Unitarization and Simplified Models for Vector Boson Scattering

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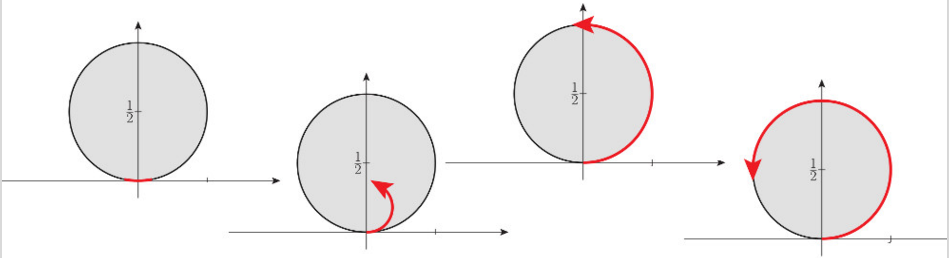
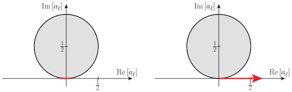
Institute of Theoretical Physics (ITP)

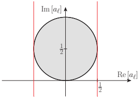


Outline

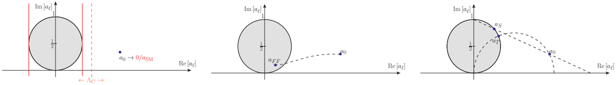
1. Introduction



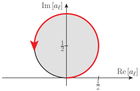
2. Unitarity



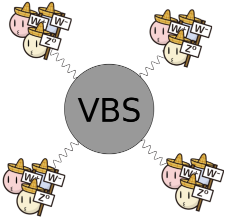
3. Unitarization



4. Simplified Models



VBS and the Standard Model



higgstan.com

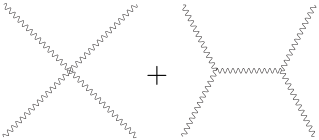
2014: Vector boson scattering is observed

$\Rightarrow$ The Higgs mechanism works as expected

Longitudinal vector boson self interaction

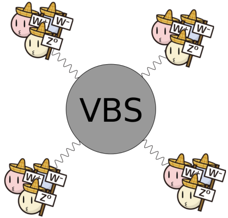
VBS amplitude rises with energy

$\Rightarrow$ violation of unitarity



$$\mathcal{O}\left(E^4\right) + \mathcal{O}\left(E^4\right) = \mathcal{O}\left(E^2\right)$$

VBS and the Standard Model



higgstan.com

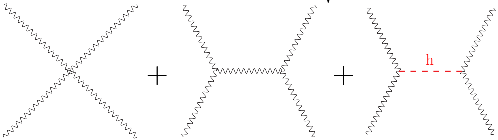
2014: Vector boson scattering is observed

$\Rightarrow$ The Higgs mechanism works as expected

VBS in the SM

Higgs exchange cancels the energy rise in VBS

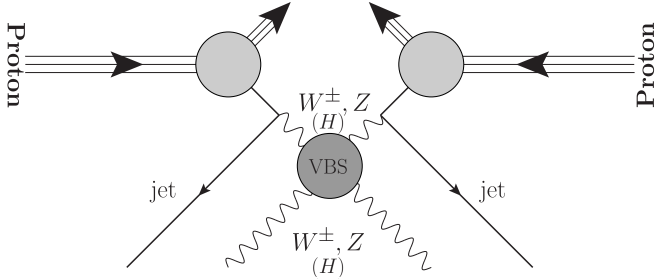
$\Rightarrow$ restores unitarity if $m_h \leq \sqrt{4\pi\sqrt{2}/G_F}$ Lee,Quigg,Thacker 1977



$+ \quad + \quad = \mathcal{O}\left(E^0\right)$

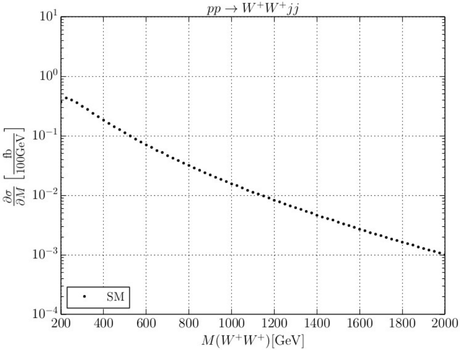
$\mathcal{O}\left(E^4\right) \qquad \mathcal{O}\left(E^4\right) \qquad \mathcal{O}\left(E^2\right)$

VBS at the LHC



- Two energetic jets in the forward and backward direction ($p_T > 20$ GeV)
- Large rapidity seperation and large invariant mass of the two tagging jets ($m_{jj} > 500$ GeV, $|\Delta y_{jj}| > 2.4$)

VBS at the LHC



- VBS amplitude is bounded (weakly int.)

$\Rightarrow$ Cross section suppressed by PDF

- Look for deviation from the SM prediction

$\rightarrow$ Sensitive test of new physics contributions

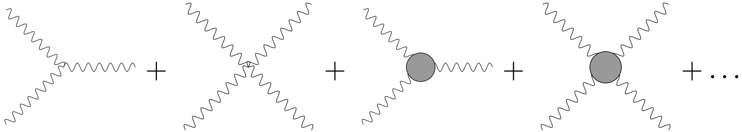
Modelling Physics beyond the SM

Desirable features of a generic SM extension

- Recovers the SM in an appropriate limit
- Respects established symmetries: $SU(3)_C \times SU(2)_L \times U(1)_Y$
- Captures any new physics
  (+ guidance where physics impact is large)
- Possibility to calculate radiative corrections

Effective Field Theory (EFT)

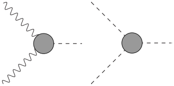
$$\mathcal{L} = \mathcal{L}_{SM} + \sum_{d\geq 4} \sum_i \frac{C_i}{\Lambda^{d-4}} \mathcal{O}_i^d$$



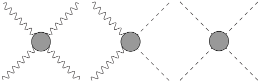
Longitudinal EFT Operators

Anomalous couplings effecting longitudinal VBS

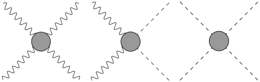
$$\mathcal{L}_{HD} = F_{HD}\ \mathrm{tr}\left[\mathbf{H}^\dagger\mathbf{H} - \frac{v^2}{4}\right] \cdot \mathrm{tr}\left[\left(\mathbf{D}_\mu\mathbf{H}\right)^\dagger\left(\mathbf{D}^\mu\mathbf{H}\right)\right]$$



$$\mathcal{L}_{S,0} = F_{S,0}\ \mathrm{tr}\left[\left(\mathbf{D}_\mu\mathbf{H}\right)^\dagger\mathbf{D}_\nu\mathbf{H}\right] \cdot \mathrm{tr}\left[\left(\mathbf{D}^\mu\mathbf{H}\right)^\dagger\mathbf{D}^\nu\mathbf{H}\right]$$



$$\mathcal{L}_{S,1} = F_{S,1}\ \mathrm{tr}\left[\left(\mathbf{D}_\mu\mathbf{H}\right)^\dagger\mathbf{D}^\mu\mathbf{H}\right] \cdot \mathrm{tr}\left[\left(\mathbf{D}_\nu\mathbf{H}\right)^\dagger\mathbf{D}^\nu\mathbf{H}\right]$$

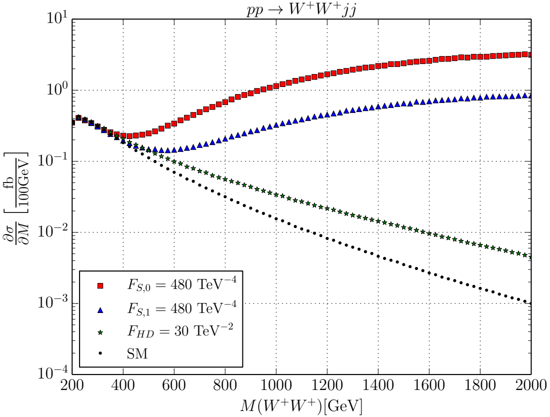


Linear Higgs matrix representation

$$\mathbf{H} = \frac{1}{2}\begin{pmatrix} v + h - \mathrm{i}w^3 & -\mathrm{i}\sqrt{2}w^+ \\ -\mathrm{i}\sqrt{2}w^- & v + h + \mathrm{i}w^3 \end{pmatrix}$$

$$F_{HD} = f_{HD}/\Lambda^2$$
$$F_{S,0} = (f_{S,0} + f_{S,2})/\Lambda^4$$
$$F_{S,1} = f_{S,1}/\Lambda^4$$

Differential cross section at LHC (14 TeV)



- AQGC amplitudes rise with energy $\sim E^4$

$\rightarrow$ $D = 8$ Operators cancel the PDF suppression

- Unitarity obviously violated (at which energy?)

Unitarity

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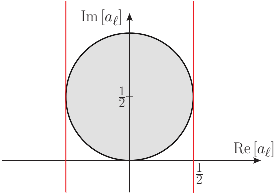
1. Unitarity of scattering matrix $S = \mathbb{1} + i\mathbf{T} : \rightarrow i\left(\mathbf{T} - \mathbf{T}^\dagger\right) = \mathbf{T}\mathbf{T}^\dagger$
2. Angular momentum conservation:
   conventionally normalized partial wave amplitudes a_ℓ
3. Unitarity implies

Argand-circle condition

$$\left| a_\ell(s) - \frac{i}{2} \right| \leq \frac{1}{2}$$



$\rightarrow$ Outside: unitarity broken
$\rightarrow$ Inside/On: unitarity fulfilled

inside: inelastic scattering ($<$)
on: elastic scattering ($=$)

Bound on real part

$$\left|\mathrm{Re}\left(a_\ell(s)\right)\right| \leq \frac{1}{2}$$

$\Rightarrow$ Conservative EFT validity bound s_{max}

Isospin-Spin Eigenamplitudes

- Weak boson interaction matrix has non-diagonal elements (GBET):

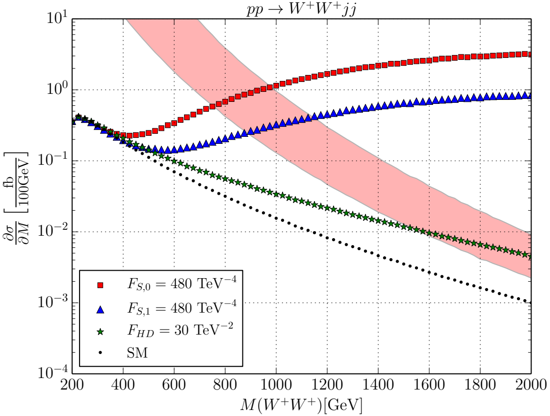
$$\begin{pmatrix} w^+w^+\to w^+w^+ & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & w^+z\to w^+z & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & w^+h\to w^+h & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & zh\to zh & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & w^+w^-\to w^+w^- & w^+w^-\to zz & w^+w^-\to hh \\ 0 & 0 & 0 & 0 & zz\to w^+w^- & zz\to zz & zz\to hh \\ 0 & 0 & 0 & 0 & hh\to w^+w^- & hh\to zz & hh\to hh \end{pmatrix}$$

$\Rightarrow$ Use isospin $SU_C(2)$ to diagonalize interaction matrix

- Partial wave decomposition into isospin-spin eigenamplitudes $a_{I\ell}$
- All $a_{I\ell}$ have to fulfill the Argand-circle condition
- Example for $a(W^+W^+ \to W^+W^+)$

$$a(w^+w^+ \to w^+w^+) = a_{20}(s) - 10a_{22}(s) - 15a_{22}(s)\frac{t^2+u^2}{s^2}$$

Unitarity bounds of Dim-8 Operators



- EFT for AQGC at current experimental bounds violates unitarity below 1 TeV (red band: variation of $a_{I\ell}$)

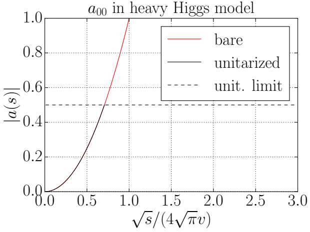
$\Rightarrow$ Naive EFT description is unphysical within LHC energy reach

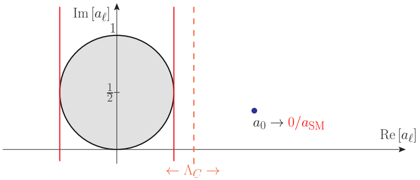
- Extrapolation?

Cut-Off Method

- Cut-Off function: $\Theta\left(\Lambda_C^2 - s\right)$
  1. On data and complete MC-simulated events
     - Requirement: experimental reconstruction of s
  2. Only on EFT part in MC-simulation
     - Creates unphysical kink in exp. accessible region
     - ! Beware of using Neural Network etc to improve sensitivity
- Choosing Λ_C:
  1. $\Lambda_C = s_{max}$
  2. Scan over Λ_C for different UV-complete models

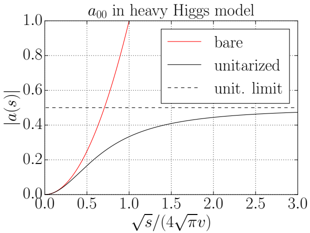
Contino, Falkowski et al 2016

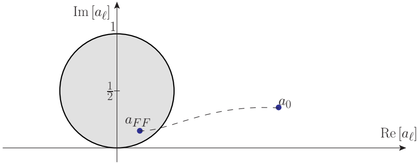




Dipole Form Factor

- Form-Factor: $\left(1 + \frac{s}{\Lambda_{FF}^2}\right)^{-p}$ Baur,Zeppenfeld 1988
- p is chosen accordingly to the EFT-operator dimension
- Λ_{FF} set to highest possible value that satisfy real unitarity bound (0th)
- Can be easily implemented for arbitrary anomalous operator
- Needs "Fine Tuning"
- Complete amplitude receives suppression factor

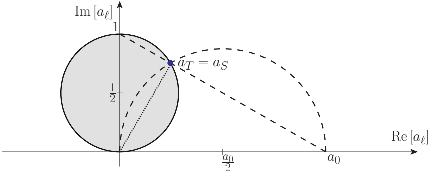




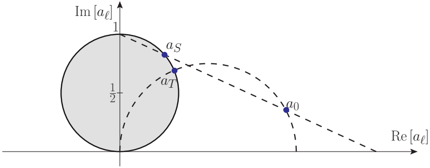
Direct T-Matrix Unitarization

1. Linear construction "Stereographic": $T = \frac{\mathrm{Re}\, T_0}{\mathbf{1} - \frac{i}{2} T_0^\dagger}$
2. Circular construction "Thales": $T = \frac{1}{\mathrm{Re}\left(\frac{1}{T_0}\right) - \frac{i}{2}\mathbf{1}}$

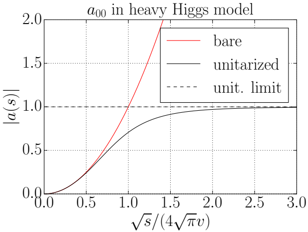
Start from real amplitude a_0:



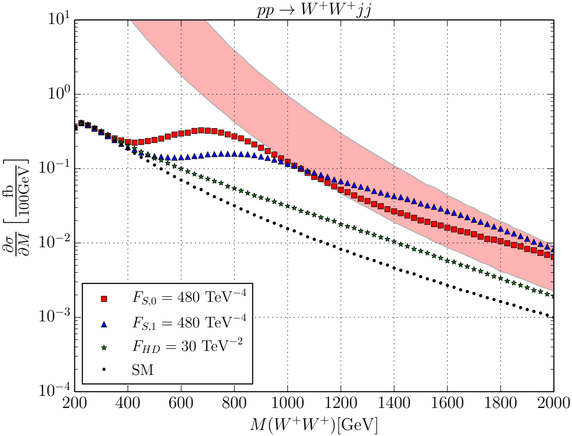
Start from complex amplitude a_0:



- Unitary amplitude left invariant
- But scheme dependence for complex a_0
- Example: Higgs-less amplitude



T-matrix Unitarization

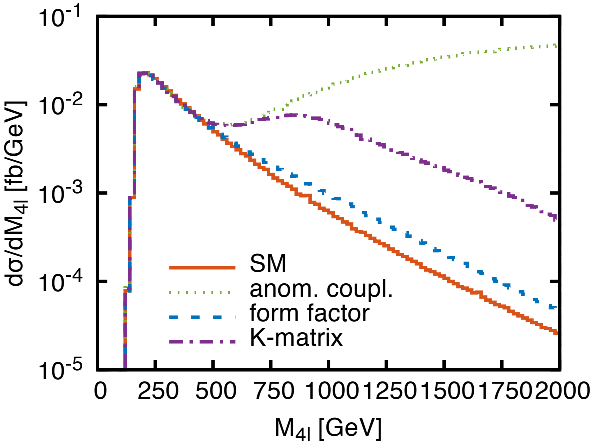


⇒ Saturation of isospin-spin amplitudes at their unitarity limit

- Leaves scattering matrices, which satisfy unitarity, invariant

! Introducing model dependence

Comparison for $pp \to e^+\nu_e\mu^+\nu_\mu jj$

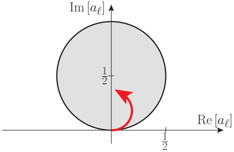


provided by M.Rauch (VBFNLO)

- EFT parameters: $F_{S,1} = 400\,TeV^{-4} \Rightarrow s_{max} = 780\ \mathrm{GeV}$
- FF parameters: $p = 2,\ \Lambda_{FF} = 832\ \mathrm{GeV}$

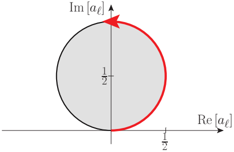
Scenarios for New Physics at High Energies

❶ Inelastic



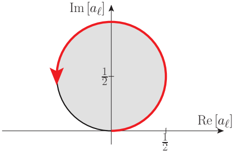
EFT+ Form-factor

❷ Saturation



EFT elastic + T-Matrix

❸ Resonance



Adding additional resonances

The rise of an amplitude (AQGC) may be an expansion of a resonance

Resonances in VBS: Quantum Numbers

Spin

- Just consider Spin 0,2
- Spin 1 has different pheno (W/Z-mixing)

Symmetry

$$SU(2)_L \times SU(2)_R \to SU(2)_C$$

$$(\mathbf{0},\mathbf{0}) \to 0$$

$$(\mathbf{1},\mathbf{1}) \to 2+1+0$$

	isoscalar	isotensor
scalar	σ^0	$\begin{pmatrix} \phi_t^{--}, \phi_t^{-}, \phi_t^{0}, \phi_t^{+}, \phi_t^{++} \\ \phi_v^{-}, \phi_v^{0}, \phi_v^{+} \\ \phi_s^{0} \end{pmatrix}$
tensor	f^0	$\begin{pmatrix} X_t^{--}, X_t^{-}, X_t^{0}, X_t^{+}, X_t^{++} \\ X_v^{-}, X_v^{0}, X_v^{+} \\ X_s^{0} \end{pmatrix}$
...	...	...

Integrate out Isoscalar-scalar Resonance

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- Simple example: Extension via scalar singlet σ:

$$\mathcal{L}_\sigma = -\frac{1}{2}\sigma\left(m_\sigma^2 + \partial^2\right)\sigma + \sigma J_\sigma$$

$$J_\sigma = F_\sigma \,\mathrm{tr}\left[\left(\mathbf{D}_\mu \mathbf{H}\right)^\dagger \mathbf{D}^\mu \mathbf{H}\right] \qquad \text{where} \quad F_\sigma \propto \frac{1}{\Lambda}$$

- Scalar mass is beyond experimental energy reach
- Integrate out heavy scalar resonance

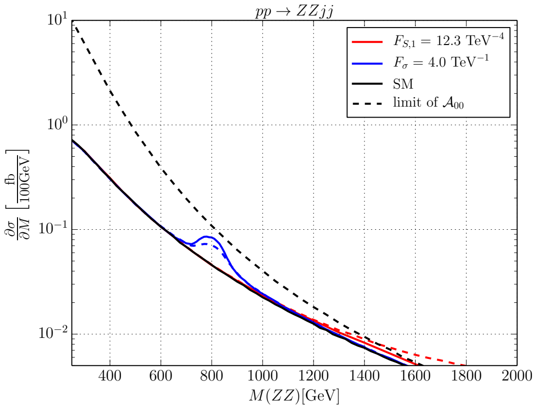
⇒ Effective Lagrangian

$$\mathcal{L}_\sigma^{\mathrm{eff}} = \frac{F_\sigma^2}{2m_\sigma^2}\,\mathrm{tr}\left[\left(\mathbf{D}_\mu \mathbf{H}\right)^\dagger \mathbf{D}^\mu \mathbf{H}\right]\mathrm{tr}\left[\left(\mathbf{D}_\nu \mathbf{H}\right)^\dagger \mathbf{D}^\nu \mathbf{H}\right]$$

- Leads to following AQGC

$$F_{S,1} = \frac{F_\sigma^2}{2m_\sigma^2}, \qquad \mathbf{H} = \frac{1}{2}\begin{pmatrix} v + h + \mathrm{i}w^3 & -\mathrm{i}\sqrt{2}w^+ \\ -\mathrm{i}\sqrt{2}w^- & v + h + \mathrm{i}w^3 \end{pmatrix}$$

Comparison of Simplified Models and EFT



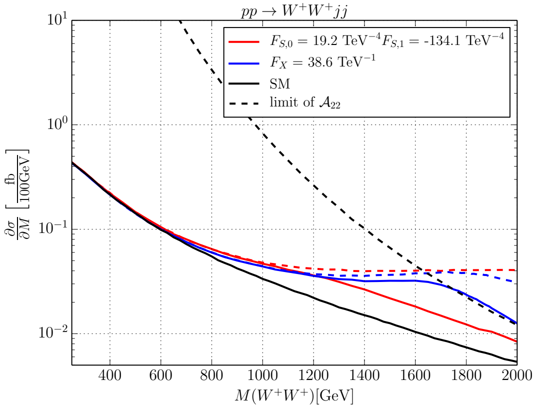
- EFT fails at resonance
- AQGC describes the rise of a resonance
- Energy validity range of theory is increased

Calculation: `WHIZARD`

$$m_\sigma = 800\ \mathrm{GeV}, \quad \Gamma_\sigma / m_\sigma = 0.1$$

$$F_{S,1} = \frac{1}{2}\frac{F_\sigma^2}{m_\sigma^2}$$

Comparison of Simplified Models and EFT



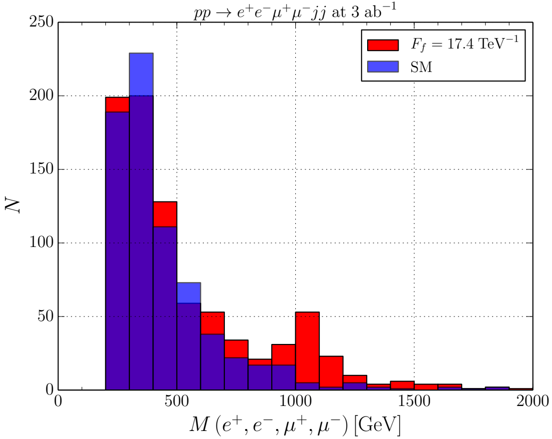
Calculation: `WHIZARD`

$$m_X = 1800\ \text{GeV},\ \Gamma_X/m_X = 0.4$$

$$F_{S,0} = \frac{1}{2}\frac{F_f^2}{m_f^2},\ F_{S,1} = -\frac{1}{6}\frac{F_f^2}{m_f^2}$$

- EFT fails at resonance
- AQGC describes the rise of a resonance
- Energy validity range of theory is increased

Complete LHC process at $\sqrt{s} = 14$ TeV



Simulation: `WHIZARD`

$$m_f = 1.0 \text{ TeV}, \Gamma_f / m_f = 0.1$$

$$F_{S,0} = \frac{1}{2}\frac{F_f^2}{m_f^2}, F_{S,1} = -\frac{1}{6}\frac{F_f^2}{m_f^2}$$

Summary and Outlook

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- Effective theory: limited applicability for quartic gauge couplings
- Scheme to avoid unitarity violation: Θ, Form-Factor or T-Matrix
- Frameworks for quantitative tests of the SM version of electroweak interactions which matches the low-energy EFT
  - ✓ $\mathcal{O}_{HD}$, $\mathcal{O}_{S,0}$,$\mathcal{O}_{S,1}$, $\mathcal{O}_{T,0}$, $\mathcal{O}_{T,1}$ and $\mathcal{O}_{T,2}$
- Realization: generic resonances $\rightarrow$ simplified model

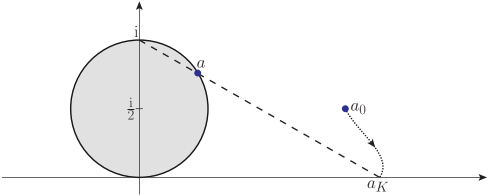
$\Rightarrow$ Extension for EFT by resonances

	isoscalar	isotensor
scalar	✓	✓
tensor	✓	✓

- Working: Implementation of T-matrix for generic EFT operators and resonances within VBS
- Working: Unitarization for $V \rightarrow VVV$

Backup Slides

K Matrix



Cayley Transform

Heitler, 1941

$$S = \frac{\mathbf{1} + \mathrm{i}K/2}{\mathbf{1} - \mathrm{i}K/2},$$

where $K = K^\dagger$

and $S = \mathbf{1} + \mathrm{i}T$

Original K Matrix algorithm

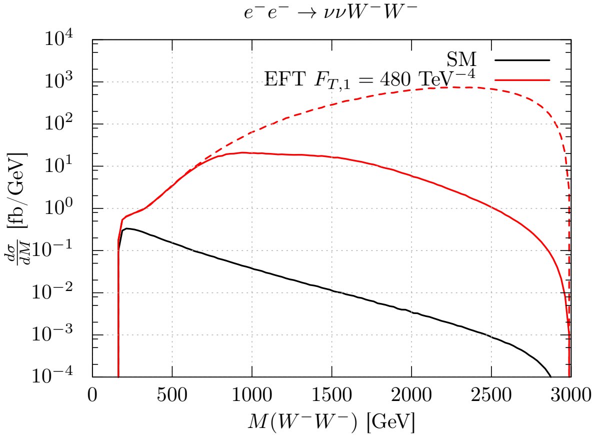
Gupta, 1951/1981

- a_0: Compute T_0 matrix perturbatively
- a_K: Reconstruct K matrix order by order
- a: Insert into S matrix formula, without expanding again
  $a = \frac{a_k}{\mathbb{1} - \mathrm{i}a_k}$

Relies on perturbation theory

$\Rightarrow$ Compute unitarized T matrix directly

T-Matrix for transversal couplings

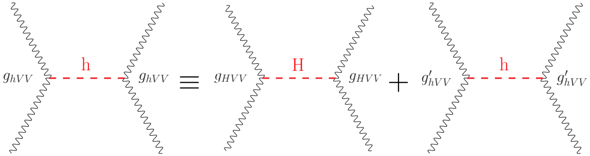


provided by C. Fleper(WHIZARD)

- Implementation of transversal couplings in validation
- Example: $\mathcal{L}_{T,1} = g^4\, \mathrm{tr}\left[\mathbf{W}_{\alpha\mu}\mathbf{W}^{\mu\beta}\right] \mathrm{tr}\left[\mathbf{W}_{\beta\nu}\mathbf{W}^{\nu\alpha}\right]$

Adding additional heavy Higgs

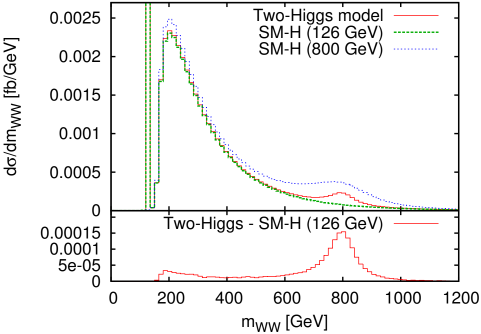
1. Adding additional heavy Higgs with mass m_H and coupling g_{HVV}
2. To satisfy unitarity: $g_{hVV} = g'_{hVV} + g_{HVV}$



3. Unitarity gives bounds to $m_H \leq m_H^B \left(g_{HVV}, m_h\right)$

- Modified coupling for light Higgs as compared to SM
- For generic resonances $\rightarrow$ Simplified models

Adding additional heavy Higgs



provided by D. Zeppenfeld (VBFNLO)

- Modified coupling for light Higgs as compared to SM
- For generic resonances → Simplified models

Guideline for Simplified Models

Introduction of generic resonances

- Use EFT-framework
- Introducing custodial $SU(2)_C$ symmetry $m_Z \approx m_W$
- Allow resonances in all accessible spin/isospin channels (here: only Higgs sector)
- Include extra anomalous couplings (reproduce unitary two Higgs model with $F_{HD} = -\frac{2}{v^2}\left(1 \pm \sqrt{\frac{v^2}{4}F_\sigma^2 + 1}\right)$)
- Beyond the resonance, the amplitude may eventually rise

$\Rightarrow$ Apply T-matrix unitarization scheme

Resonance width and corresponding AQGC

- Use width as parameter instead coupling ($\times m^3/(32\pi)F^2$)

	σ	ϕ	f	X
Γ	1	1/4	1/30	1/120

- Corresponding AQGC ($\times 32\pi\Gamma/m^5$)
  (transversal spin-2 coupling supressed)

	σ	ϕ	f	X
$F_{S,0}$	–	2	15	5
$F_{S,1}$	$\frac{1}{2}$	$-\frac{1}{2}$	-5	-35

Overview of WHIZARD-models for VBS

Karlsruhe Institute of Technology

- Model including all dim 6 operators of Warsaw basis: SM_dim6
- Models with T-matrix for longitudinal (transversal) couplings:

Model	SM-Higgs	Resonances	EFT representation
NoH_rx	X	Form factor	Non-linear
SM_rx	✓	Form factor	Non-linear
AltH	X	Fields	Non-linear
SSC	✓	Fields	Non-linear
SSC_2	✓	Fields	Linear
SSC_AltT	✓	Fields	Linear

! Resonances described by Form factos will neglect the induced transversal couplings of spin 2 particles (scalars are ok)

- The linear EFT-representation will give rise to couplings between Higgs and resonances or anomalous Higgs-VB and 4-Higgs couplings
- Model to calculate Isospin-Spin bounds: SM_ul

Custodial Symmetry

$$\beta' \frac{v^2}{8} \operatorname{tr}\left[T\mathbf{V}_\mu\right] \operatorname{tr}\left[T\mathbf{V}^\mu\right]$$

- Free parameter $\beta' = \beta'(\rho_*)$
- Experimental data constrains $\rho_* = \frac{m_W^2}{c_w^2 m_Z^2}$:

$\rightarrow$ $\beta'(\rho_* \equiv 1) = 0$

- Impose approximate symmetry to forbid above term

$\Rightarrow$ $SU(2)_L \times U(1)_Y \rightarrow SU(2)_L \times SU(2)_R$

Fermionic sector

Very strong violation due large top mass

Bosonic sector

- Broken by coupling $B\tau_3 U \propto s_w^2$

$\Rightarrow$ Only small violation of $M_W = M_Z$

- Higgs mechanism: $SU(2)_L \times SU(2)_R \rightarrow SU(2)_C$

Lagrangian of Resonances

$$\mathcal{L}_\sigma = -\frac{1}{2}\sigma\left(M_\sigma^2 + \partial^2\right)\sigma + \sigma J_\sigma$$

$$\mathcal{L}_\phi = \frac{1}{2}\left[-\frac{1}{2}\,\mathrm{tr}\left[\Phi\left(m_\phi + \partial^2\right)\Phi\right] + \mathrm{tr}\left[\Phi J_\phi\right]\right]$$

$$\mathcal{L}_f = \mathcal{L}_{kin} - \frac{m_f^2}{2} f_{\mu\nu} f^{\mu\nu} + f_{\mu\nu} J_f^{\mu\nu}$$

$$\mathcal{L}_x = \mathcal{L}_{kin} - \frac{m_X^2}{4}\,\mathrm{tr}\left[\mathbf{X}_{\mu\nu}\mathbf{X}^{\mu\nu}\right] + \frac{1}{2}\,\mathrm{tr}\left[\mathbf{X}_{\mu\nu}\mathbf{J}_X^{\mu\nu}\right]$$

$$J_\sigma = F_\sigma^{||}\,\mathrm{tr}\left[\left(\mathbf{D}_\mu\mathbf{H}\right)^\dagger \mathbf{D}^\mu\mathbf{H}\right]$$

$$J_\phi = F_\phi^{||}\left[\left(\mathbf{D}_\mu\mathbf{H}\right)^\dagger \otimes \mathbf{D}^\mu\mathbf{H} - \frac{\tau^{aa}}{6}\,\mathrm{tr}\left[\left(\mathbf{D}_\mu\mathbf{H}\right)^\dagger \mathbf{D}^\mu\mathbf{H}\right]\right]$$

$$J_f^{\mu\nu} = F_f^{||}\left(\mathrm{tr}\left[\left(\mathbf{D}^\mu\mathbf{H}\right)^\dagger \mathbf{D}^\nu\mathbf{H}\right] - \frac{c_f^{||}}{4} g^{\mu\nu}\,\mathrm{tr}\left[\left(\mathbf{D}_\rho\mathbf{H}\right)^\dagger \mathbf{D}^\rho\mathbf{H}\right]\right)$$

$$J_X^{\mu\nu} = F_X^{||}\left[\frac{1}{2}\left(\left(\mathbf{D}^\mu\mathbf{H}\right)^\dagger \otimes \mathbf{D}^\nu\mathbf{H} + \left(\mathbf{D}^\nu\mathbf{H}\right)^\dagger \otimes \mathbf{D}^\mu\mathbf{H}\right) - \frac{c_X^{||}}{4} g^{\mu\nu}\left(\mathbf{D}_\rho\mathbf{H}\right)^\dagger \otimes \mathbf{D}^\rho\mathbf{H}\right.$$
$$\left. - \frac{\tau^{aa}}{6}\left(\mathrm{tr}\left[\left(\mathbf{D}^\mu\mathbf{H}\right)^\dagger \mathbf{D}^\nu\mathbf{H}\right] - \frac{c_X^{||}}{4} g^{\mu\nu}\,\mathrm{tr}\left[\left(\mathbf{D}_\rho\mathbf{H}\right)^\dagger \mathbf{D}^\rho\mathbf{H}\right]\right)\right]$$

Isospin-Spin Eigenamplitudes

$$
\begin{aligned}
a(w^+w^+ \to w^+w^+) &= a_{02}(s) - 10a_{22}(s) \\
&\quad + 15a_{22}(s)\frac{t^2+u^2}{s^2} \\
a(w^+w^- \to zz) &= \frac{1}{3}\left(a_{00}(s) - a_{20}(s)\right) - \frac{10}{3}\left(a_{02}(s) - a_{22}(s)\right) \\
&\quad + 5\left(a_{02}(s) - a_{22}(s)\right)\frac{t^2+u^2}{s^2} \\
a(w^+z \to w^+z) &= \frac{1}{2}a_{20}(s) - 5a_{22}(s) \\
&\quad + \left(-\frac{3}{2}a_{11}(s) + \frac{15}{2}a_{22}(s)\right)\frac{t^2}{s^2} \\
&\quad + \left(\frac{3}{2}a_{11}(s) + \frac{15}{2}a_{22}(s)\right)\frac{u^2}{s^2}
\end{aligned}
$$

Isospin-Spin Eigenamplitudes

$$a(w^+w^- \to w^+w^-) = \frac{1}{6}\left(2a_{00}(s) + a_{20}(s)\right) - \frac{5}{3}\left(2a_{02}(s) + a_{22}(s)\right)$$
$$+ \left(5a_{02}(s) - \frac{3}{2}a_{11}(s) + \frac{5}{2}a_{22}(s)\right)\frac{t^2}{s^2}$$
$$+ \left(5a_{02}(s) + \frac{3}{2}a_{11}(s) + \frac{5}{2}a_{22}(s)\right)\frac{u^2}{s^2}$$
$$a(zz \to zz) = \frac{1}{3}\left(a_{00}(s) + 2a_{20}(s)\right) - \frac{10}{3}\left(a_{02}(s) + 2a_{22}(s)\right)$$
$$+ 5\left(a_{02}(s) + 2a_{22}(s)\right)\frac{t^2 + u^2}{s^2}$$

Bounds on Eigenamplitudes

AQGC amplitudes (GBET):

$$a_{00}(s) = \frac{1}{6}(7F_{S,0} + 11F_{S,1})\,s^2$$
$$a_{02}(s) = \frac{1}{30}(2F_{S,0} + F_{S,1})\,s^2$$
$$a_{11}(s) = \frac{1}{12}(F_{S,0} - 2F_{S,1})\,s^2$$
$$a_{20}(s) = \frac{1}{3}(2F_{S,0} + F_{S,1})\,s^2$$
$$a_{22}(s) = \frac{1}{60}(2F_{S,0} + F_{S,1})\,s^2$$

a_{20} bounds

$$F_{S,0} = F_{S,1} = 480\ \text{TeV}^{-4} = (0.214\ \text{TeV})^{-4}$$

$$\sqrt{s} \lesssim 2.95 \cdot F_{S,0}^{-\frac{1}{4}} \approx 0.65\ \text{TeV}$$
$$\sqrt{s} \lesssim 3.50 \cdot F_{S,1}^{-\frac{1}{4}} \approx 0.75\ \text{TeV}$$

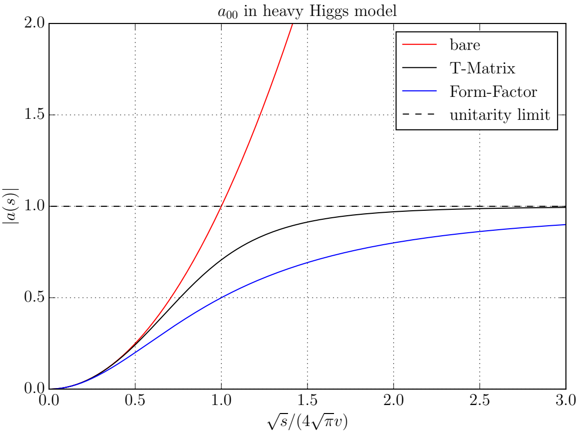
- Bounds depend on linear combination of AQGC (Assumption: Isospin/$SU(2)_C$ is preserved)

Algorithm for T-matrix

- Start with input model
- $\mathcal{L}_{S,1} = F_{S,1}\ \mathrm{tr}\left[(\mathbf{D}_\mu \mathbf{H})^\dagger\, \mathbf{D}^\mu \mathbf{H}\right] \cdot \mathrm{tr}\left[(\mathbf{D}_\nu \mathbf{H})^\dagger\, \mathbf{D}^\nu \mathbf{H}\right]$
- leads to the Feynman rules in unitary gauge
  $W^+_{\mu_1} W^+_{\mu_2} W^-_{\mu_3} W^-_{\mu_4}: \quad \frac{\mathrm{i} g^4 v^4}{8}\left[F_{S,1}\left(g_{\mu_1\mu_3} g_{\mu_2\mu_4} + g_{\mu_1\mu_4} g_{\mu_2\mu_3}\right)\right]$
- Extract strong-interaction part in Goldstone limit (Feynman Rules)
  $z(p_1) z(p_2) w^+(p_3) w^-(p_4): \quad 2\mathrm{i} F_{S,1}\,(p_1 \cdot p_2)\,(p_3 \cdot p_4)$
- Use of custodial/crossing symmetry to calculate $a^0_{I\ell}$
- Unitarize via T Matrix projection: $a_{I\ell}(s) = \left[\mathrm{Re}\left(a^0_{I\ell}(s)^{-1}\right) - \mathrm{i}\right]^{-1}$
- Calculate counter terms: $\Delta a_{I\ell} = a_{I\ell} - a^0_{I\ell}$
- Re-insert s-channel correction as form factor into Feynman rules

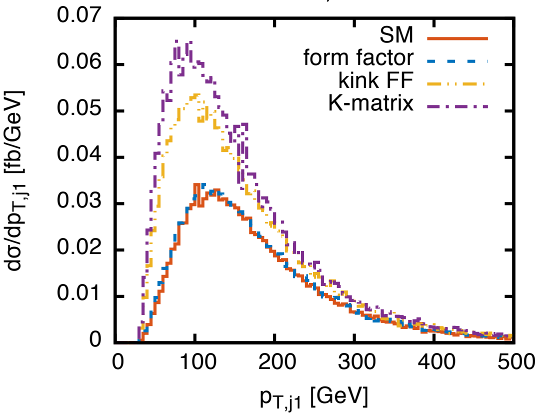
\+ Extrapolate off-shell
  $W^\pm_{\mu_1} W^\pm_{\mu_2} \to W^\pm_{\mu_3} W^\pm_{\mu_4}: \quad 8\pi g^4 v^4 \left[(\Delta a_{02}(s) - 10\Delta a_{22}(s))\, \frac{g_{\mu_1\mu_2} g_{\mu_3\mu_4}}{s^2} + 15\Delta a_{22}(s)\, \frac{g_{\mu_1\mu_3} g_{\mu_2\mu_4} + g_{\mu_1\mu_4} g_{\mu_2\mu_3}}{s^2}\right]$

Comparison of Higgsless amplitude



! To calculate Λ_{FF}, the limit $|a_\ell| < 1$ was used instead of the conventional $\mathrm{Re}\,(a_\ell) < 0.5$

Comparison for $pp \to e^+\nu_e\mu^+\nu_\mu jj$ (p_T)



provided by M.Rauch (VBFNLO)

- EFT parameters: $F_{S,1} = 400\,TeV^{-4}$
- FF parameters: $p = 2,\ \Lambda_{FF} = 832\ \mathrm{GeV}$
- Kink: $\Theta\left(\Lambda_{FF} - M_{4l}\right) + \frac{\Lambda_{FF}^4}{M_{4l}^4}\Theta\left(M_{4l} - \Lambda_{FF}\right)$

Conversions

Non-linear representation

Applequist, Bernard 1980

$$\mathcal{L}_{\alpha_4} = \alpha_4 \, \mathrm{tr}\left[\mathbf{V}_\mu \mathbf{V}_\nu\right] \mathrm{tr}\left[\mathbf{V}^\mu \mathbf{V}^\nu\right]$$
$$\mathcal{L}_{\alpha_5} = \alpha_5 \, \mathrm{tr}\left[\mathbf{V}_\mu \mathbf{V}^\mu\right] \mathrm{tr}\left[\mathbf{V}_\nu \mathbf{V}^\nu\right]$$

Higgs-Doublet representation

Rauch, Zeppenfeld

$$\mathcal{O}_{S,0} = \frac{f_{S,0}}{\Lambda^4}\left[\left(\mathbf{D}_\mu\Phi\right)^\dagger \mathbf{D}_\nu\Phi\right]\left[\left(\mathbf{D}^\mu\Phi\right)^\dagger \mathbf{D}^\nu\Phi\right]$$
$$\mathcal{O}_{S,1} = \frac{f_{S,1}}{\Lambda^4}\left[\left(\mathbf{D}_\mu\Phi\right)^\dagger \mathbf{D}^\mu\Phi\right]\left[\left(\mathbf{D}_\nu\Phi\right)^\dagger \mathbf{D}^\nu\Phi\right]$$
$$\mathcal{O}_{S,0} = \frac{f_{S,0}}{\Lambda^4}\left[\left(\mathbf{D}_\mu\Phi\right)^\dagger \mathbf{D}_\nu\Phi\right]\left[\left(\mathbf{D}^\nu\Phi\right)^\dagger \mathbf{D}^\mu\Phi\right]$$

Conversions

$$F_{S,0} = 16\frac{\alpha_4}{v^4} = \frac{f_{S,0} + f_{S,2}}{\Lambda^4}, \quad \text{with } f_{S,0} = f_{S,2}$$
$$F_{S,1} = 16\frac{\alpha_5}{v^4} = \frac{f_{S,1}}{\Lambda^4}$$

Keep in mind: S_0 (S_2) and S_1 contribute also to anomalous *VVHH* and *HHHH* couplings!