A]
-0.02
-0.03
-0.04
-0.05
-0.06
-0.07
-0.08
-0.09
-0.1
-0.11
-0.12
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9


Relatve Occupation Number of HFS a1
Current into the Sona Coil [A]
-0.07
-0.075
-0.08
-0.085
-0.09
-0.095
0.66
0.665
0.67
0.675
0.68
0.685
0.69

OUTLOOK

- Adjustment of the free parameter

$$\vec{B}(r,t),\lambda(v)$$

$$\frac{g_J(H,2S_{1/2})}{g_e} = 1 - 4.426352(11) \times 10^{-6}$$

- QED-corrections

$$\frac{g_I(H,2S_{1/2})}{g_p} = 1 - 23(14) \times 10^{-9}$$

- New method of neV energy spectroscopy

➔ Also for molecules $H_2{}^+, D_2{}^+, HD^+$

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