Axion dark matter from topological defects

Ken'ichi Saikawa (DESY)

Based on

T. Hiramatsu, M. Kawasaki, KS, T. Sekiguchi, PRD85, 105020 (2012) [1202.5851]
T. Hiramatsu, M. Kawasaki, KS, T. Sekiguchi, JCAP01, 001 (2013) [1207.3166]
M. Kawasaki, KS, T. Sekiguchi, PRD91, 065014 (2015) [1412.0789]
A. Ringwald, KS, PRD93, 085031 (2016) [1512.06436]

QCD axion as dark matter candidate

- Motivated by Pecccei-Quinn mechanism Peccei and Quinn (1977) as a solution of the strong CP problem

- Spontaneous breaking of continuous Peccei-Quinn symmetry at

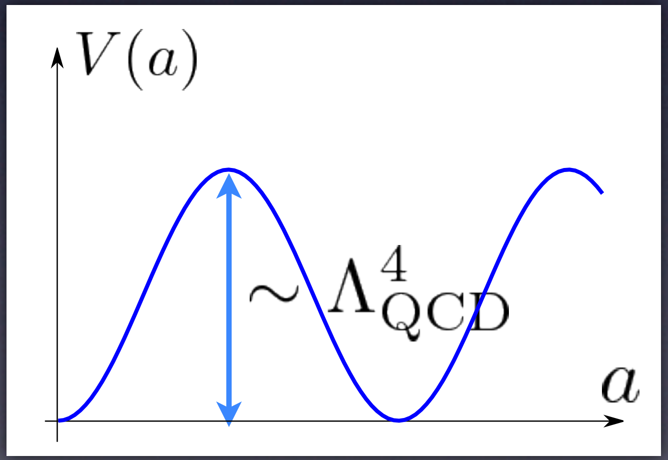
$$T \simeq F_a \simeq 10^{8-11}\mathrm{GeV}$$ "axion decay constant"

  - Nambu-Goldstone theorem
    → emergence of the (massless) particle ≡ axion
    Weinberg(1978), Wilczek(1978)

- Axion has a small mass (QCD effect)
  → pseudo-Nambu-Golstone boson

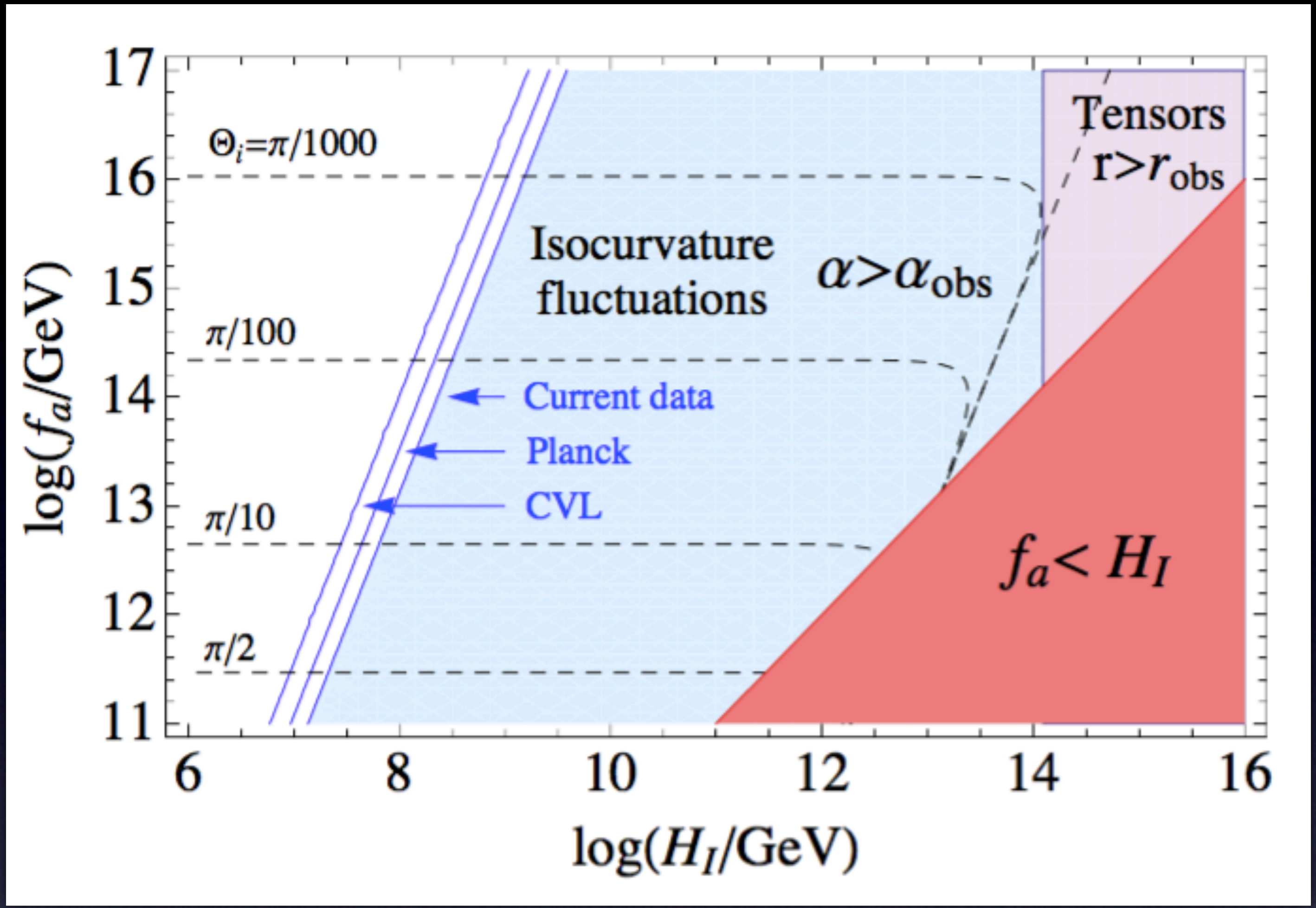
$$m_a \sim \frac{\Lambda_{\mathrm{QCD}}^2}{F_a} \sim 6 \times 10^{-5}\mathrm{eV}\left(\frac{10^{11}\mathrm{GeV}}{F_a}\right)$$

$$\Lambda_{\mathrm{QCD}} \simeq \mathcal{O}(100)\mathrm{MeV}$$



- Tiny coupling with matter + non-thermal production
  → good candidate of cold dark matter

Axions in the inflationary universe: two scenarios



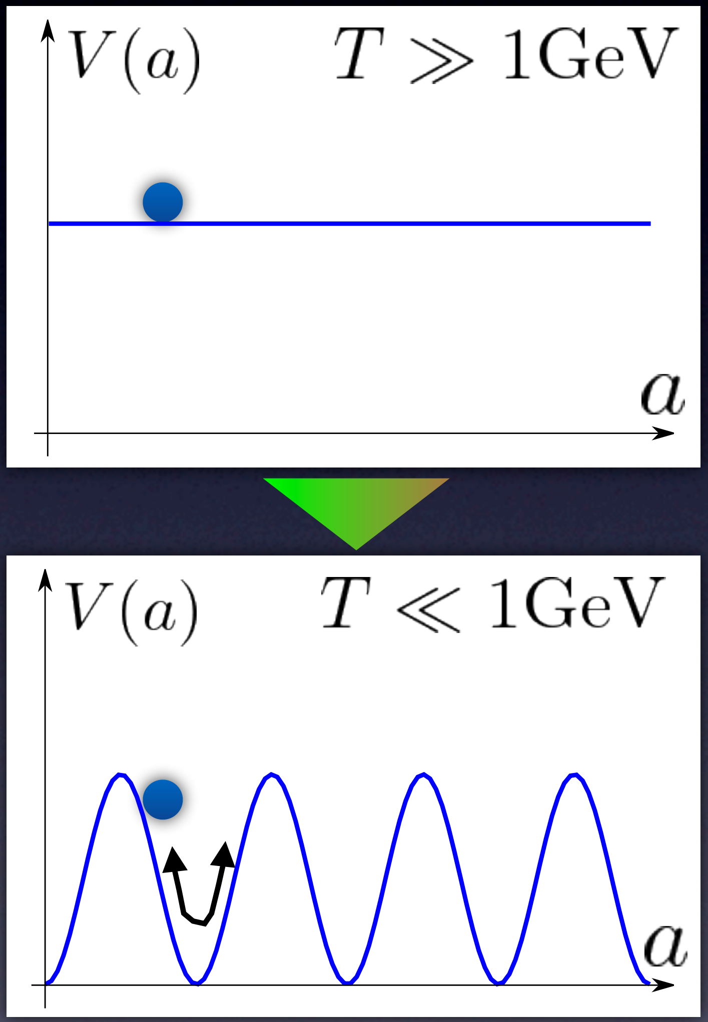
Hamann, Hannestad, Raffelt and Wong (2009)

- Pre-inflationary PQ symmetry breaking
  - Severe isocurvature constraints
  - Tuning of the initial field value ("anthropic window")
- Post-inflationary PQ symmetry breaking ← this talk
  - Formation of topological defects

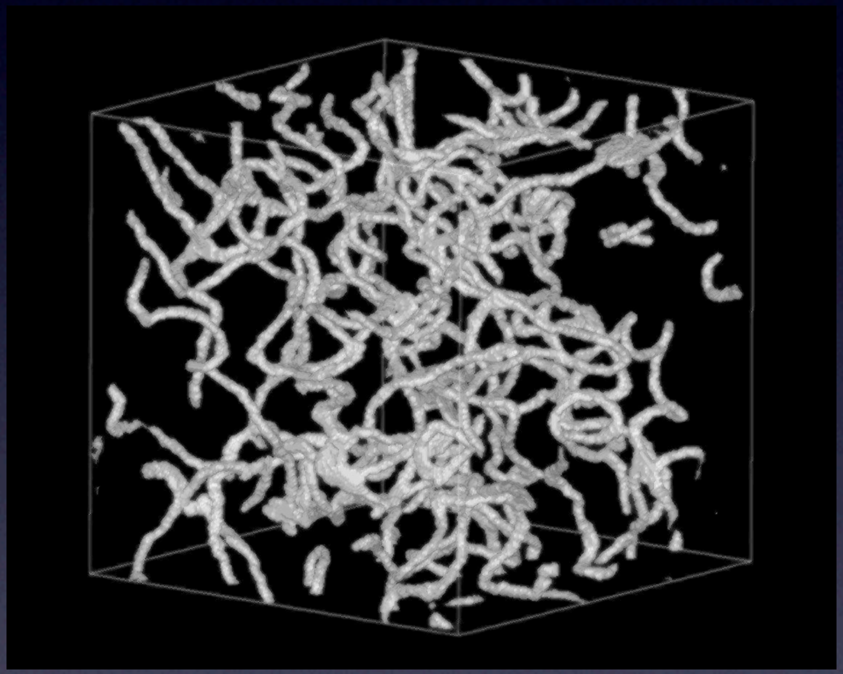
How axions are produced ?

If PQ symmetry is broken after inflation,
there are three contributions

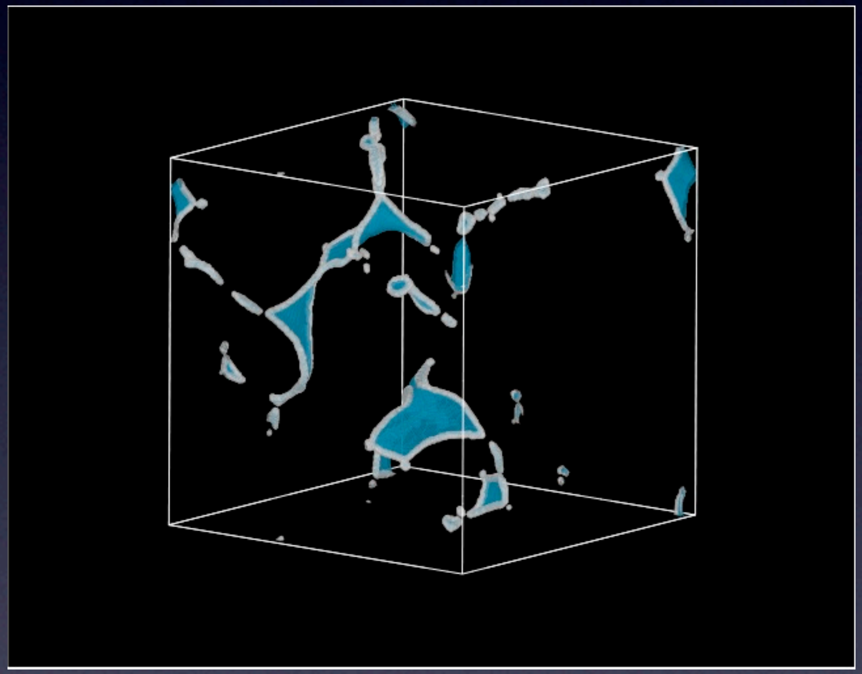
(1) Re-alignment mechanism



(2) Radiation from strings



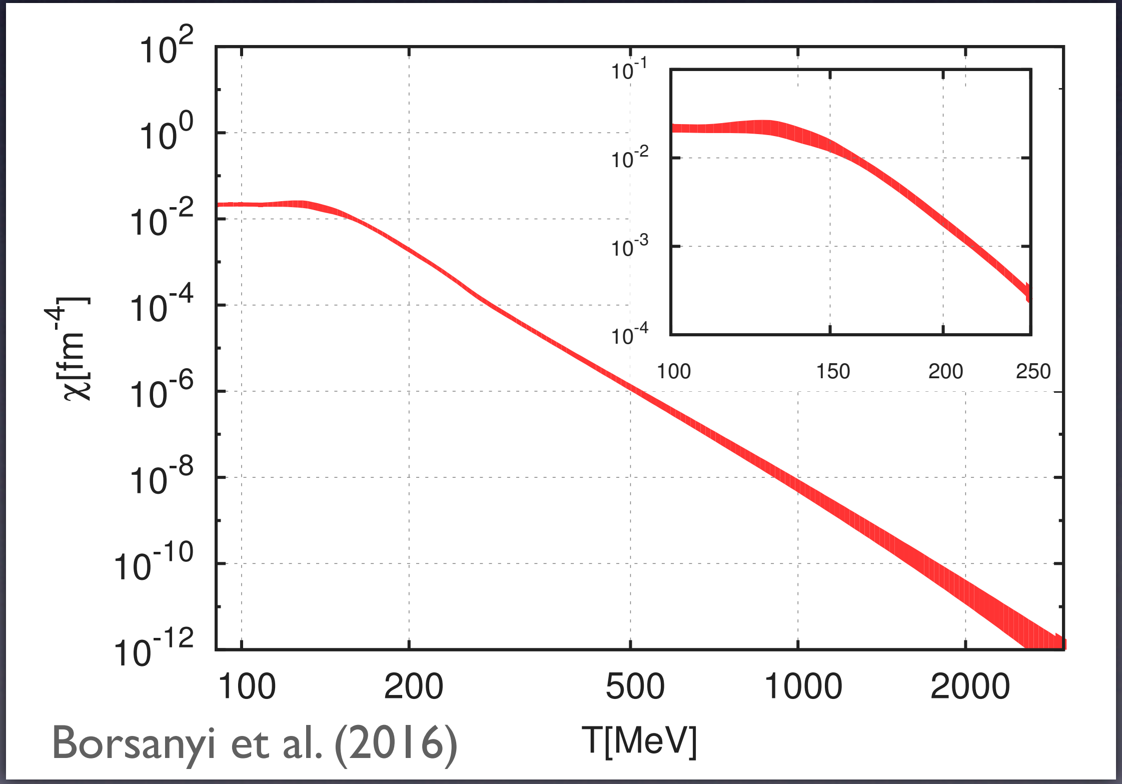
(3) Collapse of string-wall systems

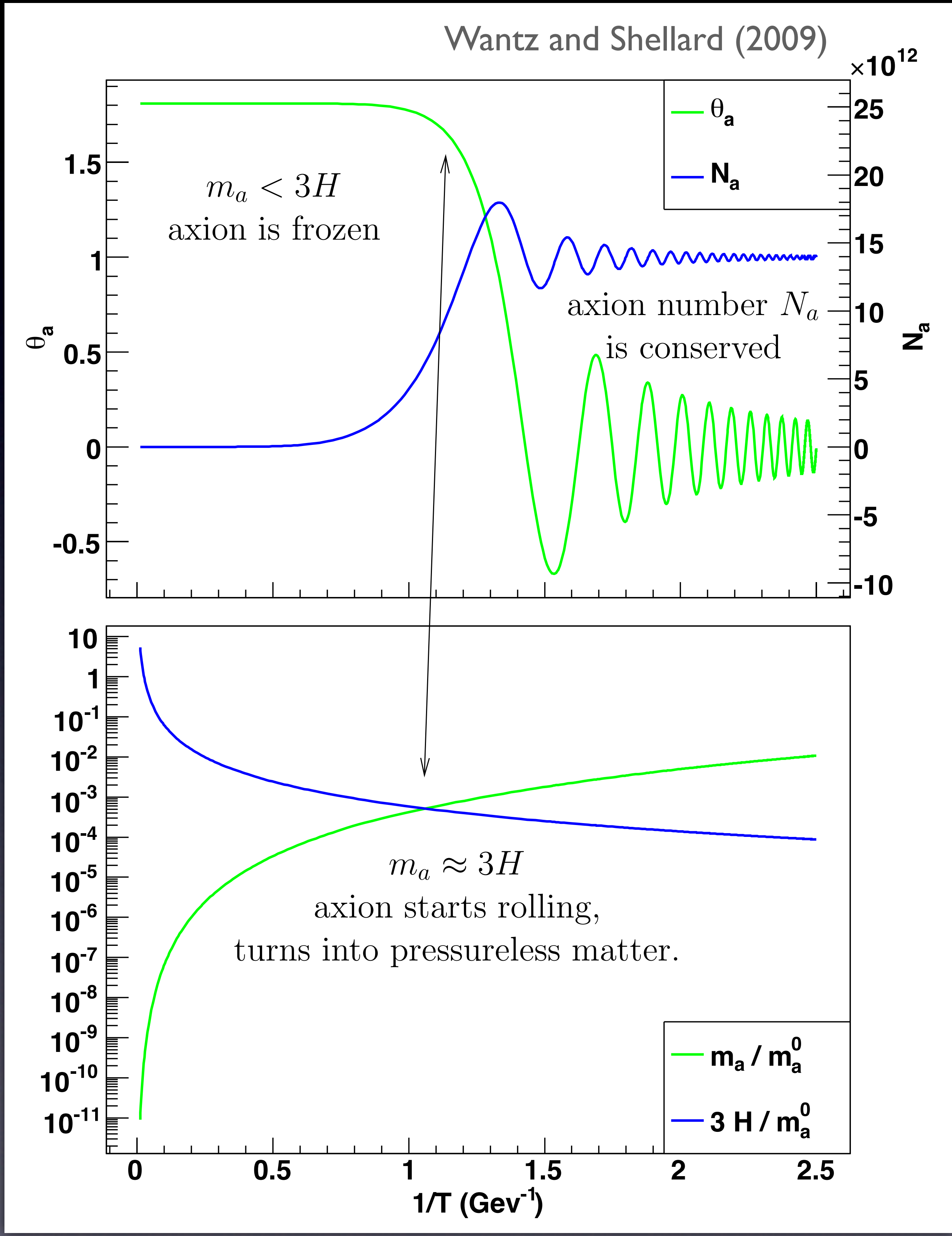


- Total abundance is sum of all these contributions
- All these effects have to be quantitatively taken into account

Re-alignment mechanism

- Axion field starts to oscillate at $$m_a(T_{\rm osc}) \approx 3H(T_{\rm osc})$$
- Temperature dependence of axion mass is important $$m_a(T)F_a = \sqrt{\chi(T)}$$
- Recently, the lattice calculations of χ in full QCD became available





Axionic string and axionic domain wall

Peccei-Quinn field (complex scalar field)

$$\Phi = |\Phi| e^{ia(x)/\eta}$$

$a(x)$: axion field

$$F_a = \eta / N_{\mathrm{DW}}$$

String formation $T \lesssim F_a$

Spontaneous breaking of U(1)PQ

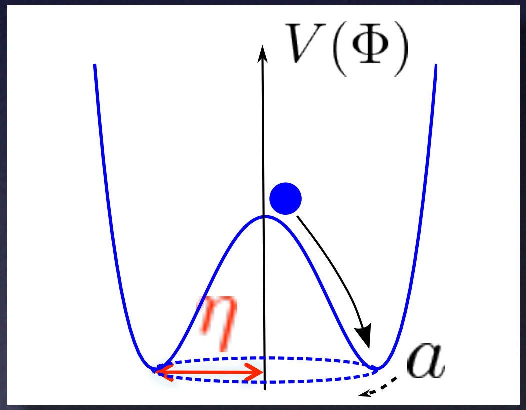
$$V(\Phi) = \frac{\lambda}{4}(|\Phi|^2 - \eta^2)^2$$

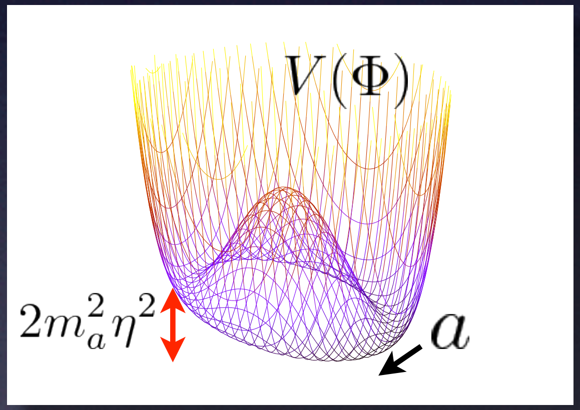
Domain wall formation $T \lesssim 1\mathrm{GeV}$

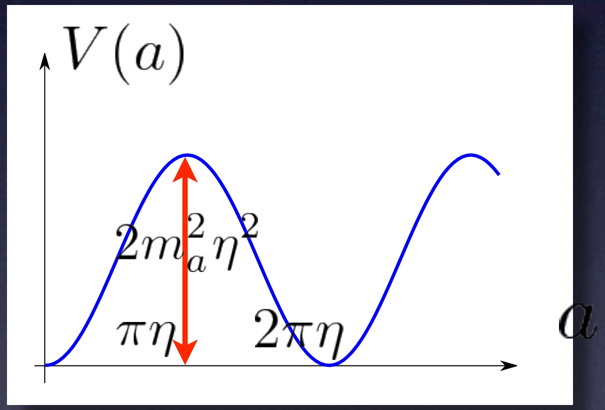
QCD effect

$$V(\Phi) = \frac{\lambda}{4}(|\Phi|^2 - \eta^2)^2 + m_a^2 \eta^2 (1 - \cos(a/\eta))$$

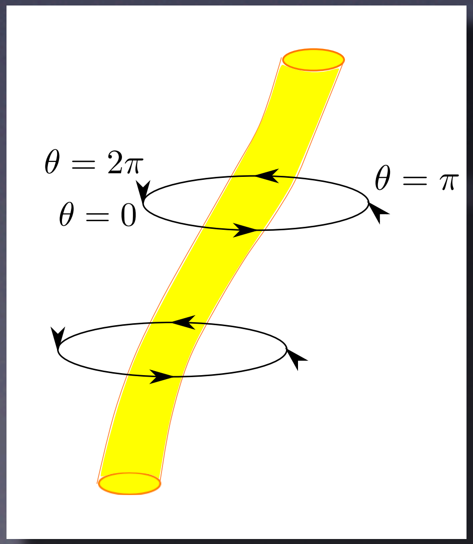
field space

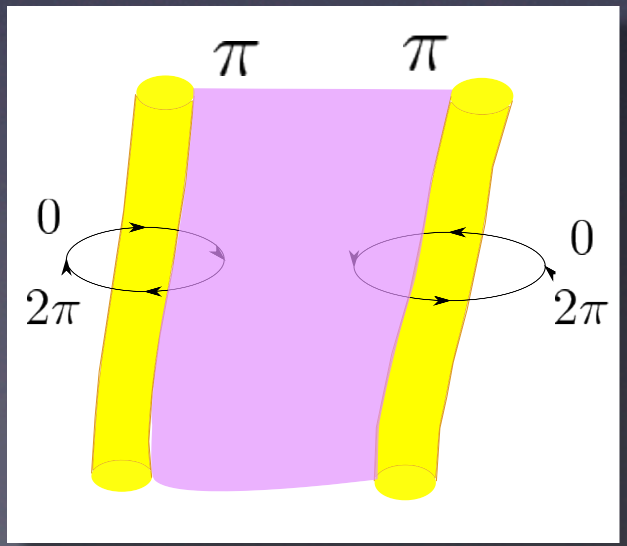






coordinate space





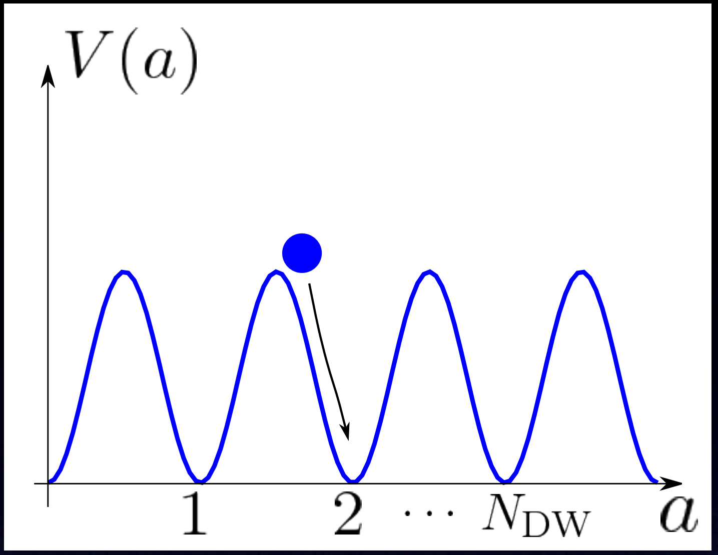
strings attached by domain walls

Domain wall problem

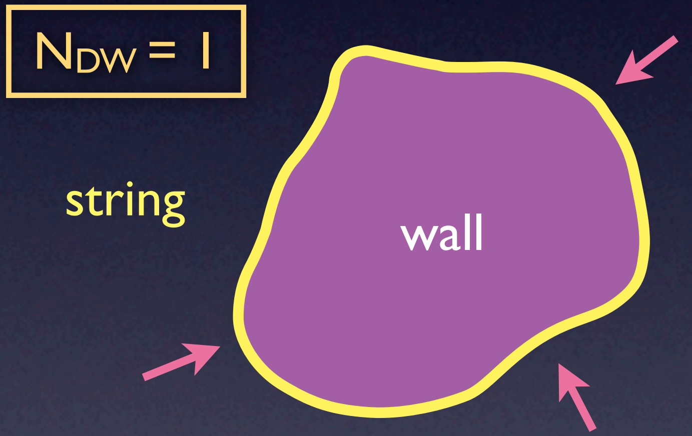
- Domain wall number N_{DW}
  - N_{DW} degenerate vacua

$$V(a) = \frac{m_a^2 \eta^2}{N_{\rm DW}^2}(1 - \cos(N_{\rm DW} a/\eta))$$

$N_{\rm DW}$: integer determined by QCD anomaly



- If N_{DW} = 1, string-wall systems are unstable
  - Collapse soon after the formation

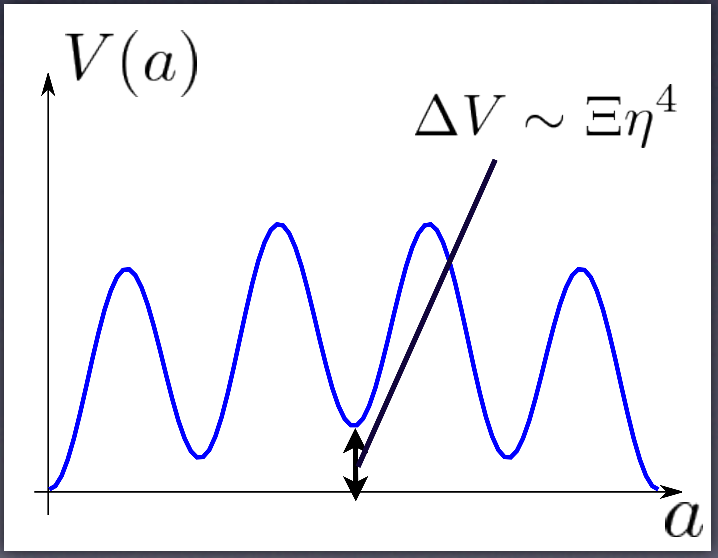


- If N_{DW} > 1, string-wall systems are stable
  - coming to overclose the universe

Zel'dovich, Kobzarev and Okun (1975)

  - We may avoid this problem by introducing an explicit symmetry breaking term Sikivie (1982) (walls become unstable)

$$V(\Phi) = \frac{m_a^2 \eta^2}{N_{\rm DW}^2}(1 - \cos(N_{\rm DW} a/\eta)) - \Xi \eta^3 (\Phi e^{-i\delta} + {\rm h.c.})$$



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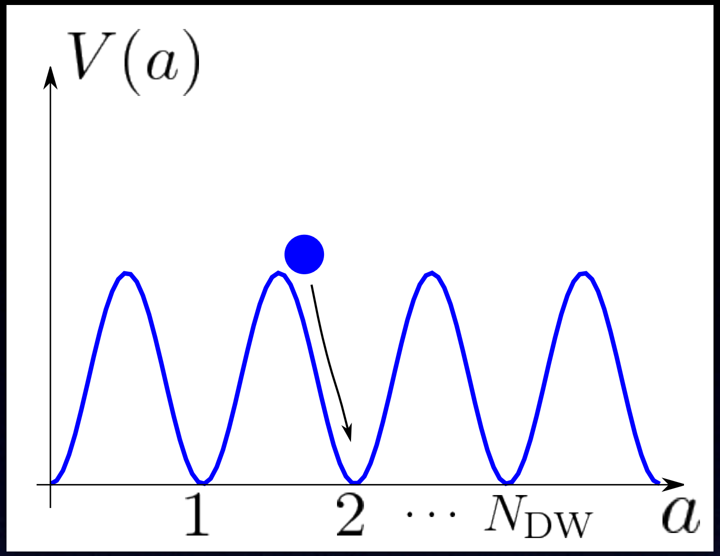

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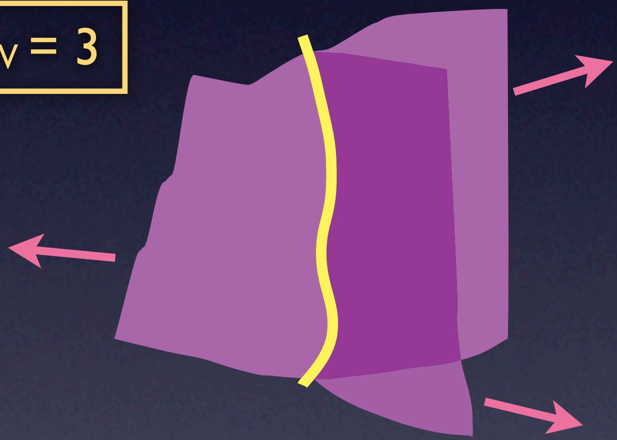
  - Collapse soon after the formation



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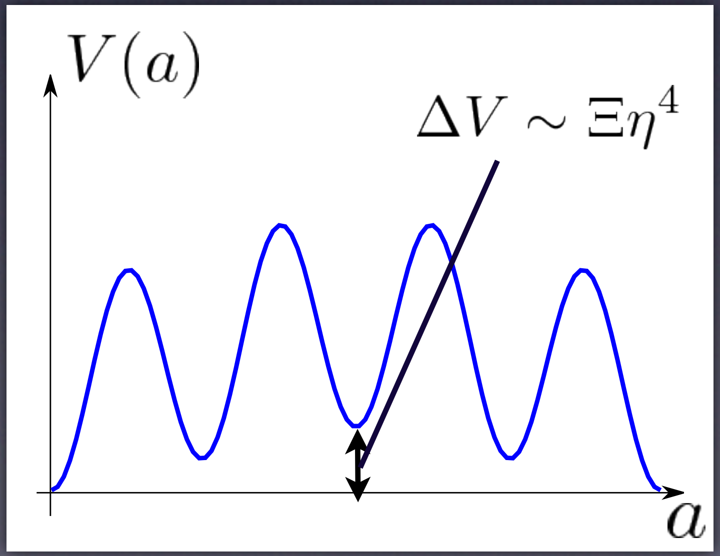
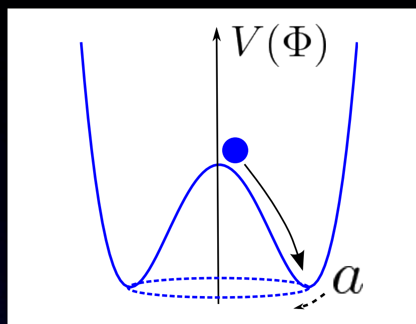


Production of axions in the early universe

(post-inflationary PQ symmetry breaking scenario)

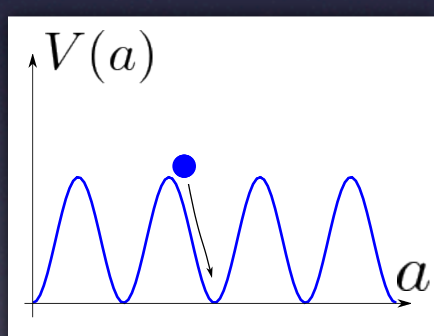
Inflation



$T \lesssim F_a \simeq 10^{8-11}\,\mathrm{GeV}$

PQ symmetry breaking
- Formation of strings

$T \lesssim 1\,\mathrm{GeV}$



QCD phase transition
- Axion acquires a mass
- Formation of domain walls

$N_{DW} = 1$ — soon after formation

Collapse of string-wall systems

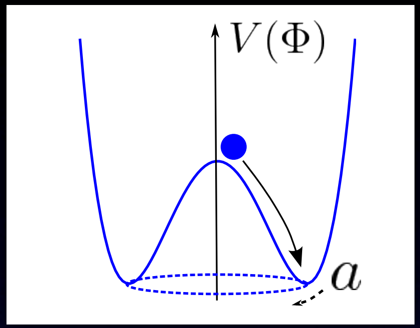
$N_{DW} > 1$ — String-wall networks exist for a long time

Annihilation of domain walls before they overclose the universe

Production of axions in the early universe

(post-inflationary PQ symmetry breaking scenario)

Inflation



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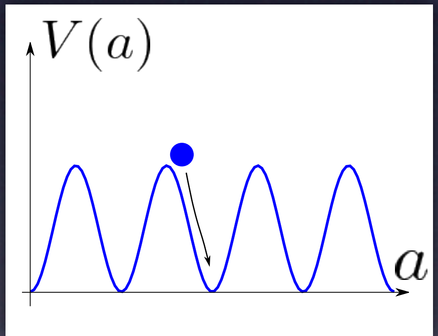

QCD phase transition
- Axion acquires a mass
- Formation of domain walls

(i) Coherent oscillation (re-alignment mechanism)

$\Omega_{a,\mathrm{real}}$

$N_{DW} = 1$ — soon after formation

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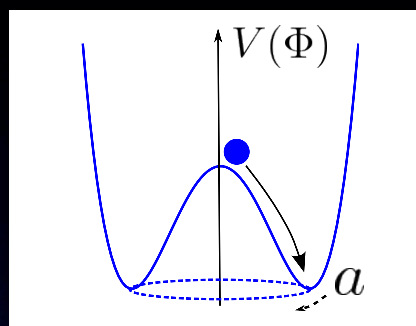

$T \lesssim F_a \simeq 10^{8-11}\,\mathrm{GeV}$

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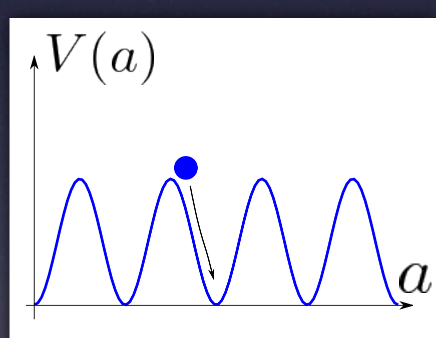


Inflation

PQ symmetry breaking
- Formation of strings

QCD phase transition
- Axion acquires a mass
- Formation of domain walls

(ii) Radiation from strings $\Omega_{a,\mathrm{string}}$

(i) Coherent oscillation (re-alignment mechanism) $\Omega_{a,\mathrm{real}}$

$N_{DW} = 1$ — soon after formation

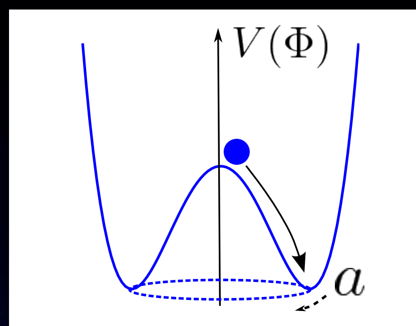
Collapse of string-wall systems

$N_{DW} > 1$ — String-wall networks exist for a long time

Annihilation of domain walls before they overclose the universe

Production of axions in the early universe

(post-inflationary PQ symmetry breaking scenario)



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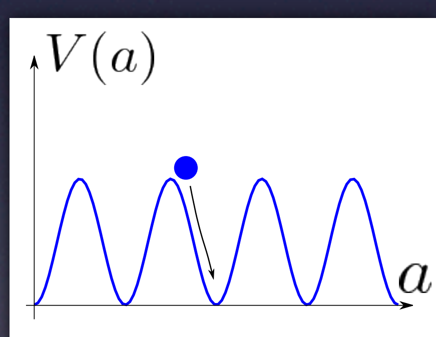


Inflation

PQ symmetry breaking
- Formation of strings

QCD phase transition
- Axion acquires a mass
- Formation of domain walls

(ii) Radiation from strings
$\Omega_{a,\mathrm{string}}$

(i) Coherent oscillation
(re-alignment mechanism)
$\Omega_{a,\mathrm{real}}$

(iii) Wall decay
$\Omega_{a,\mathrm{dec}}$

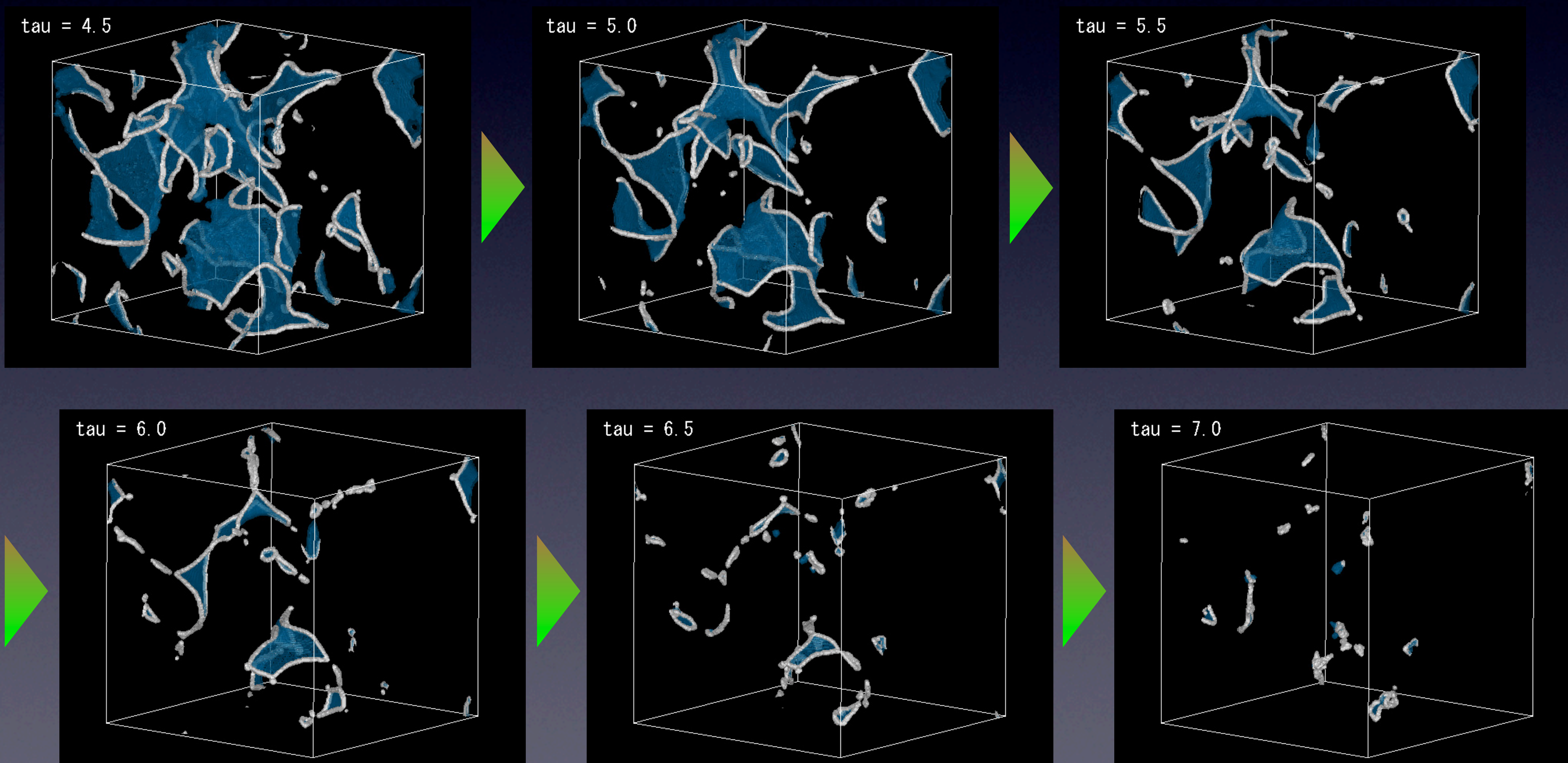
$N_{DW} = 1$ — soon after formation → **Collapse of string-wall systems**

$N_{DW} > 1$ — String-wall networks exist for a long time → **Annihilation of domain walls before they overclose the universe**

Numerical simulation : $N_{DW} = 1$

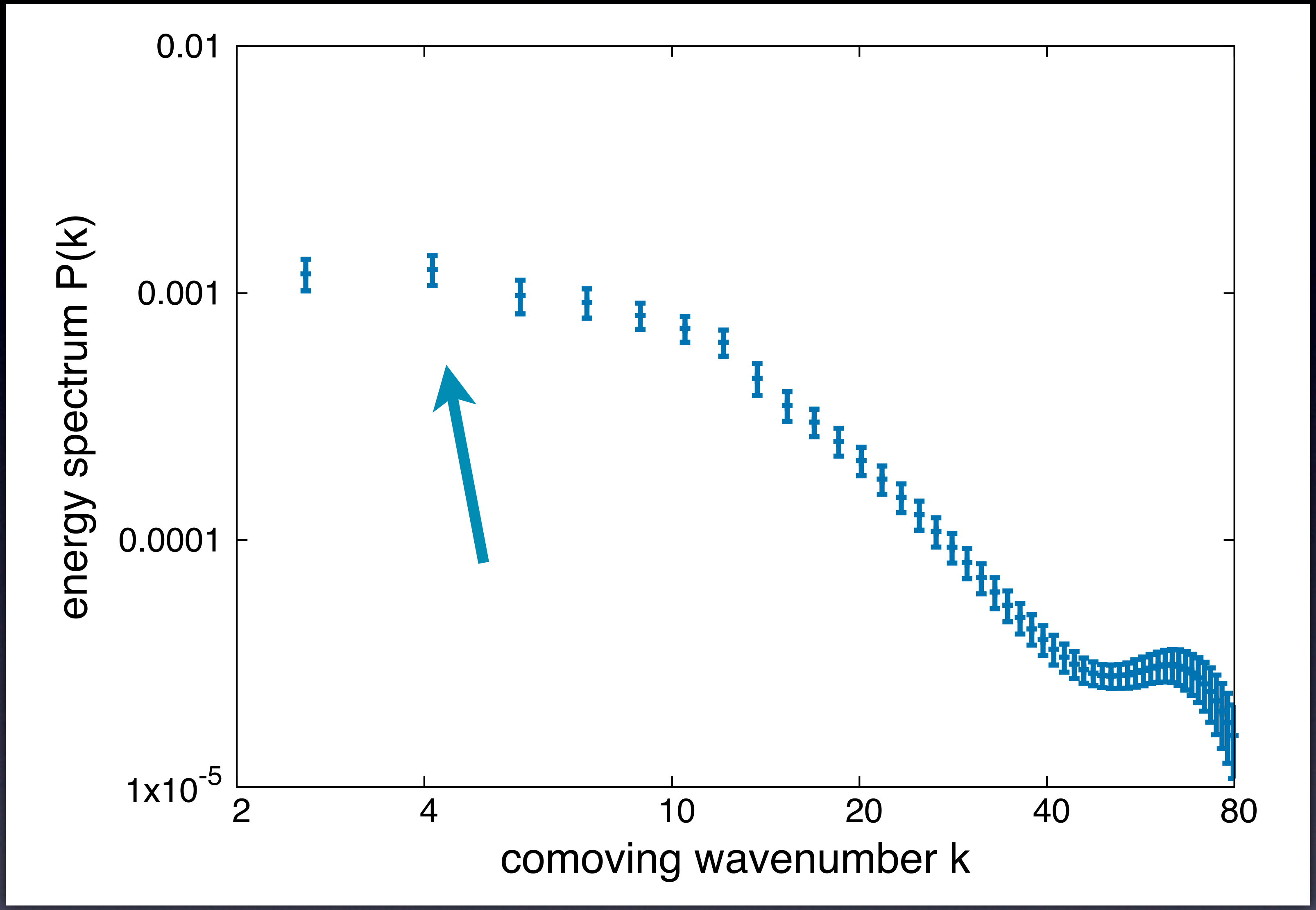
Hiramatsu, Kawasaki, KS and Sekiguchi (2012)

- Solve the classical field equations on lattice
- Number of grids in simulation box: $N^3 = 512^3$



Spectrum of radiated axions

Hiramatsu, Kawasaki, KS and Sekiguchi (2012)
Kawasaki, KS and Sekiguchi (2015)



Mean energy

$$\frac{\langle \omega_a \rangle}{m_a}(t_{\rm decay}) = 3.23 \pm 0.18$$

Contribution to relic abundance

$$\rho_a(t_{\rm today}) = m_a \frac{\rho_a(t_{\rm decay})}{\langle \omega_a \rangle} \left(\frac{R(t_{\rm decay})}{R(t_{\rm today})} \right)^3$$

$$\rho_a(t_{\rm decay}) \approx \rho_{\rm defects}(t_{\rm decay})$$

Axion dark matter abundance ($N_{DW} = 1$)

- Re-alignment mechanism Borsanyi et al. (2016), Ballesteros, Redondo, Ringwald and Tamarit (2016)

$$\Omega_{a,\mathrm{real}}h^2 \approx (3.8 \pm 0.6) \times 10^{-3} \left(\frac{F_a}{10^{10}\,\mathrm{GeV}}\right)^{1.165}$$

- Production from string-wall systems

$$\Omega_{a,\mathrm{string\text{-}wall}}h^2 \approx 1.2^{+0.9}_{-0.7} \times 10^{-2} \left(\frac{F_a}{10^{10}\,\mathrm{GeV}}\right)^{1.165}$$

- Total axion abundance

$$\Omega_{a,\mathrm{tot}}h^2 \approx 1.6^{+1.0}_{-0.7} \times 10^{-2} \left(\frac{F_a}{10^{10}\,\mathrm{GeV}}\right)^{1.165}$$

$$\Omega_{a,\mathrm{tot}} \leq \Omega_{\mathrm{CDM}}$$

$$\Omega_{\mathrm{CDM}}h^2 \simeq 0.12$$

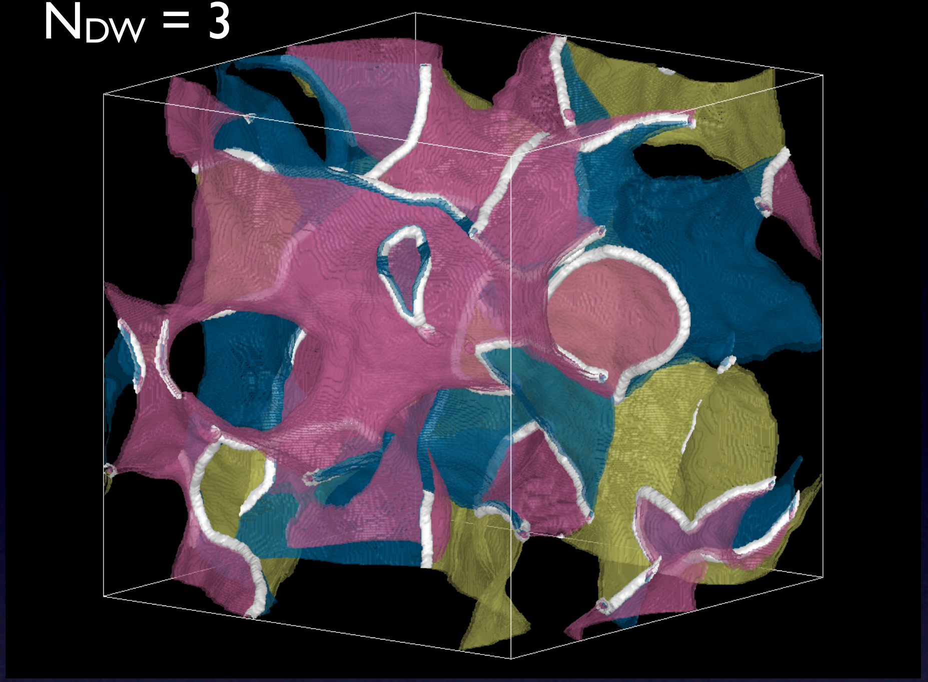
⇒

$$F_a \lesssim (3.8\text{–}9.9) \times 10^{10}\,\mathrm{GeV}$$

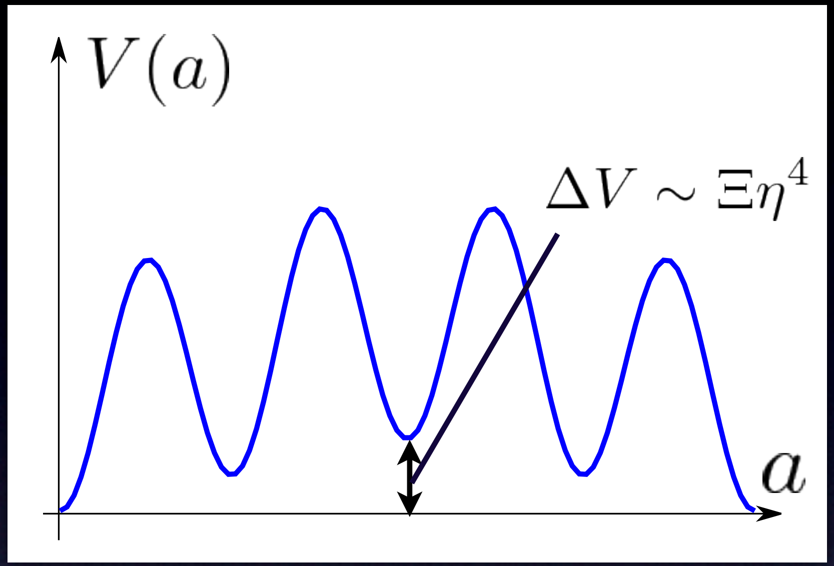
$$m_a \gtrsim (0.6\text{–}1.5) \times 10^{-4}\,\mathrm{eV}$$

- Large uncertainty comes from estimation of the string density in numerical simulations

Models with $N_{DW} > 1$



Hiramatsu, Kawasaki, KS and Sekiguchi (2013), Kawasaki, KS and Sekiguchi (2015), Ringwald and KS (2016)



- Domain walls are long-lived and decay due to the explicit symmetry breaking term

$$\Delta V = -\Xi\eta^3\left(\Phi e^{-i\delta} + \text{h.c.}\right)$$

- The contribution from long-lived domain walls leads to the possibility that axions explain CDM at lower F_a or larger m_a

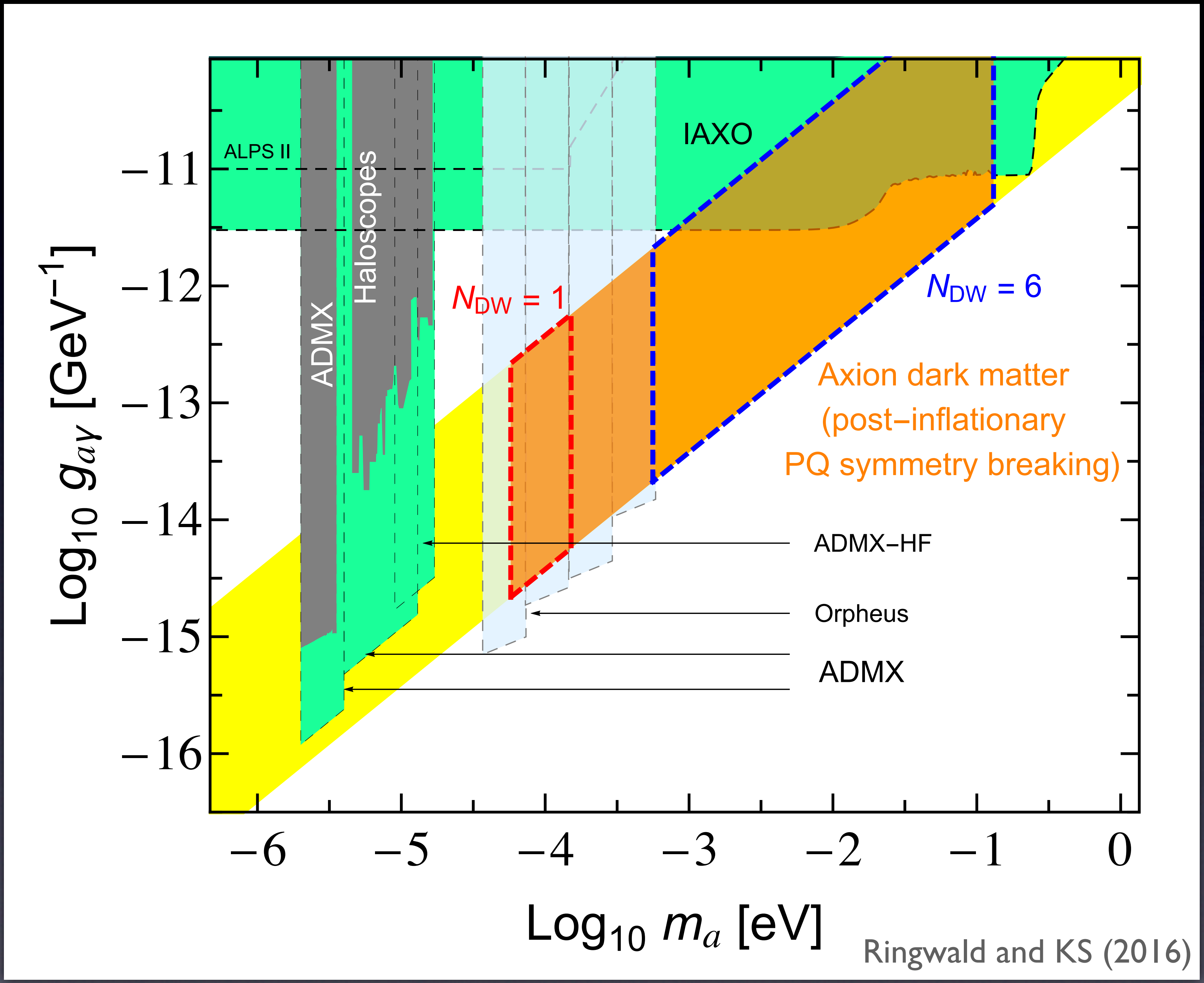
$$\Omega_{a,\text{wall}}h^2 \approx (0.09\text{–}0.17) \times \left(\frac{\Xi}{10^{-52}}\right)^{-1/2}\left(\frac{F_a}{10^9\,\text{GeV}}\right)^{-1/2} \quad (\text{for } N_{\text{DW}} = 6)$$

- Several constraints on the explicit symmetry breaking parameter Ξ

➡ Poster presentation
"Axion dark matter in the post-inflationary Peccei-Quinn symmetry breaking scenario"

Search for axion dark matter

Search space in photon coupling $g_{a\gamma} \sim \alpha/(2\pi F_a)$ vs. mass m_a



Mass ranges predicted in the post-inflationary PQ symmetry breaking scenario can be probed by various future experimental studies

Summary

- The scenario where PQ symmetry is broken after inflation is investigated
- Radiation from string-wall systems gives additional contribution to the CDM abundance
- Axion can be dominant component of dark matter if

$$F_a \simeq (3.8\text{–}9.9) \times 10^{10}\,\mathrm{GeV}$$
$$m_a \simeq (0.6\text{–}1.5) \times 10^{-4}\,\mathrm{eV}$$

for $N_{DW} = 1$

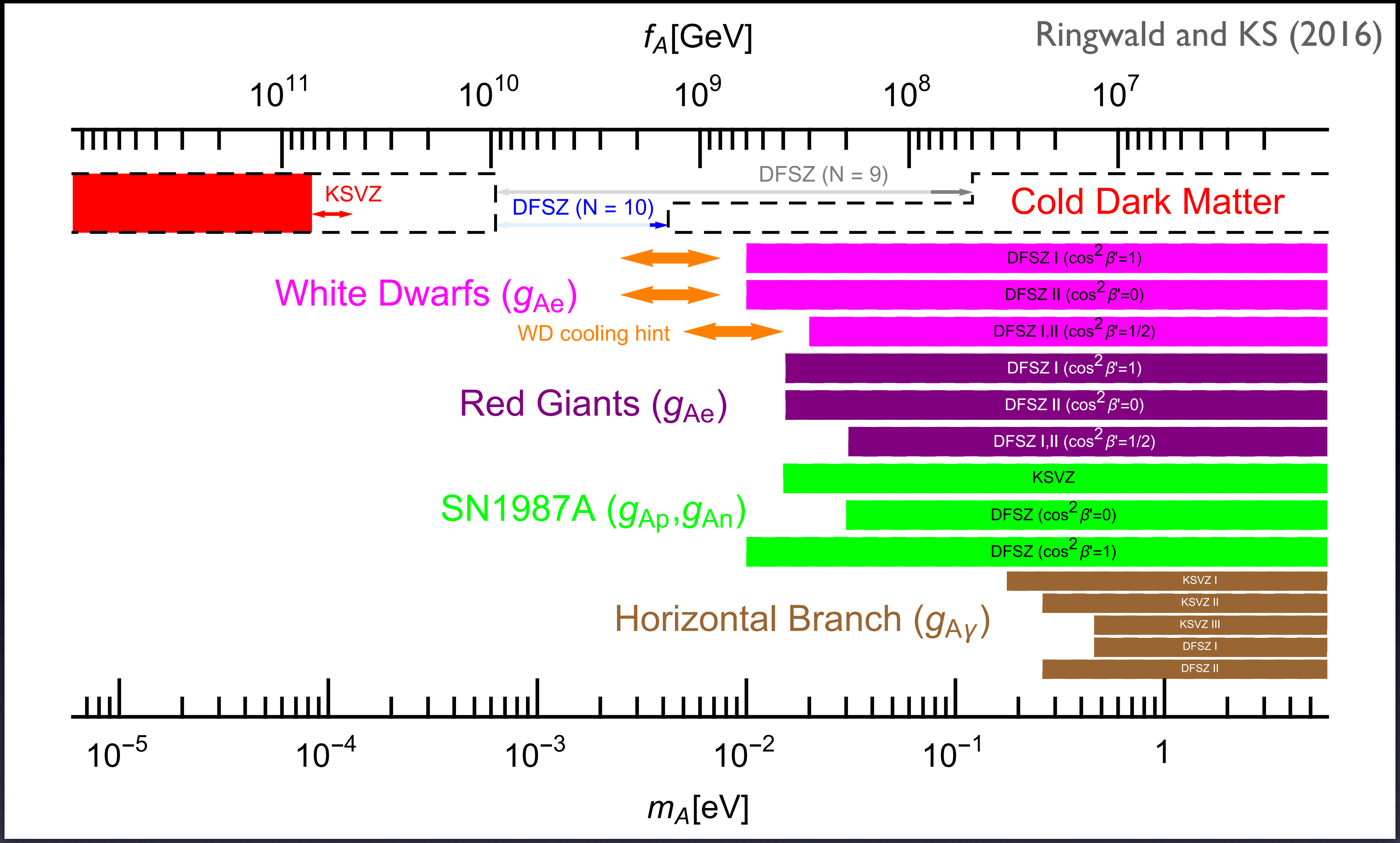
$$F_a \simeq \mathcal{O}(10^{8}\text{–}10^{10})\,\mathrm{GeV}$$
$$m_a \simeq \mathcal{O}(10^{-4}\text{–}10^{-2})\,\mathrm{eV}$$

for $N_{DW} > 1$

- Mass ranges can be probed in the future experiments

Backup slides

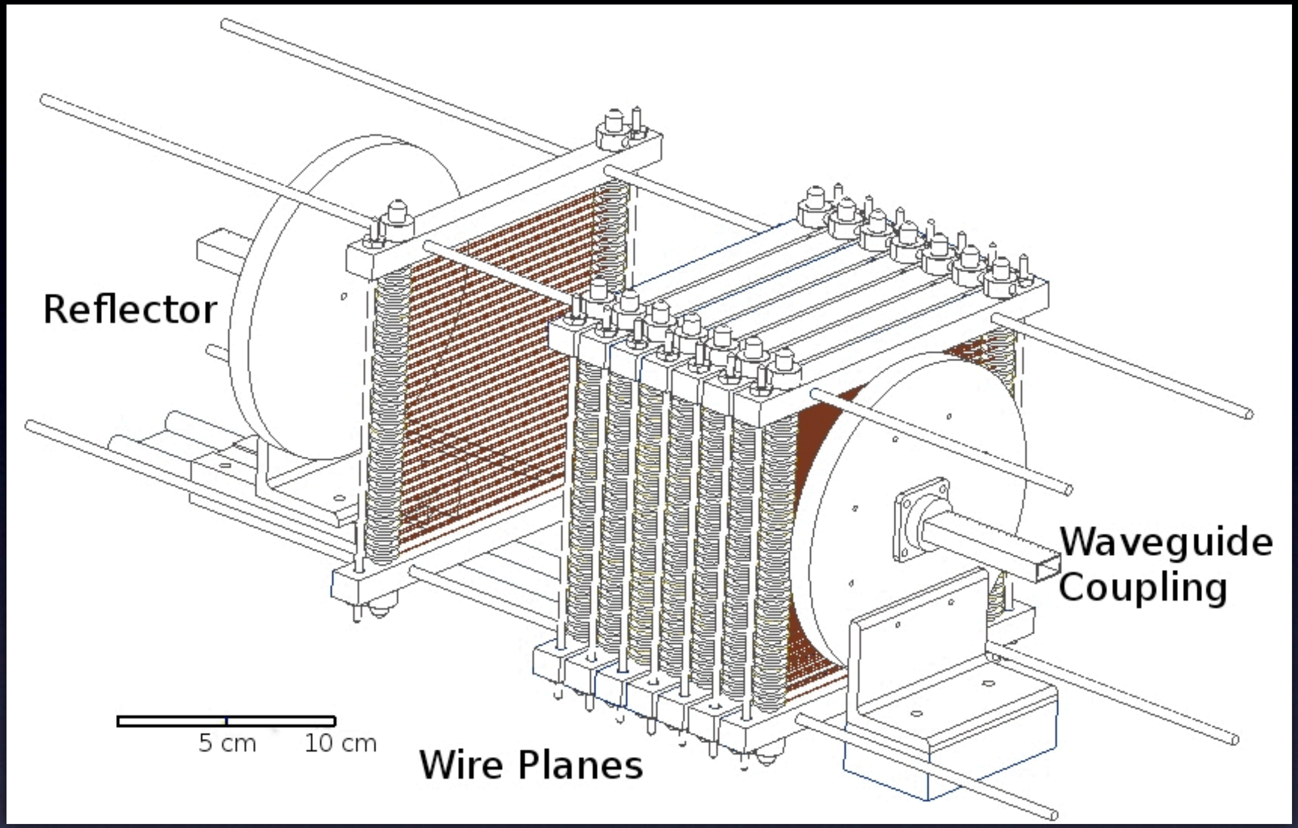
Astrophysical and cosmological constraints

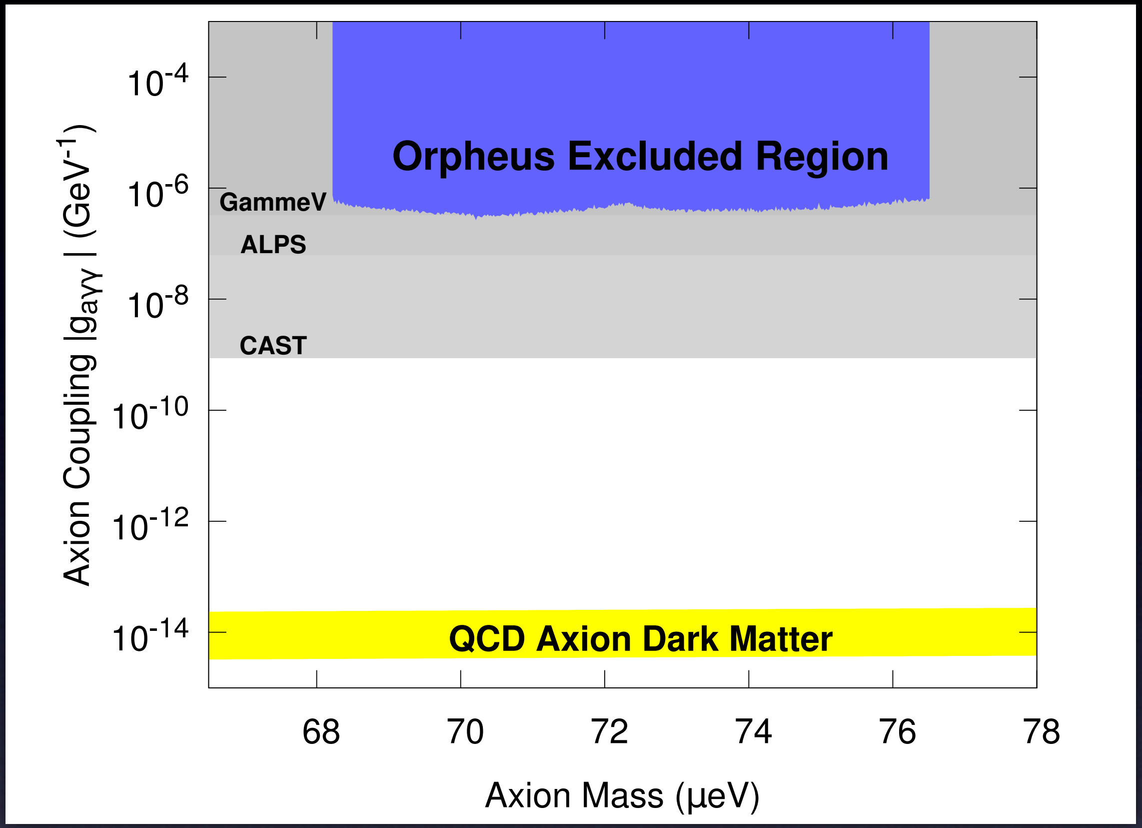


- Astrophysical observations give lower (upper) bounds on F_a (m_a)
- Dark matter abundance gives upper (lower) bounds on F_a (m_a) [and also a lower (upper) bound for DFSZ models]
- DFSZ models can explain CDM abundance at lower F_a (higher m_a) due to the additional contribution from long-lived string-wall systems

Orpheus

Rybka, Wagner, Patel, Percival, Ramos and Brill (2015)



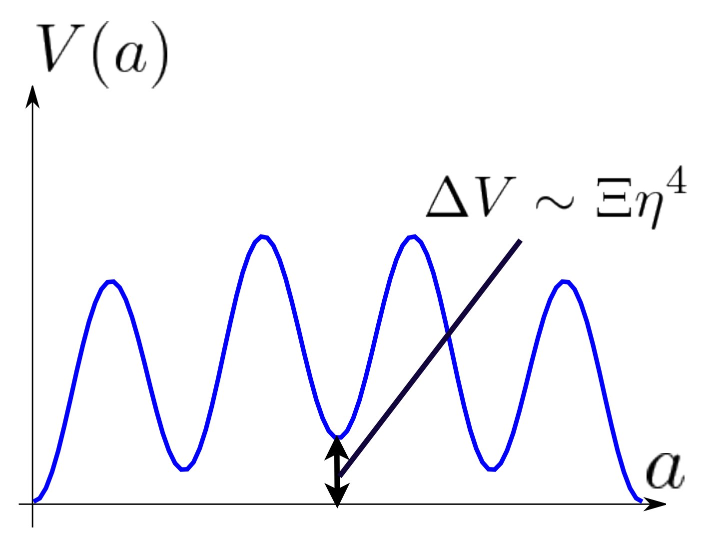


- Open Fabry-Perot resonator and a series of current-carrying wire planes
- Searches for axion like particles in the 68.2-76.5μeV mass range were demonstrated
- Potentially searches in the mass range 40-400μeV in the future

$N_{DW} > 1$: long-lived domain walls

- Domain walls are long-lived and decay due to the bias term

$$V_{\rm bias}(\Phi) = -\Xi\eta^3(\Phi e^{-i\delta} + {\rm h.c.})$$



- For small bias

  Long-lived domain walls emit a lot of axions which might exceed the observed matter density

Cosmology → large bias is favored

- For large bias

  Bias term shifts the minimum of the potential and might spoil the original Peccei-Quinn solution to the strong CP problem

$$\bar{\theta} = \frac{2\Xi N_{\rm DW}^3 F_a^2 \sin\delta}{m_a^2 + 2\Xi N_{\rm DW}^2 F_a^2 \cos\delta} < 7 \times 10^{-12}$$

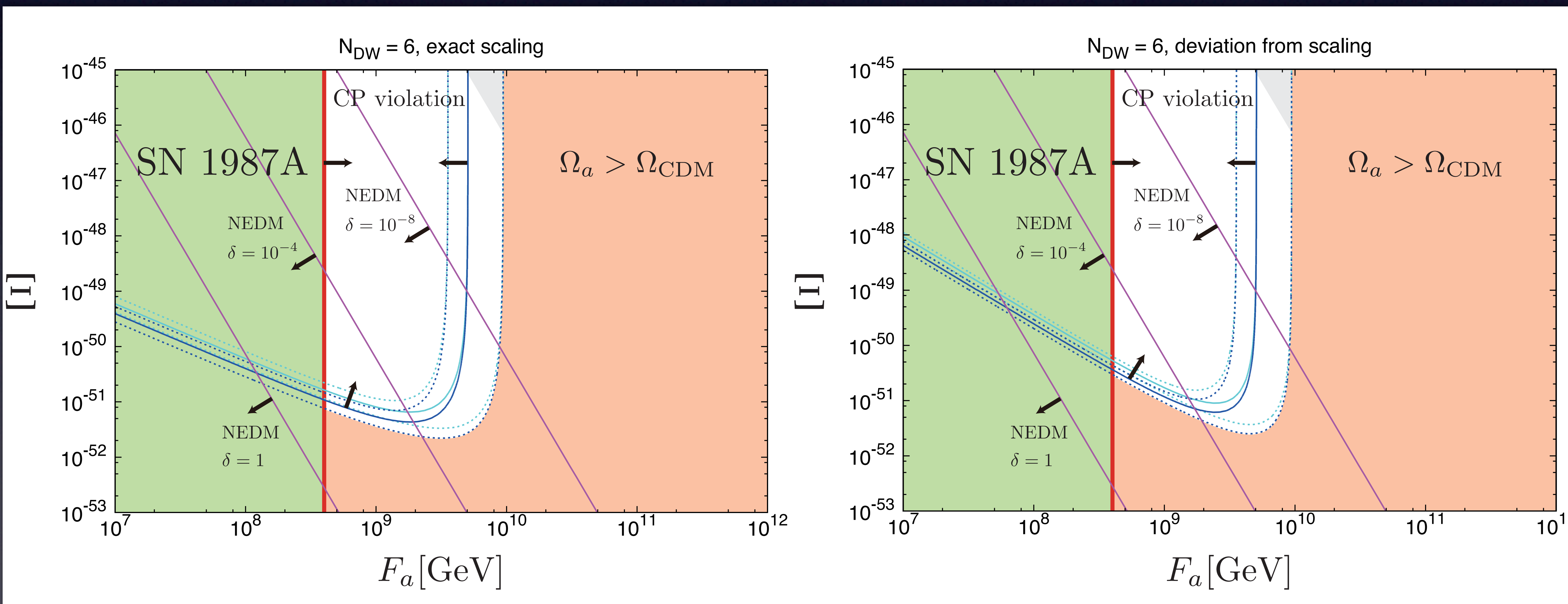
δ : phase of bias term

CP → small bias is favored

- Consistent parameters ?

Constraints

- Axion density $\Omega_{a,\mathrm{mis}} + \Omega_{a,\mathrm{string}} + \Omega_{a,\mathrm{dec}} \leq \Omega_{\mathrm{CDM}}$
- Neutron electric dipole moment (NEDM) $\bar{\theta} < 0.7 \times 10^{-11}$
- Astrophysical constraint (SN1987A) $F_a > 4 \times 10^{8}\mathrm{GeV}$



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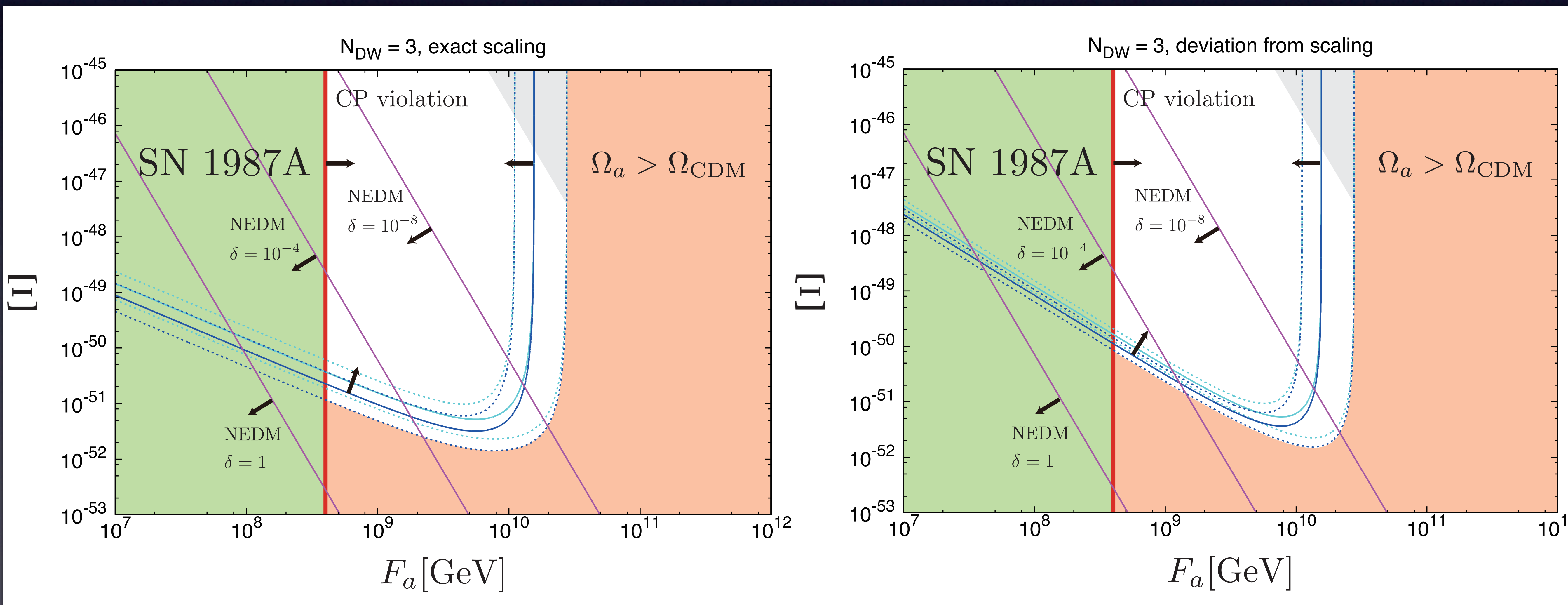


Numerical simulation

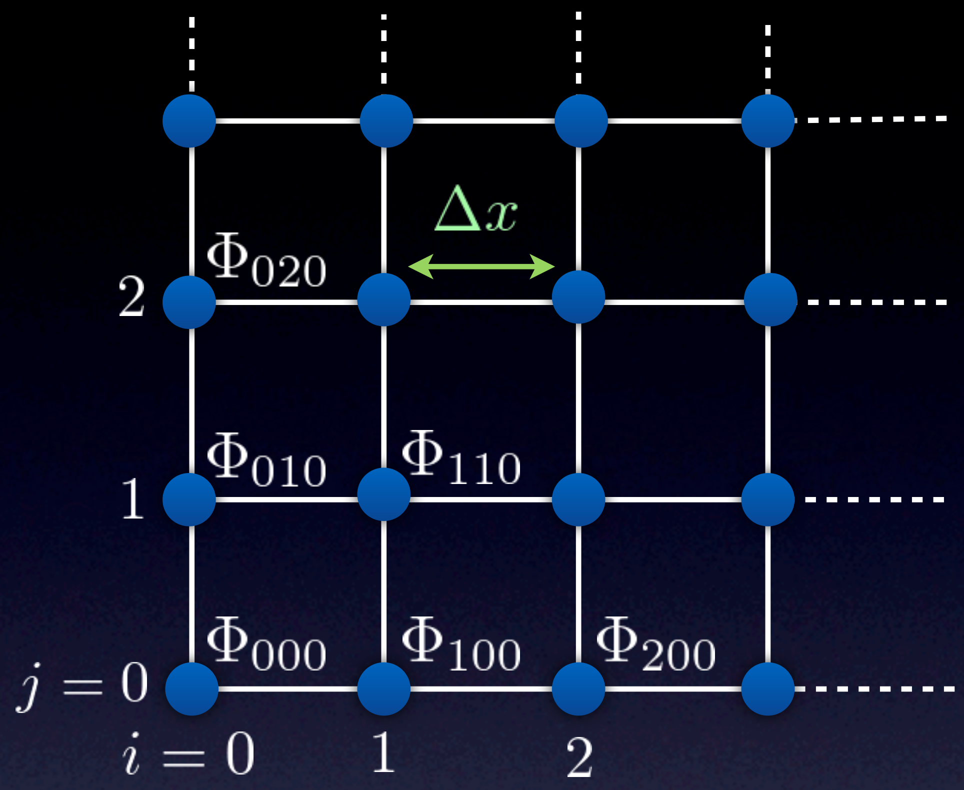
- Discretize the spatial coordinate

$$\vec{\mathbf{x}} \quad \rightarrow \quad (i, j, k)$$

$$i, j, k = 0, 1, \ldots, N-1$$

$$\Phi(\vec{\mathbf{x}}) \quad \rightarrow \quad \Phi_{i,j,k}$$

$$\nabla^2\Phi(\vec{\mathbf{x}}) \;\rightarrow\; (\nabla^2\Phi)_{i,j,k}$$



$$= \frac{1}{12(\Delta x)^2}\left[16(\Phi_{i+1,j,k} + \Phi_{i-1,j,k} + \Phi_{i,j+1,k} + \Phi_{i,j-1,k} + \Phi_{i,j,k+1} + \Phi_{i,j,k-1}) - (\Phi_{i+2,j,k} + \Phi_{i-2,j,k} + \Phi_{i,j+2,k} + \Phi_{i,j-2,k} + \Phi_{i,j,k+2} + \Phi_{i,j,k-2}) - 90\Phi_{i,j,k}\right]$$

- Solve the classical EOM for complex scalar $\Phi = \phi_1 + i\phi_2$ on lattice

$$\ddot{\phi}_i + 3H\dot{\phi}_i - \frac{\nabla^2}{R^2(t)}\phi_i = -\frac{\partial V}{\partial \phi_i} \qquad i = 1, 2$$

- Number of grids in simulation box : $N^3 = 512^3$

Numerical simulations: N_{DW} = 1

- Solve the classical EOM for complex scalar $\Phi = \phi_1 + i\phi_2$ on 3D lattice

$$\ddot{\phi}_1 + 3H\dot{\phi}_1 - \frac{\nabla^2}{a^2}\phi_1 = -\lambda\phi_1(|\Phi|^2 - \eta^2) - \frac{\lambda}{3}T^2\phi_1 + m_a^2\eta$$

$$\ddot{\phi}_2 + 3H\dot{\phi}_2 - \frac{\nabla^2}{a^2}\phi_2 = -\lambda\phi_2(|\Phi|^2 - \eta^2) - \frac{\lambda}{3}T^2\phi_2$$

- Include temperature dependence of axion mass Wantz & Shellard, PRD82, 123508 (2010)

$$m_a(T)^2/F_a^2 = c_T\kappa^{n+4}\left(\frac{T}{F_a}\right)^{-n} \qquad n = 6.68$$

$$m_a(0)^2/F_a^2 = c_0\kappa^4 \quad \text{for} \quad m_a(T) > m_a(0)$$

$$\kappa \equiv \Lambda_{QCD}/F_a = \Lambda_{QCD}/\eta$$

c_T	6.26
c_0	1.0
λ	1.0
κ	0.2-0.4

- Number of grids in simulation box : $N^3 = 512^3$
- (Comoving) Box size : $L = 20$ ($\Delta x = L/N \simeq 0.039$)
- Numerical computation is carried out in SR16000 at the Yukawa Institute Computer Facility

Numerical simulations: $N_{DW} > 1$

- Solve the classical EOM for complex scalar field

$$\Phi = \phi_1 + i\phi_2$$

$$\ddot{\phi}_1 + 3H\dot{\phi}_1 - \frac{\nabla^2}{a^2(t)}\phi_1 = -\lambda\phi_1(\phi_1^2 + \phi_2^2 - \eta^2) - \frac{\partial V_a}{\partial \phi_1}$$

$$\ddot{\phi}_2 + 3H\dot{\phi}_2 - \frac{\nabla^2}{a^2(t)}\phi_2 = -\lambda\phi_2(\phi_1^2 + \phi_2^2 - \eta^2) - \frac{\partial V_a}{\partial \phi_2}$$

$$\frac{\partial V_a}{\partial \phi_1} = -\frac{m^2\eta}{N_{\rm DW}^2}(\cos\theta\cos N_{\rm DW}\theta + N_{\rm DW}\sin\theta\sin N_{\rm DW}\theta)$$

$$\frac{\partial V_a}{\partial \phi_2} = -\frac{m^2\eta}{N_{\rm DW}^2}(\sin\theta\cos N_{\rm DW}\theta - N_{\rm DW}\cos\theta\sin N_{\rm DW}\theta)$$

- Parameters

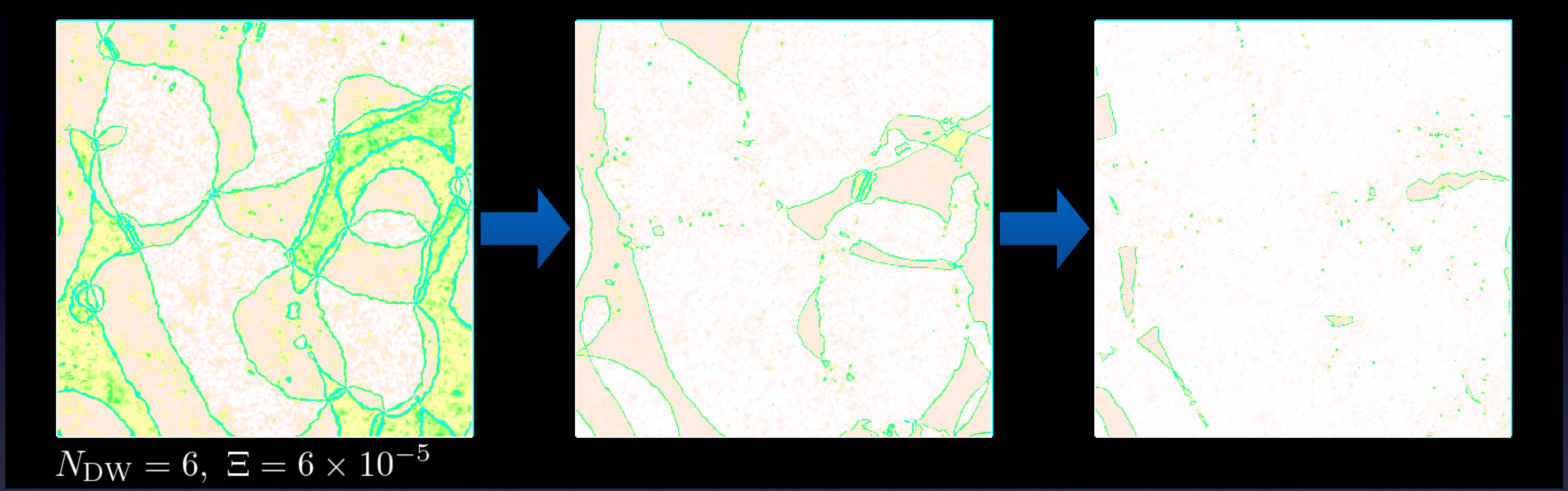
λ	0.1
m/η	0.1
$N_{\rm DW}$	2 - 6

- Number of grids in simulation box : $N^3 = 512^3$
- (Comoving) Box size : $L = 80$ $(\Delta x = L/N \simeq 0.156)$
- Note: we do not include the bias term (Ξ term)
  - evolution of the stable networks

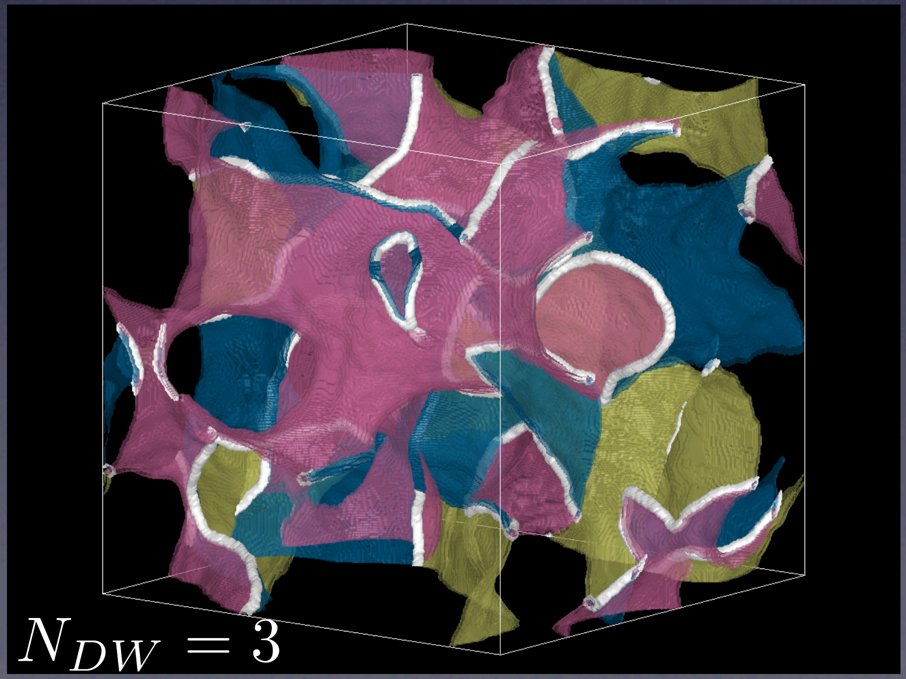
Numerical simulations : $N_{DW} > 1$

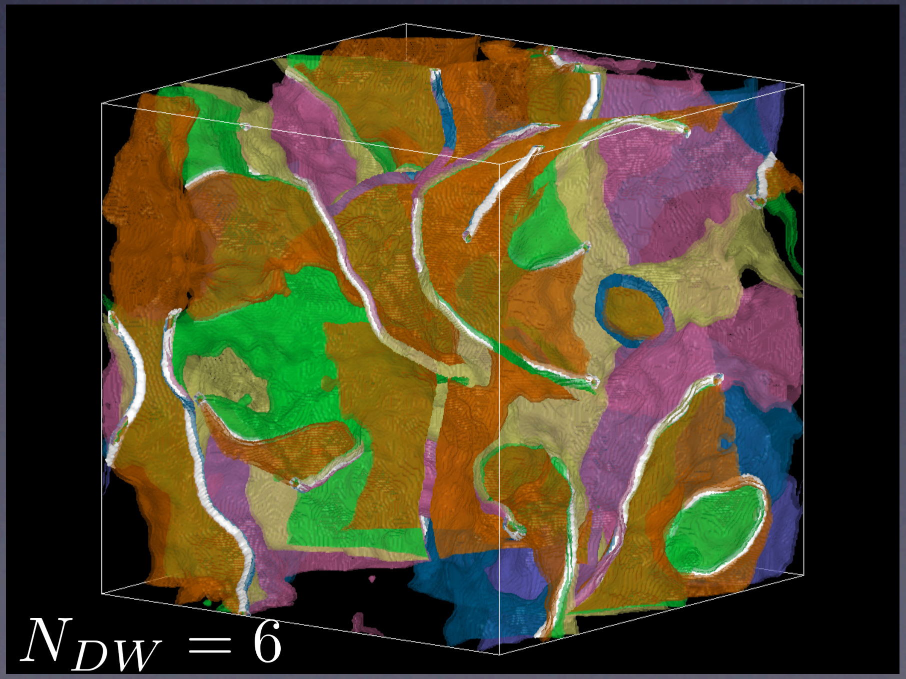
Hiramatsu, Kawasaki, KS and Sekiguchi (2012)
Kawasaki, KS and Sekiguchi (2015)

- 8192^2, 16384^2, 32768^2 (2D) → decay time of domain walls



- 512^3 (3D) → spectrum of radiated axions





Initial Conditions (N_{DW} = 1)

- Treat ϕ_1 and ϕ_2 as two independent real scalar fields with correlation function in the finite temperature

$$\phi = \phi_1 + i\phi_2$$

$$\langle \phi_i(\mathbf{k})\phi_i(\mathbf{k}')\rangle = \frac{n_k}{E_k}(2\pi)^3\delta^{(3)}(\mathbf{k}+\mathbf{k}') \quad i = 1, 2$$

$$\langle \dot\phi_i(\mathbf{k})\dot\phi_i(\mathbf{k}')\rangle = E_k n_k (2\pi)^3\delta^{(3)}(\mathbf{k}+\mathbf{k}') \qquad E_k = \sqrt{k^2+m^2}$$

$$n_k = \frac{1}{e^{E_k/T}-1}$$

- No correlation in the k space
  - Generate $\phi_i(\mathbf{k})$ as Gaussian with

$$\langle |\phi(\mathbf{k})|^2\rangle = \frac{n_k}{E_k}V_b \quad \langle |\dot\phi(\mathbf{k})|^2\rangle = E_k n_k V_b$$

$$\langle \phi(\mathbf{k})\rangle = \langle \dot\phi(\mathbf{k})\rangle = 0$$

$V_b \simeq (2\pi)^3\delta^{(3)}(0)$

: volume of the simulation box

  - Fourier transform to obtain $\phi_i(\mathbf{x})$ and $\dot\phi_i(\mathbf{x})$

Initial Conditions ($N_{DW} > 1$)

- Treat ϕ_1 and ϕ_2 as two independent real scalar fields with correlation function

$$\langle \phi_i(\mathbf{k})\phi_i(\mathbf{k}')\rangle = \frac{1}{2k}(2\pi)^3\delta^{(3)}(\mathbf{k}+\mathbf{k}') \qquad \phi = \phi_1 + i\phi_2$$

$$\langle \dot{\phi}_i(\mathbf{k})\dot{\phi}_i(\mathbf{k}')\rangle = \frac{k}{2}(2\pi)^3\delta^{(3)}(\mathbf{k}+\mathbf{k}') \qquad (i = 1, 2)$$

- No correlation in the k space
    - Generate $\phi_i(\mathbf{k})$ as Gaussian with

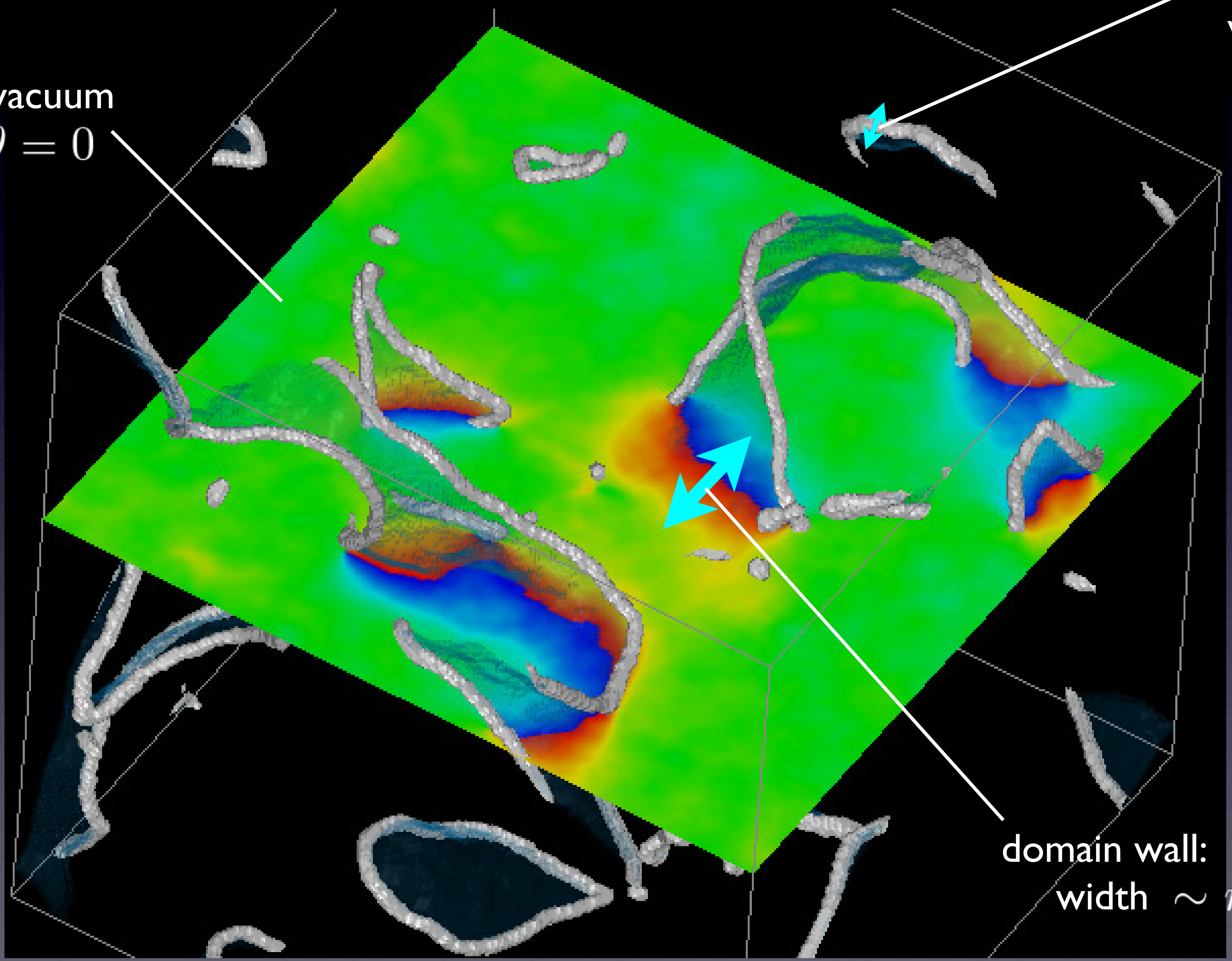
$$\langle |\dot{\phi}(\mathbf{k})|^2\rangle = \frac{k}{2}V_b \qquad \langle |\phi(\mathbf{k})|^2\rangle = \frac{1}{2k}V_b$$

$$\langle \phi(\mathbf{k})\rangle = \langle \dot{\phi}(\mathbf{k})\rangle = 0$$

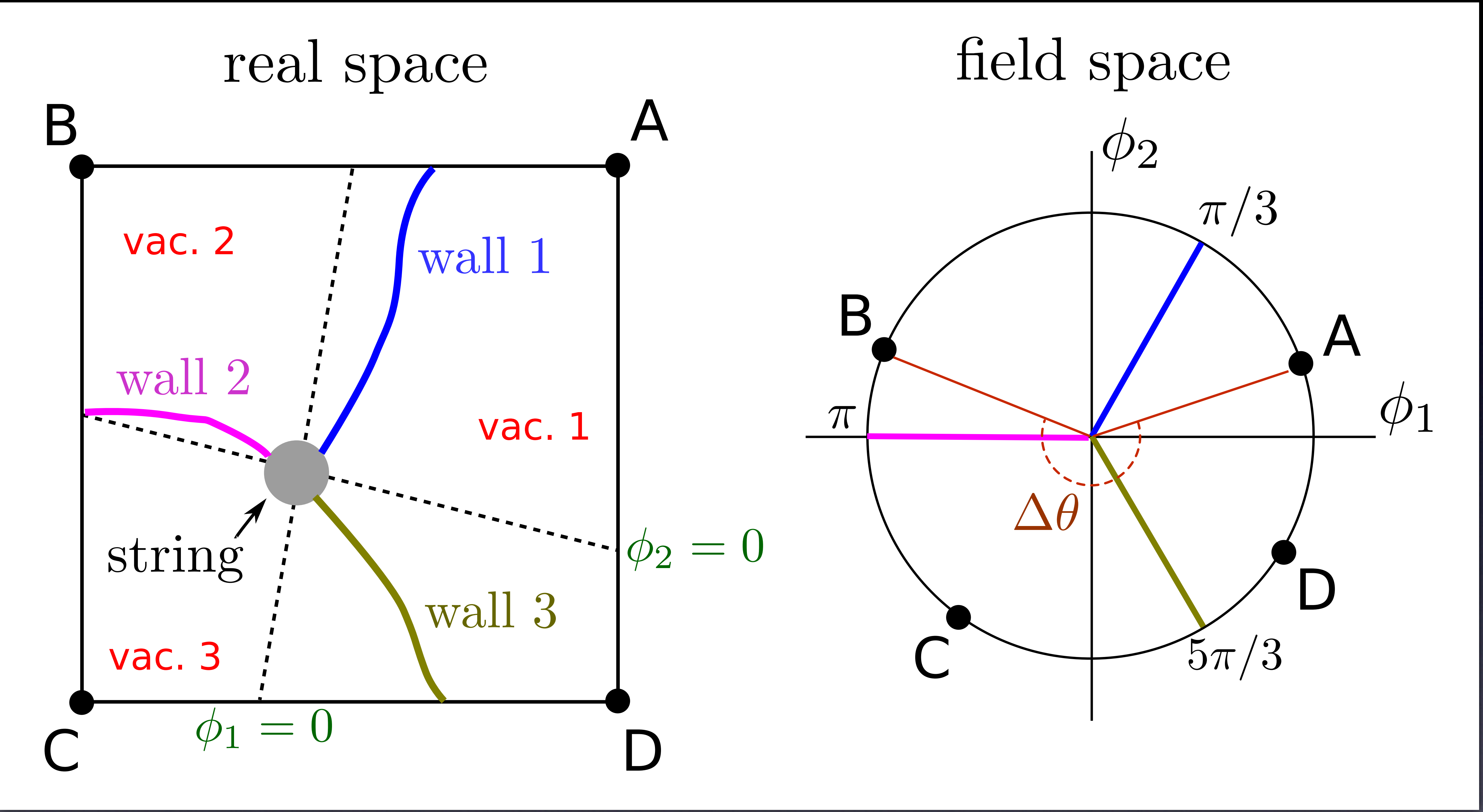
$V_b \simeq (2\pi)^3\delta^{(3)}(0)$: volume of the simulation box

- Fourier transform to obtain $\phi_i(\mathbf{x})$ and $\dot{\phi}_i(\mathbf{x})$

Map of the phase of PQ field



Identification of defects



String exists if $\Delta\theta > \pi$

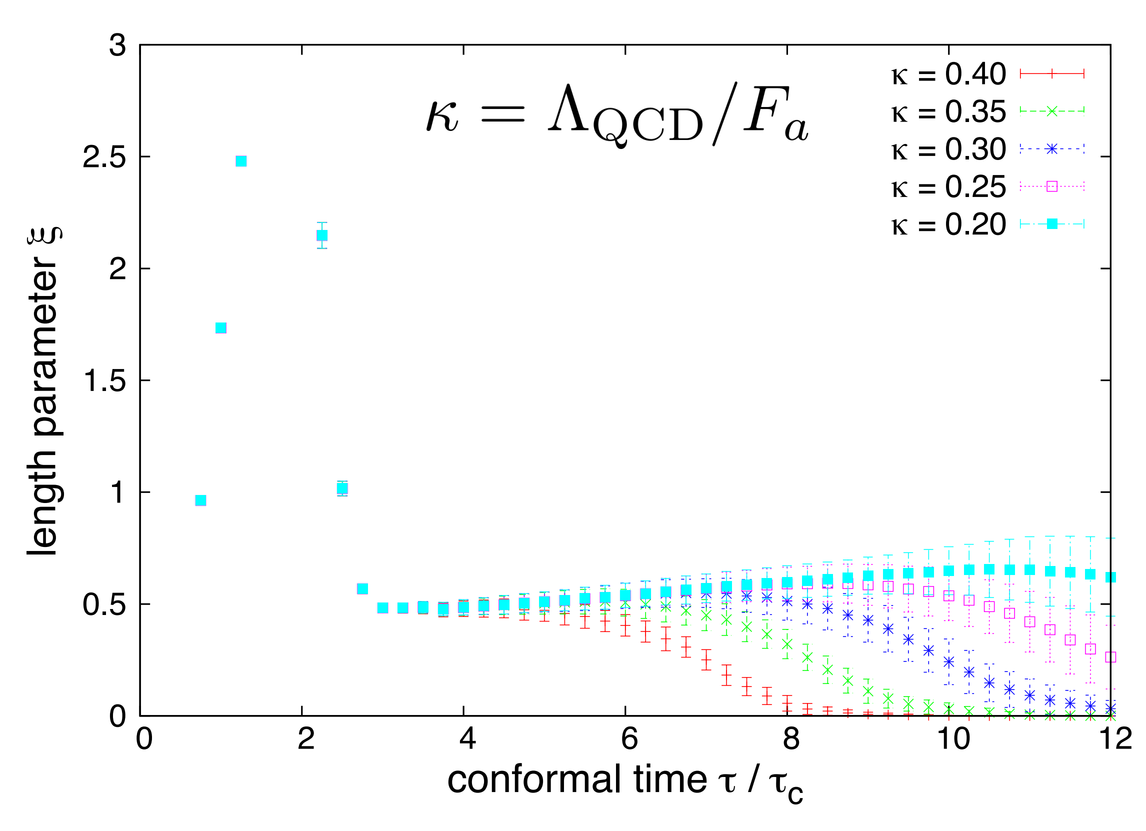
Evolution of string-wall systems

- After the production,
  stings obey scaling solution

$$\rho_{\rm string} = \xi \frac{\mu}{t^2}$$

"O(1) strings in a horizon volume"

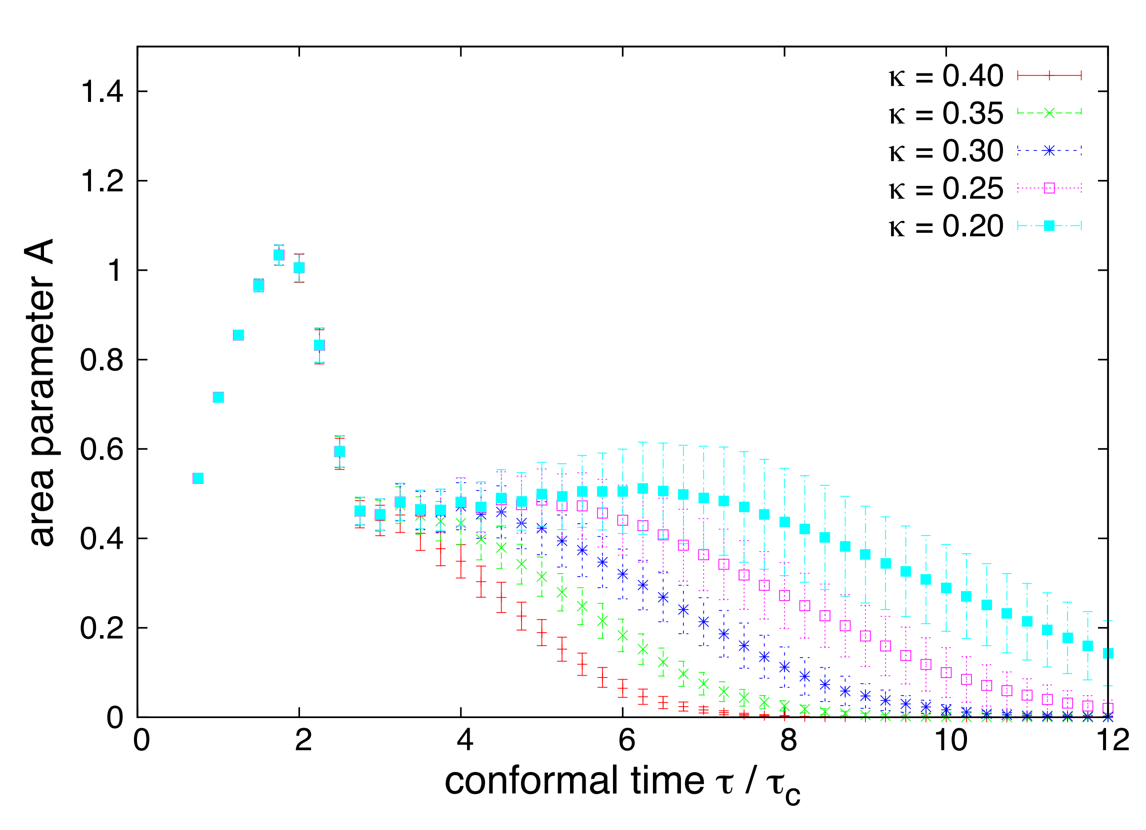
$$\mu = \pi\eta^2 \ln\left(\sqrt{\lambda}\eta t/\sqrt{\xi}\right)$$: energy per length



- Walls also obey scaling solution

$$\rho_{\rm wall} = \mathcal{A}\frac{\sigma}{t}$$

$$\sigma \sim m_a F_a^2$$: wall tension



- Scaling parameters

$$\xi, \mathcal{A} \sim \mathcal{O}(1)$$

contain relatively large uncertainties

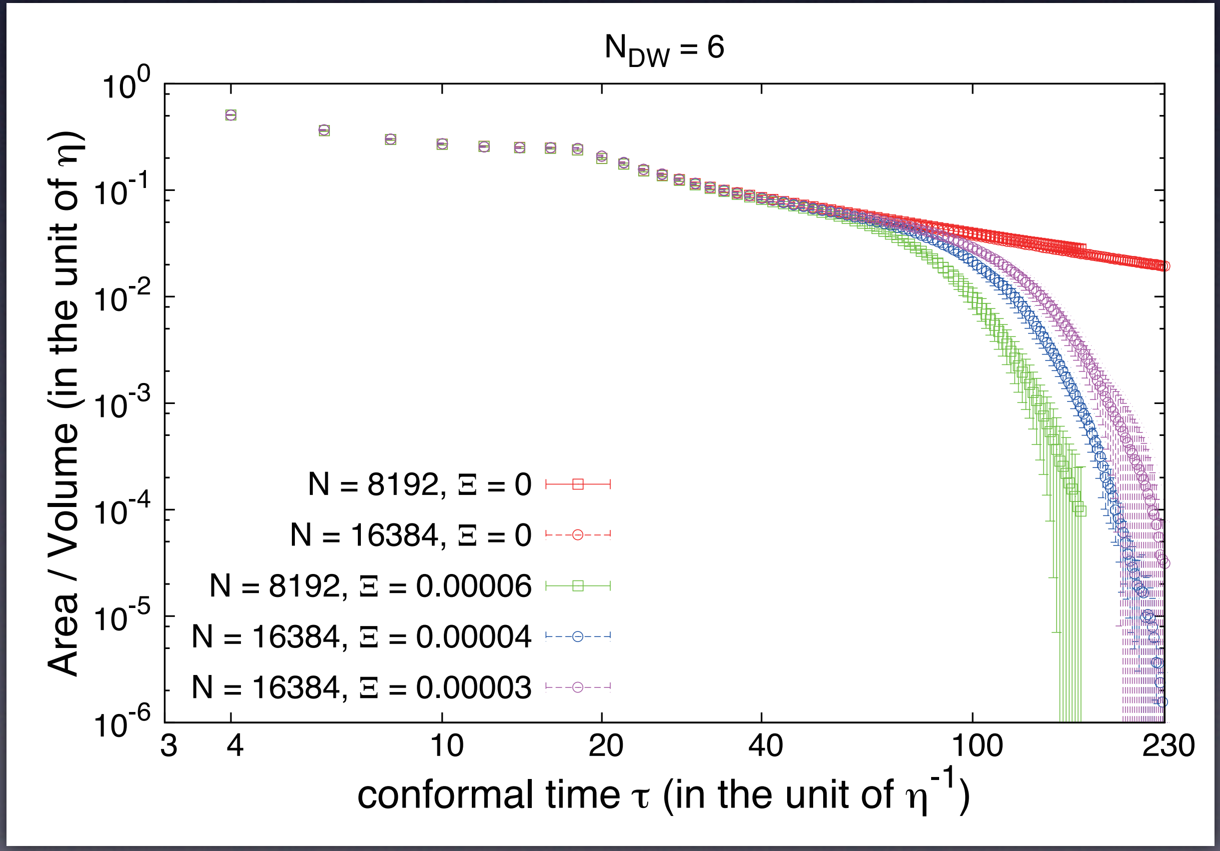
Evolution of long-lived domain walls

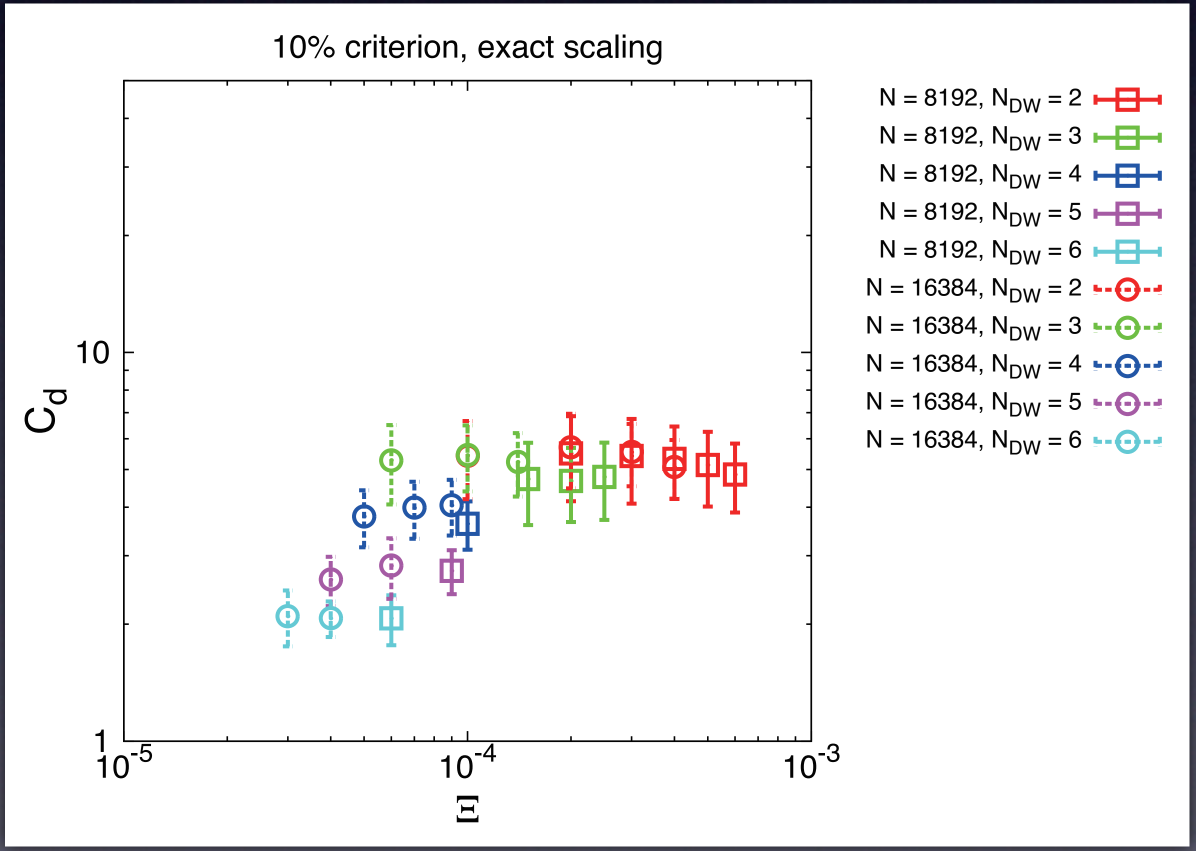
- Walls obey scaling solution if $\Xi = 0$: $\rho_{\rm wall} = \mathcal{A}\frac{\sigma}{t}$
- Decay time (estimated from the condition $\underset{\text{pressure}}{\Xi\eta^4} \gtrsim \underset{\text{tension}}{\mathcal{A}\sigma/t}$)

$$t_{\rm dec} = C_d \frac{\mathcal{A}\sigma}{\Xi\eta^4[1-\cos(2\pi N/N_{\rm DW})]}$$

- C_d is determined from numerical simulation

$$\left.\frac{A/V(\Xi)}{A/V(\Xi=0)}\right|_{t_{\rm dec}} = 0.1$$





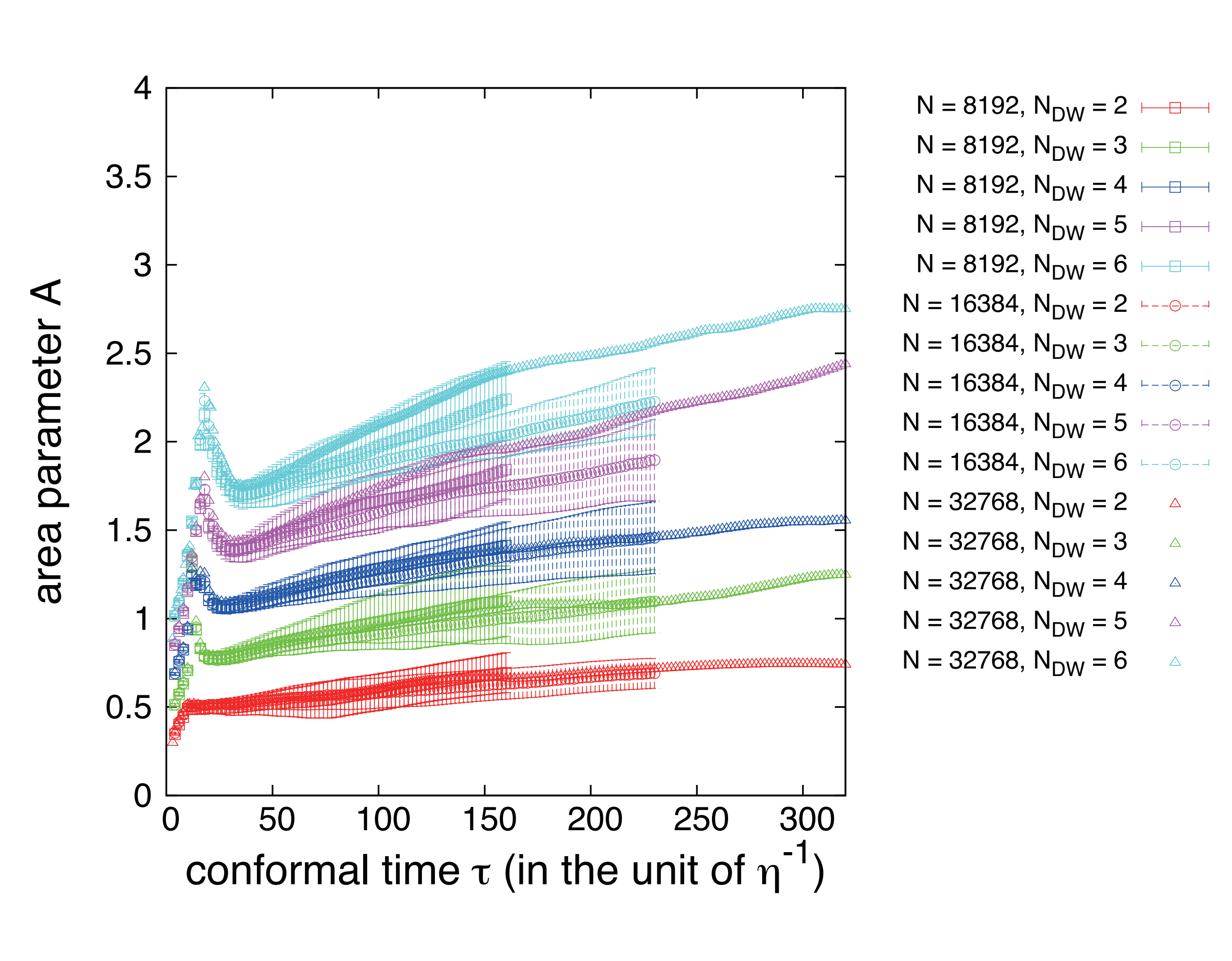
→ $C_d \simeq 2\text{–}5$

Area parameter

- Area parameters increase for large N_{DW}

$$\rho_{\rm wall} = \mathcal{A}\frac{\sigma_{\rm wall}}{t}$$

$N_{\rm DW}$	$\mathcal{A}(\tau_f)$ $(N = 8192,\ \tau_f = 160)$
2	0.694 ± 0.113
3	1.10 ± 0.20
4	1.41 ± 0.13
5	1.84 ± 0.17
6	2.24 ± 0.21



- Slightly increase with time ?

$$\mathcal{A}(\tau) = \mathcal{A}_{\rm form}\left(\frac{\tau}{\tau_{\rm form}}\right)^{2(1-p)}$$

$$p = 0.92\text{–}0.93$$

- It is not clear whether this slight increase continues in later times, so we consider both two cases, "exact scaling" (p=1) and "deviation from scaling" (p<1)

String-wall contribution to CDM abundance

- On the mean energy $\langle \omega_a \rangle$ of axions radiated from string-wall systems

Case A

$$\langle \omega_a \rangle \sim m_a$$

Nagasawa and Kawasaki (1994)

- Radiated axion is mildly relativistic
- Contribution for DM abundance can be large

Case B

$$\langle \omega_a \rangle \sim m_a \log(F_a/m_a)$$

Chang, Hagmann and Sikivie (1999)

- Spectrum is hard

$$dE/dk \sim 1/k$$

- Contribution for DM abundance is subdominant

$$\rho_a(t_{\mathrm{today}}) = m_a n_a(t_{\mathrm{today}}) = m_a \frac{\rho_a(t_{\mathrm{decay}})}{\langle \omega_a \rangle} \left(\frac{R(t_{\mathrm{decay}})}{R(t_{\mathrm{today}})} \right)^3$$

$R(t)$: scale factor of the universe

- This controversy can be resolved by field theoretic lattice simulation of defect networks

Radiation of axions

- Compute power spectrum by using data of scalar field $\Phi(t, \mathbf{x})$ obtained by simulations

$$\frac{1}{2}\langle \dot{a}(t,\mathbf{k})^* \dot{a}(t,\mathbf{k}')\rangle = \frac{(2\pi)^3}{k^2}\delta^{(3)}(\mathbf{k}-\mathbf{k}')P(k,t)$$

$$\dot{a}(t,\mathbf{k}) = \int d^3\mathbf{x} e^{i\mathbf{k}\cdot\mathbf{x}}\dot{a}(t,\mathbf{x}) \qquad \dot{a}(t,\mathbf{x}) = \mathrm{Im}\left[\frac{\dot{\Phi}}{\Phi}(t,\mathbf{x})\right]$$

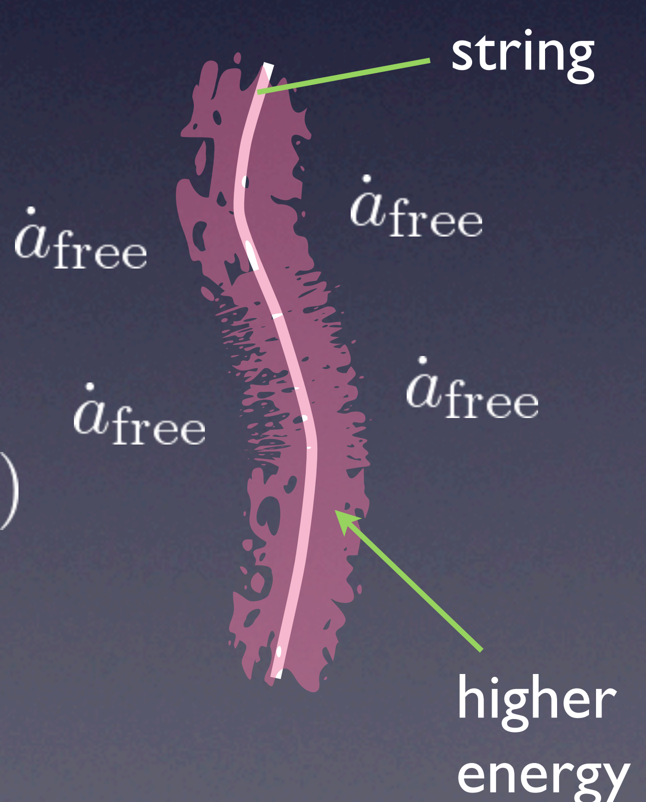
- We overestimate the energy of axions if we include data on the defects

$$\dot{a}(t,\mathbf{x})$$

$$= \dot{a}_{\mathrm{free}}(t,\mathbf{x}) + (\text{contamination from defects})$$

radiated axions

higher energy



Masking analysis

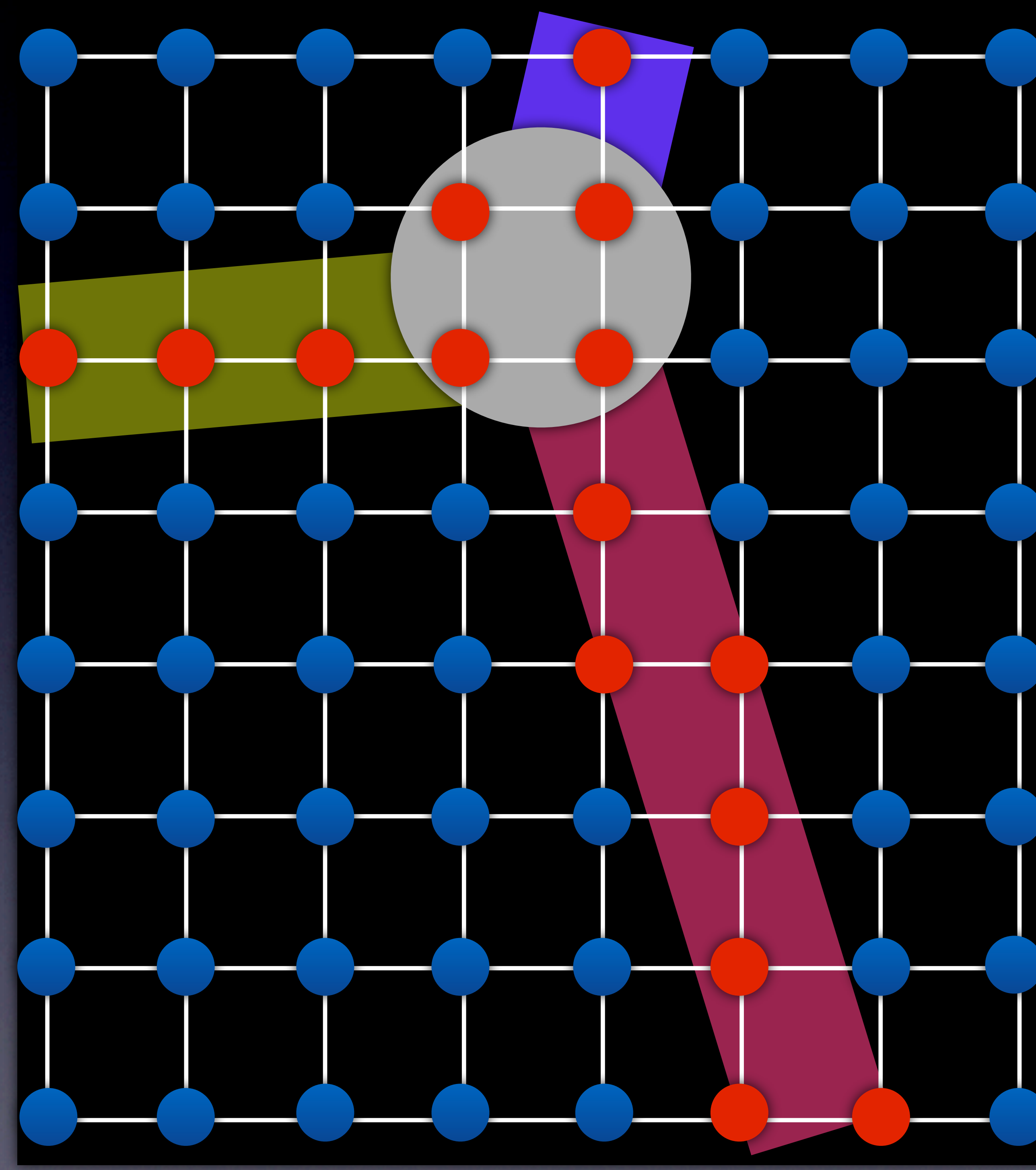
Hiramatsu, Kawasaki, Sekiguchi, Yamaguchi and Yokoyama (2011)

$a(x)$: contains contamination from defects 🔴

$a_{\rm free}(x)$: use masked data 🔵 only

compute

$$\frac{1}{2}\langle \dot{a}_{\rm free}(\mathbf{k})^* \dot{a}_{\rm free}(\mathbf{k}')\rangle = \frac{(2\pi)^3}{k^2}\delta^{(3)}(\mathbf{k}-\mathbf{k}')P(k)$$



Computation of the power spectrum (1)

- The moving defects can contamimate the spectrum

$$\dot{a}(t,\mathbf{x}) = \dot{a}_{\text{free}}(t,\mathbf{x}) + (\text{contamination from strings})$$

- Introduce window function

$$W(\mathbf{x}) = \begin{cases} 0 & (\text{near strings}) \\ 1 & (\text{elsewhere}) \end{cases}$$

- Masked axion field

$$\tilde{\dot{a}}(\mathbf{x}) \equiv W(\mathbf{x})\dot{a}(\mathbf{x}) = W(\mathbf{x})\dot{a}_{\text{free}}(\mathbf{x})$$

- We can compute the masked power spectrum

$$\tilde{P}(k) \equiv \frac{k^2}{V}\int \frac{d\Omega_k}{4\pi}\frac{1}{2}|\tilde{\dot{a}}(\mathbf{k})|^2$$

This is different from the true power spectrum $\langle \tilde{P}(k) \rangle \neq P_{\text{free}}(k)$

Computation of the power spectrum (2)

- The true power spectrum is given by

$$\frac{1}{2}\langle \dot{a}_{\rm free}(t,\mathbf{k})^* \dot{a}_{\rm free}(t,\mathbf{k}')\rangle = \frac{(2\pi)^3}{k^2}\delta^{(3)}(\mathbf{k}-\mathbf{k}')P_{\rm free}(k,t)$$

- Define PPSE of $P_{\rm free}(k)$

$$P_{\rm PPSE}(k) \equiv \frac{k^2}{V}\int \frac{dk'}{2\pi^2} M^{-1}(k,k')\tilde{P}(k')$$

with a window weight matrix

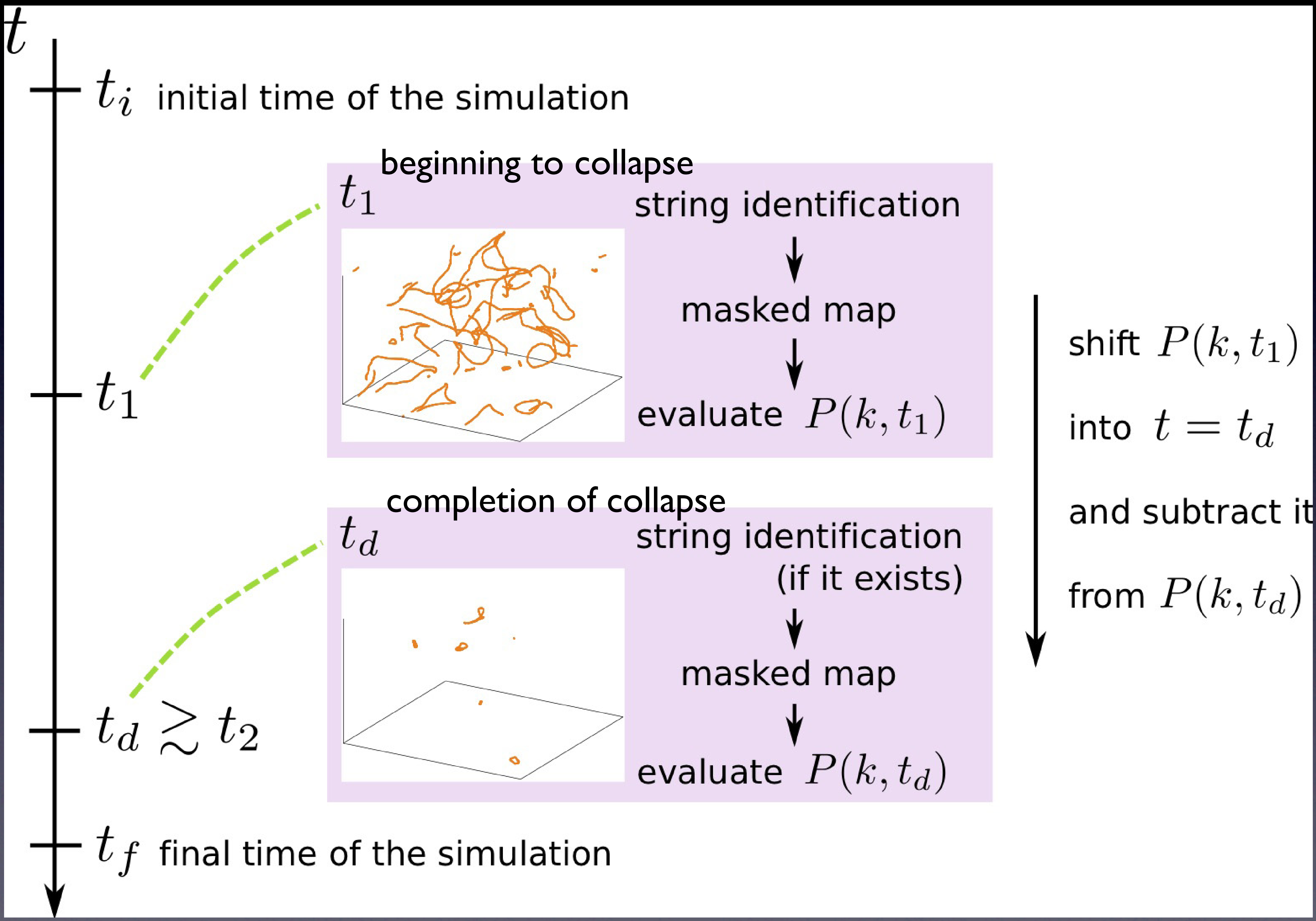
$$M(k,k') \equiv \frac{1}{V^2}\int \frac{d\Omega_k}{4\pi}\frac{d\Omega_{k'}}{4\pi}|W(\mathbf{k}-\mathbf{k}')|^2$$

$$\int \frac{k'^2 dk'}{2\pi^2} M^{-1}(k,k')M(k',k'') = \frac{2\pi^2}{k^2}\delta(k-k'')$$

- It can be shown that $\langle P_{\rm PPSE}(k)\rangle = P_{\rm free}(k)$

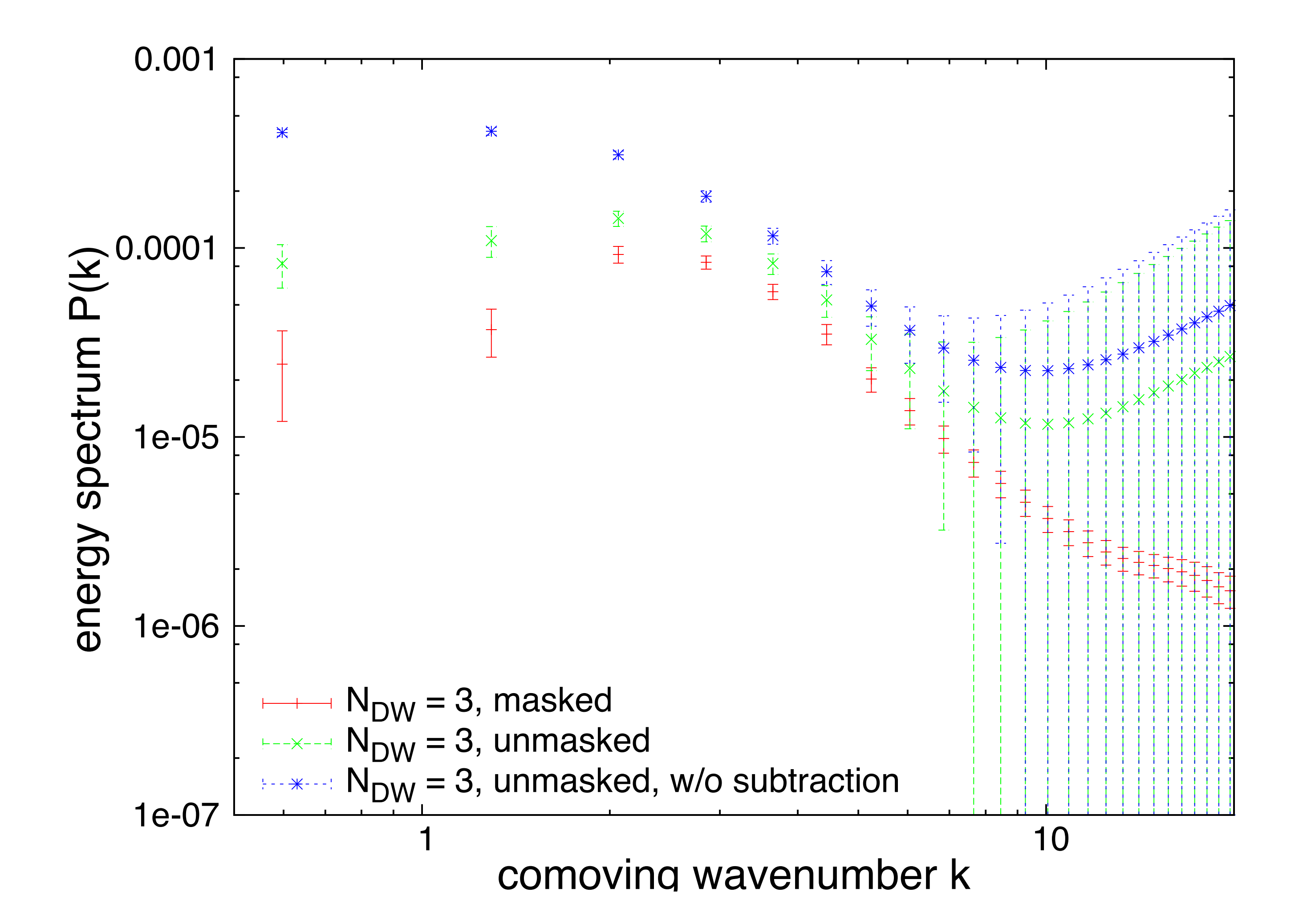
Hiramatsu, Kawasaki, Sekiguchi, Yamaguchi & Yokoyama (2011)

Procedure to estimate the power spectrum



Subtraction of pre-existing radiations

- Compute spectrum at two different times t_1 and t_2
- Subtract contributions radiated before t_1 t_1: formation time of walls

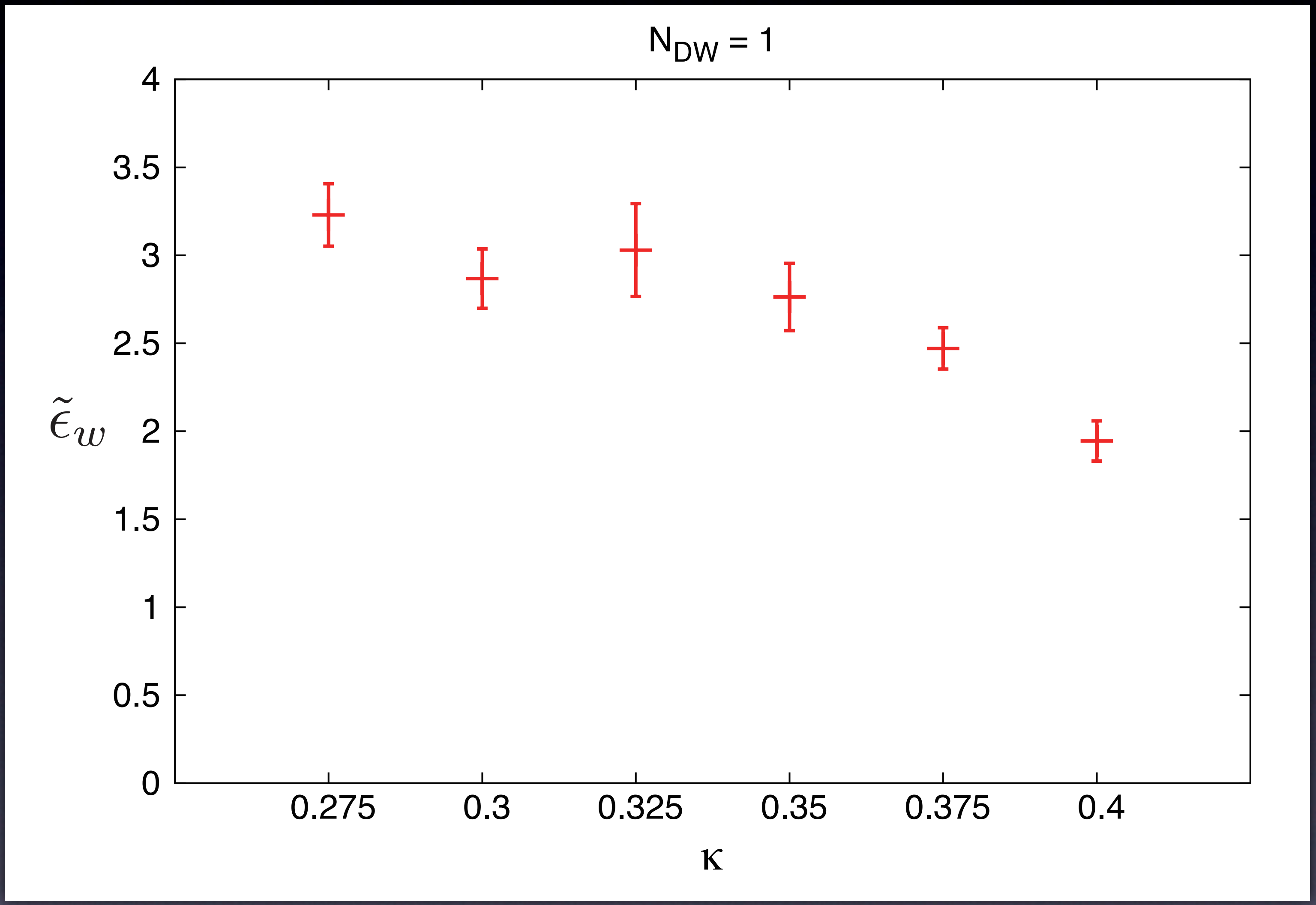


$$\Delta P(k,t_2) = P(k,t_2) - P(k,t_1)\frac{\omega_a(k,t_2)}{\omega_a(k,t_1)}\left(\frac{R(t_1)}{R(t_2)}\right)^3$$

$$\omega_a(k,t) = \sqrt{m_a^2 + k^2/R^2(t)}$$

Averaged axion energy

- Dependence on $\kappa = \Lambda_{\rm QCD}/F_a$



$$\tilde{\epsilon}_w = \frac{\omega_a}{m_a}(t_{\rm decay}) = 3.23 \pm 0.18$$