Studying generalised dark matter interactions with extended halo-independent methods.

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Based on
arXiv:1607.04418 with Sebastian Wild

HELMHOLTZ ASSOCIATION

DESY

Motivation

- Differential event rates in direct detection experiments are given by

$$\frac{\mathrm{d}R}{\mathrm{d}E_\mathrm{R}} = \frac{\rho}{m_T\, m_\chi} \int_{v_\mathrm{min}}^{\infty} v f(\boldsymbol{v} + \boldsymbol{v}_\mathrm{E}(t)) \frac{\mathrm{d}\sigma}{\mathrm{d}E_\mathrm{R}} \mathrm{d}^3 v \quad \text{with} \quad v_\mathrm{min}(E_\mathrm{R}) = \sqrt{\frac{m_T E_\mathrm{R}}{2\, \mu_{T\chi}^2}}$$

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Local DM density and velocity distribution: These are uncertainties that enter experimental predictions in a complicated way

- How reliably can we get to the particle physics properties of DM from future direct detection signals in the face of astrophysical uncertainties?

There are many models of DM

- There are many plausible models for the interactions of DM beyond standard spin-independent (SI) and spin-dependent (SD) scattering:
  - DM with an Anapole moment (AM):

    $$\mathcal{A}\,\bar{\chi}\gamma^{\mu}\gamma^{5}\chi\,\partial^{\nu}F_{\mu\nu}$$

    This is the only possible interaction of Majorana fermions with photons.
  - DM with a magnetic dipole moment (MDM):

    $$\mathcal{D}_{\mathrm{magn}}\,\bar{\chi}\sigma^{\mu\nu}\chi F_{\mu\nu}$$

    This leads to long-range interactions due to the massless photon exchange.
  - DM with a dark magnetic dipole moment (DMDM):

    $$\mathcal{D}_{\mathrm{magn}}\,\bar{\chi}\sigma^{\mu\nu}\chi F'_{\mu\nu}$$

    Same as the the MDM but for a massive hidden photon ($1/q^2 \rightarrow 1/M^2$).

Astrophysical uncertainties

- While these models yield widely different expressions for the scattering cross sections, it is always possible to factorise the result as

$$\frac{\mathrm{d}\sigma}{\mathrm{d}E_\mathrm{R}} = \frac{\mathrm{d}\sigma_1}{\mathrm{d}E_\mathrm{R}}\frac{1}{v^2} + \frac{\mathrm{d}\sigma_2}{\mathrm{d}E_\mathrm{R}}$$

- Introducing the first and second velocity integral

$$g(v_\mathrm{min}) = \int_{v_\mathrm{min}}^\infty \frac{1}{v} f(\boldsymbol{v} + \boldsymbol{v}_\mathrm{E}(t))\,\mathrm{d}^3v\,, \quad h(v_\mathrm{min}) = \int_{v_\mathrm{min}}^\infty v\, f(\boldsymbol{v} + \boldsymbol{v}_\mathrm{E}(t))\,\mathrm{d}^3v$$

we can then write the differential event rates simply as

$$\frac{\mathrm{d}R}{\mathrm{d}E_\mathrm{R}} = \frac{\rho}{m_T\, m_\chi}\left(\frac{\mathrm{d}\sigma_1}{\mathrm{d}E_\mathrm{R}} g(v_\mathrm{min}) + \frac{\mathrm{d}\sigma_2}{\mathrm{d}E_\mathrm{R}} h(v_\mathrm{min})\right)$$

- A useful observation is that the two halo integrals are related by integration by parts:

$$h(v) = -\int_{v_\mathrm{min}}^\infty v^2\, g'(v)\mathrm{d}v \; = \left[-g(v)\, v^2\right]_{v_\mathrm{min}}^\infty + \int_{v_\mathrm{min}}^\infty 2\, v\, g(v)\mathrm{d}v$$

A halo-independent approach

- In order not to make any assumptions on the DM velocity distribution, we can parameterise the velocity integral in a very general way.
- Specifically, we assume that $g(v_{\text{min}})$ is a piecewise-constant function with very many steps ($N > 30$):

$$g(v_{\text{min}}) = g_j \quad \text{for} \;\; v_{\text{min}} \in [v_j, v_{j+1}]$$

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- One can then construct a matrix $D_{ij'}$, such that the expected number of events in bin i (R_i) is given by a simple matrix multiplication:

$$R_i = \sum_j D_{ij} g_j$$

$$D_{ij} = G_{ij} + \sum_k F_{kj} H_{ik}$$

$$G_{ij} = \frac{\kappa\,\rho}{2 m_T\, m_\chi} \int_{E_j}^{E_{j+1}} \frac{\mathrm{d}\sigma_1}{\mathrm{d}E_\mathrm{R}}\,\epsilon(E_\mathrm{R}) \left[\mathrm{erf}\left(\frac{E_{i+1} - E_\mathrm{R}}{\sqrt{2}\Delta E_\mathrm{R}}\right) - \mathrm{erf}\left(\frac{E_i - E_\mathrm{R}}{\sqrt{2}\Delta E_\mathrm{R}}\right)\right] \mathrm{d}E_\mathrm{R}$$

$$H_{ij} = \frac{\kappa\,\rho}{2 m_T\, m_\chi} \int_{E_j}^{E_{j+1}} \frac{\mathrm{d}\sigma_2}{\mathrm{d}E_\mathrm{R}}\,\epsilon(E_\mathrm{R}) \left[\mathrm{erf}\left(\frac{E_{i+1} - E_\mathrm{R}}{\sqrt{2}\Delta E_\mathrm{R}}\right) - \mathrm{erf}\left(\frac{E_i - E_\mathrm{R}}{\sqrt{2}\Delta E_\mathrm{R}}\right)\right] \mathrm{d}E_\mathrm{R}$$

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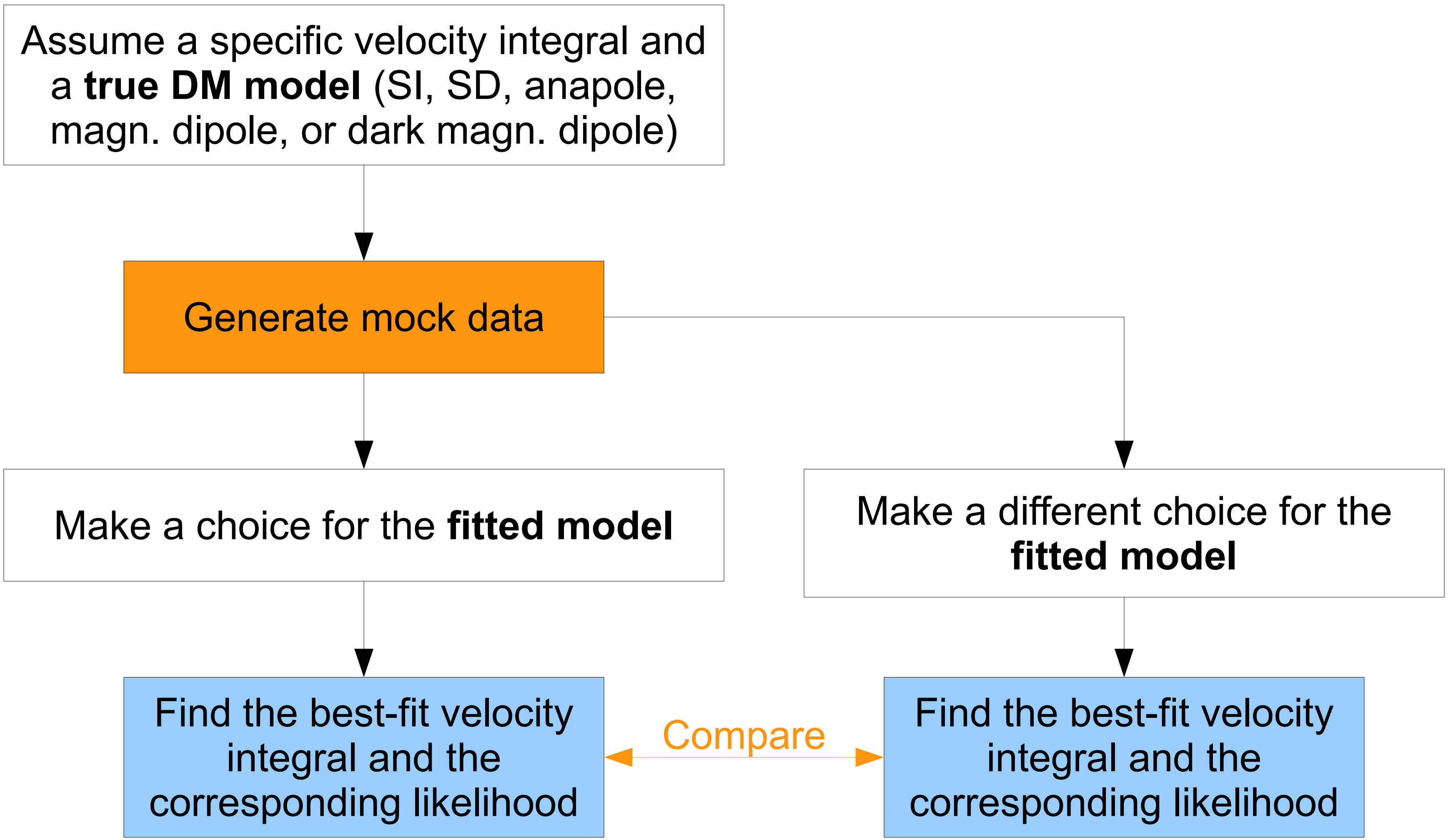
- We can then find the velocity integral that maximises the likelihood:

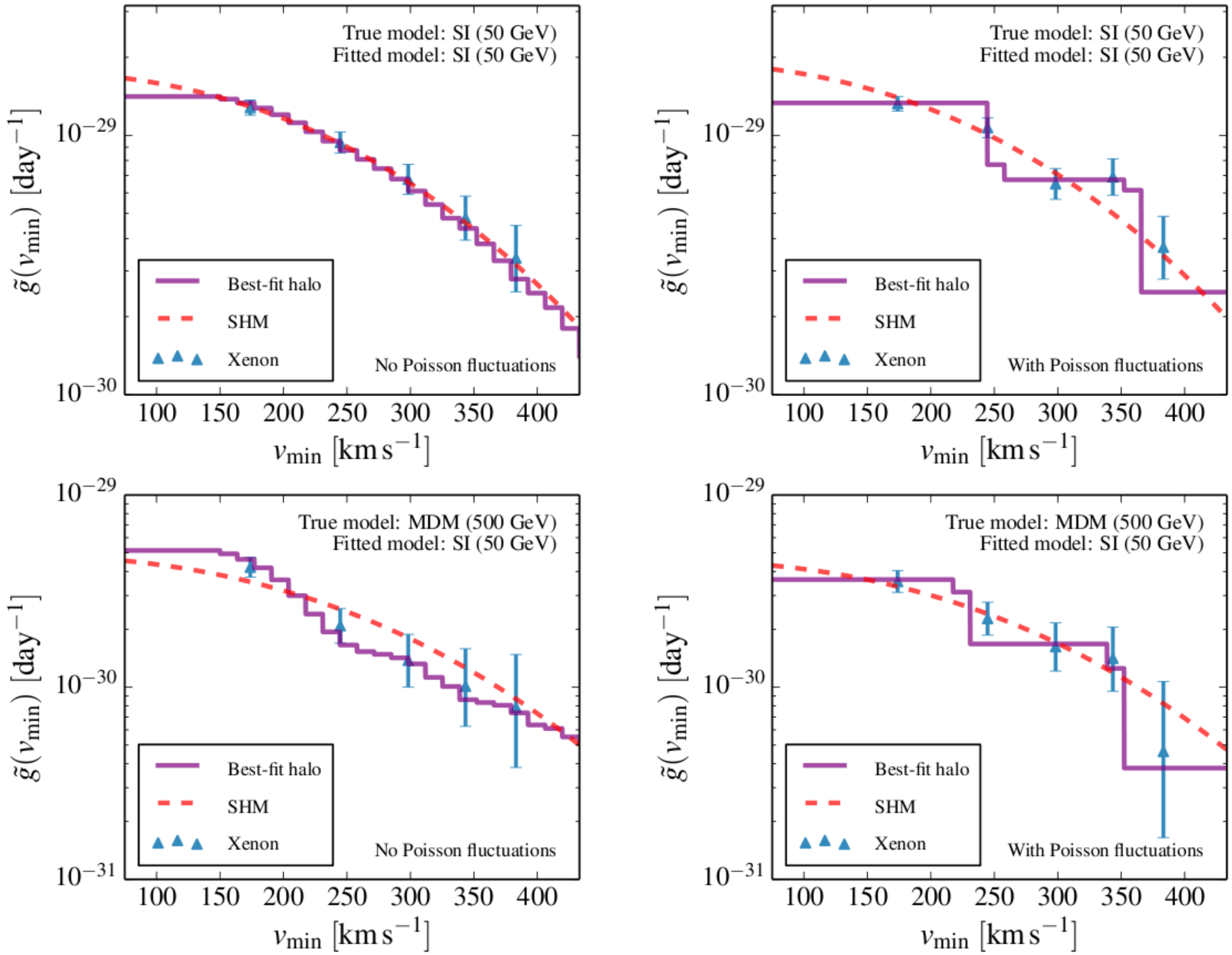
$$-2\log\mathcal{L} = 2\sum\left[R_i + B_i - N_i + N_i \log\frac{N_i}{R_i + B_i}\right]$$

R_i: Predicted signal in bin i
B_i: Predicted background in bin i
N_i: Observed events in bin i

Best-fit velocity integrals



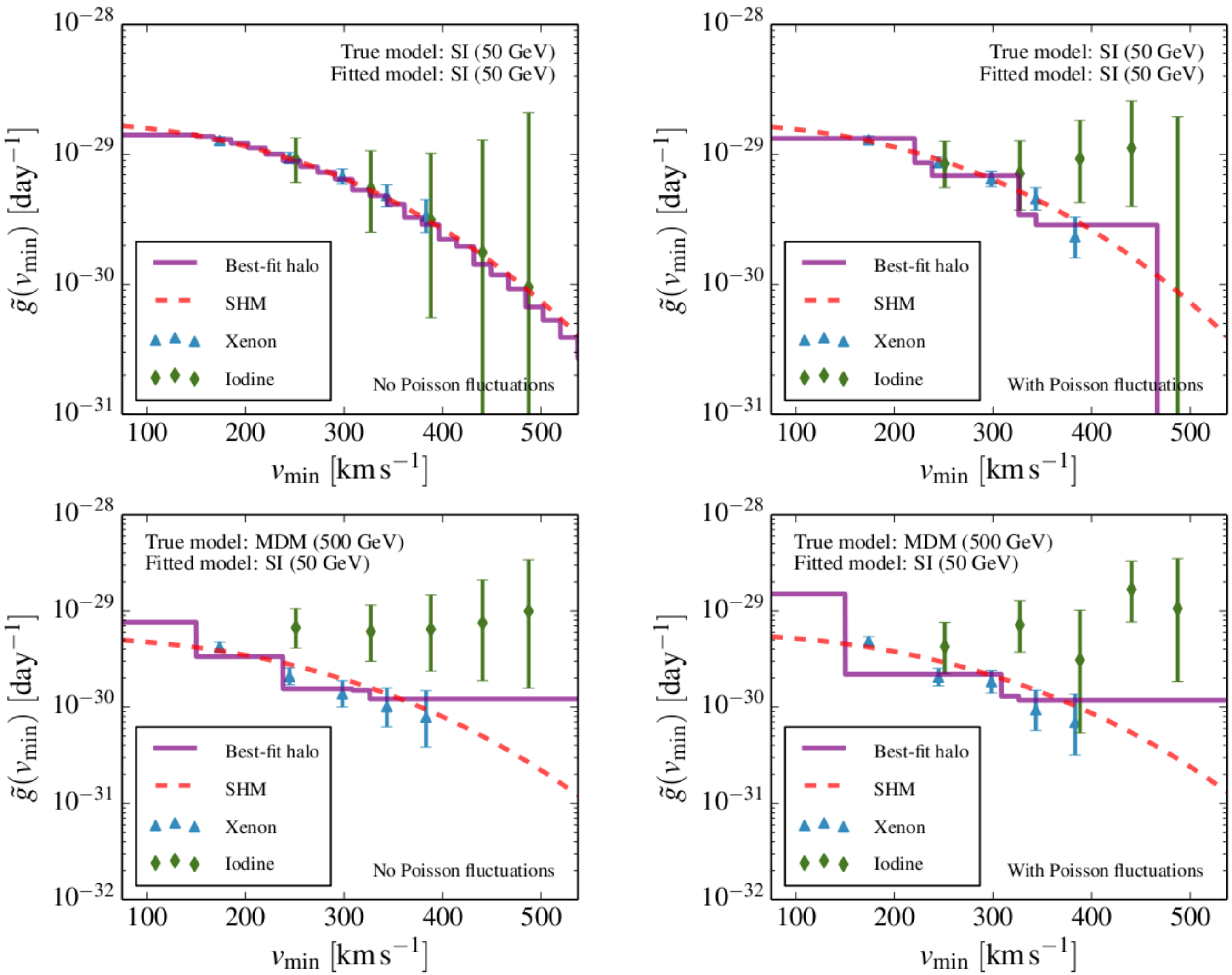
Examples for a single xenon experiment



true model
=
fitted model

true model
≠
fitted model

Examples for xenon+iodine experiments



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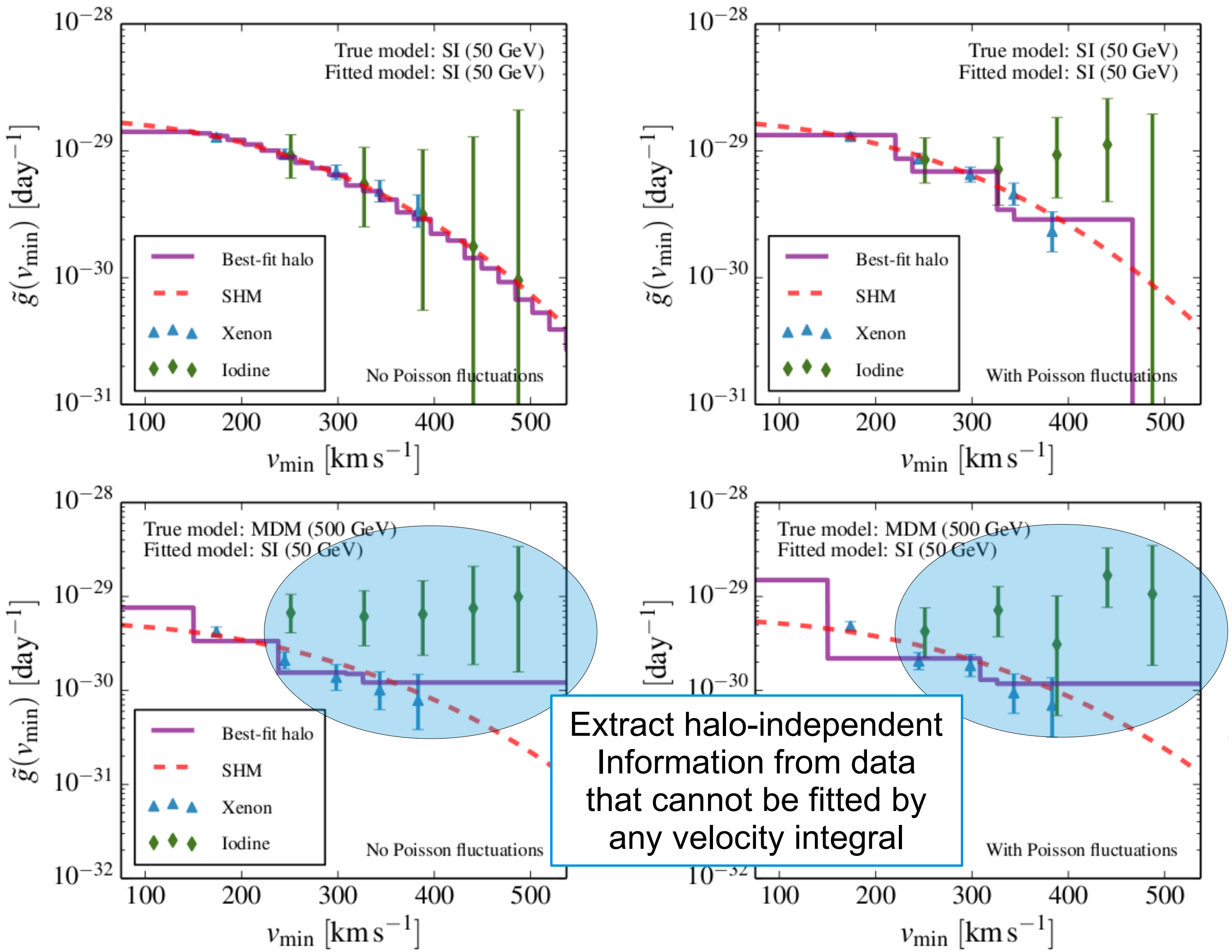
Examples for xenon+iodine experiments



Extract halo-independent Information from data that cannot be fitted by any velocity integral

true model = fitted model

true model ≠ fitted model

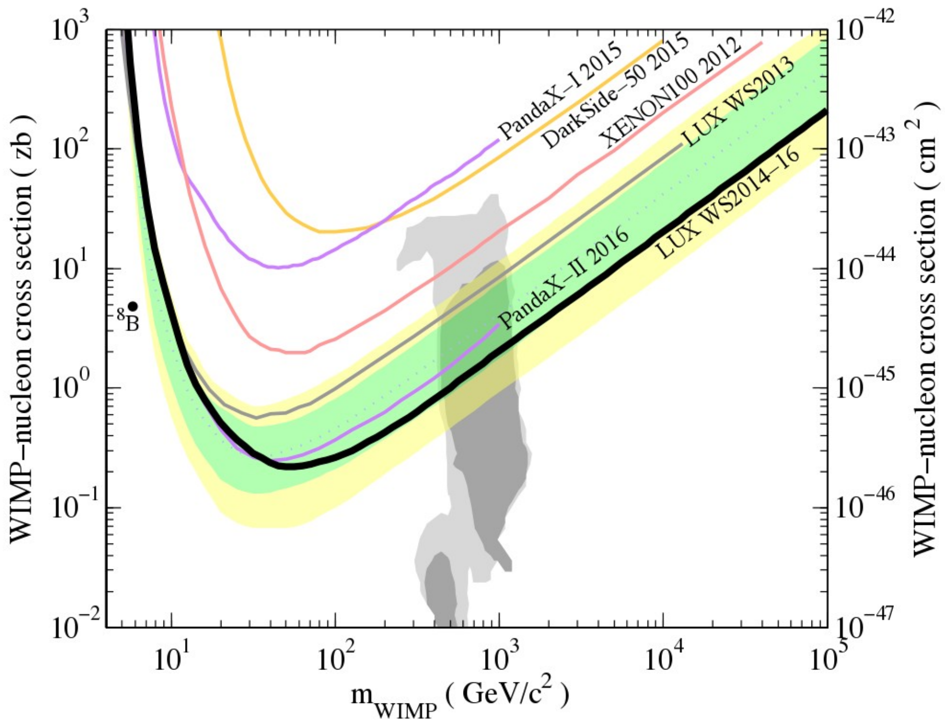
Quantifying *p*-values

- For a given set of data and a chosen **fitted DM model**, we can scan over both the velocity integral and the DM mass to minimise x_0 = -2 log L.
- Goal: Determine whether a given value of x_0 represents a good fit to the data.
- Problem: The distribution of x_0, called $\zeta(x_0)$, does not follow a χ^2-distribution
  - Need Monte Carlo simulation to determine *p*-values!
- Basic idea: Generate new mock data using the best-fit DM model from above and repeat the minimization of x_1 = -2 log L.
- Using the distribution of x_1, called $\zeta(x_1)$, we can then calculate the *p*-value of x_0:

$$p = \int_{x_0}^{\infty} \zeta(x_1) \mathrm{d}x_1$$

- Probability that a value larger than x_0 can result from random fluctuations of the data if the fitted model were to correspond to the true model of nature.

Future discoveries

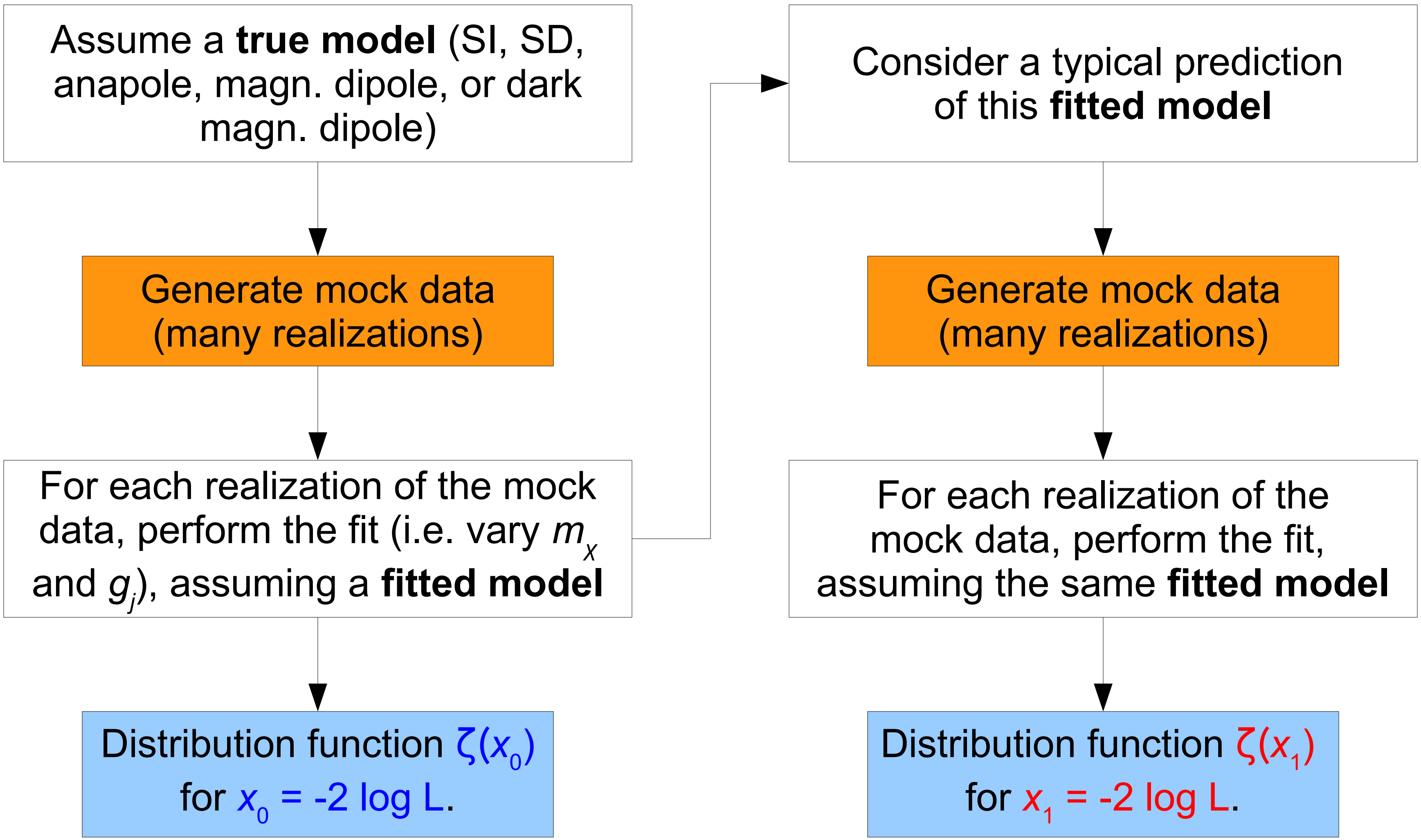


- No clear DM signal from direct detection experiments yet.

- What can we hope to learn from a signal in future experiments?
  - Can we reliably distinguish between different models of DM when accounting for astrophysical uncertainties?
  - How much data / how many experiments do we need to rule out incorrect hypotheses on the nature of DM?

DESY

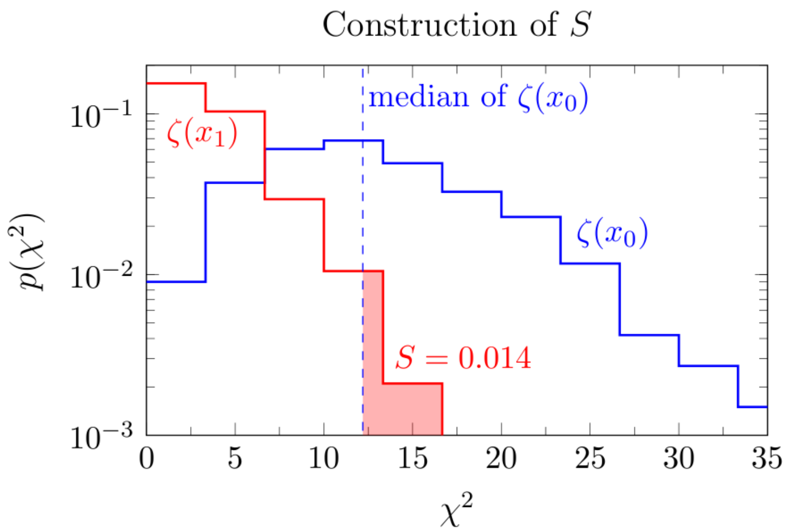
Distinguishing different DM models

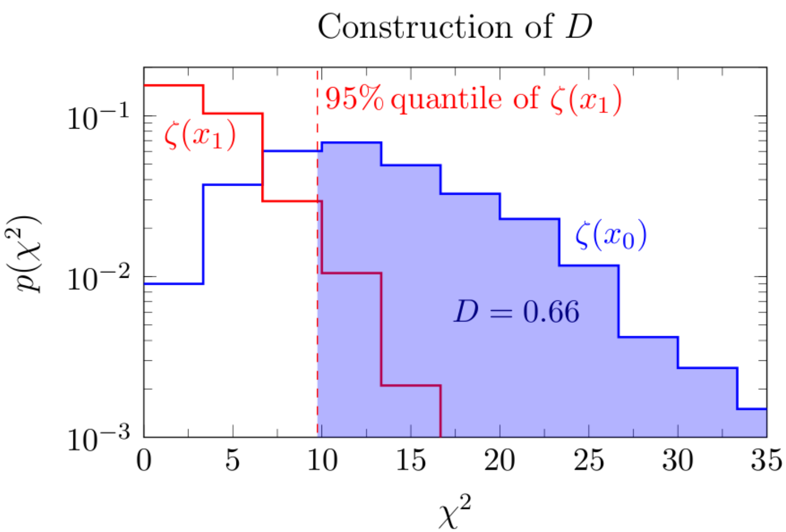


Similarity of DM models

- If the fitted DM model is similar/identical to the true DM model, the distributions $\zeta(x_0)$ and $\zeta(x_1)$ will be very similar, otherwise they will differ.

- To look at all realisations at once, we define

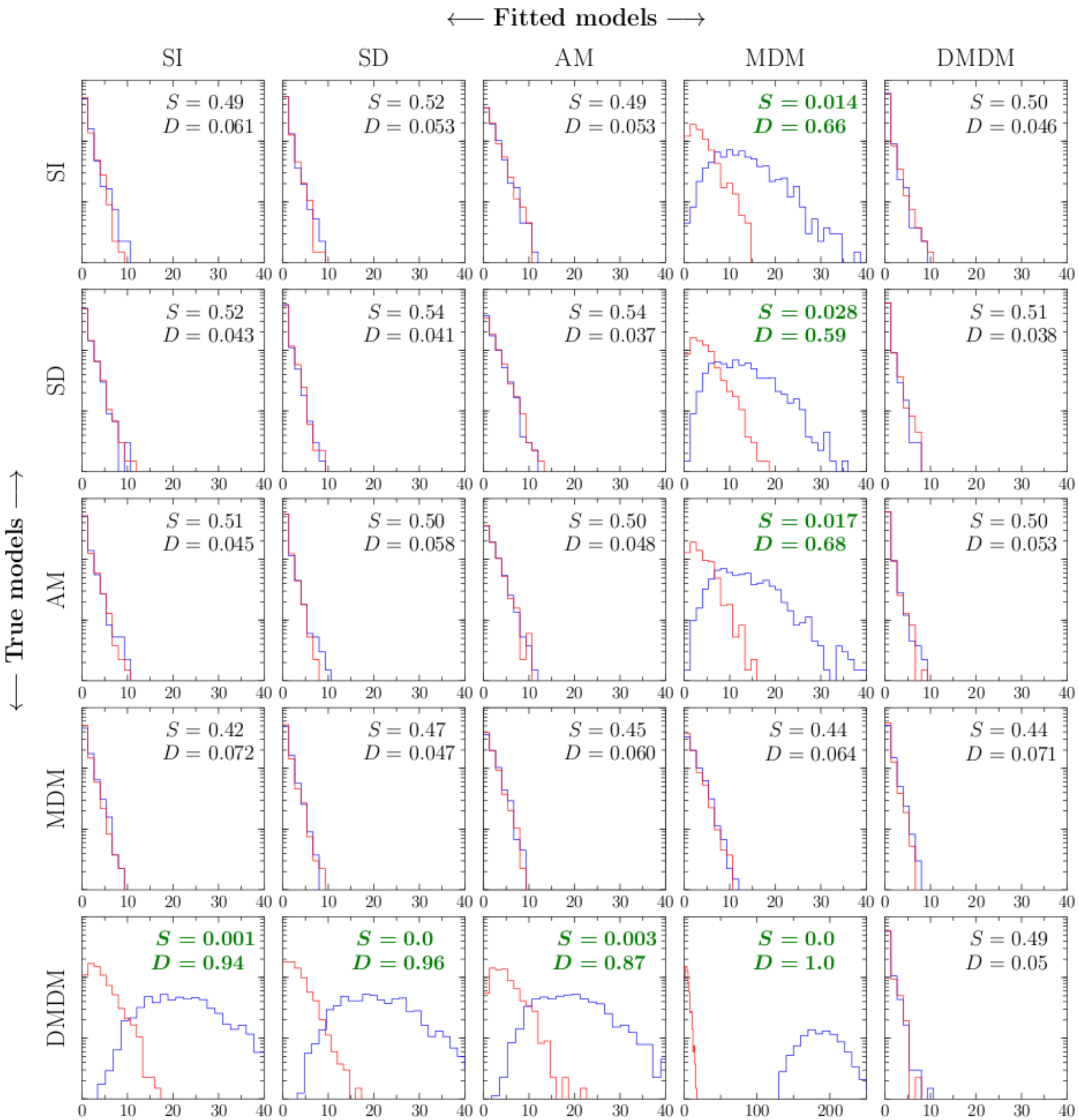




  - The similarity, *S*, as the *p*-value of a typical realisation of the assumed DM model (i.e. the median of ζ(x0))
  - The distinguishability, *D*, as the fraction of realisations of the assumed DM model, having a p-value smaller than 0.05.

- For small *S* and large *D*, the fitted model can likely be ruled out by data.

Results for a single xenon experiment



S: p-value of a typical realisation

D: fraction of realisations that are excluded at 95% CL

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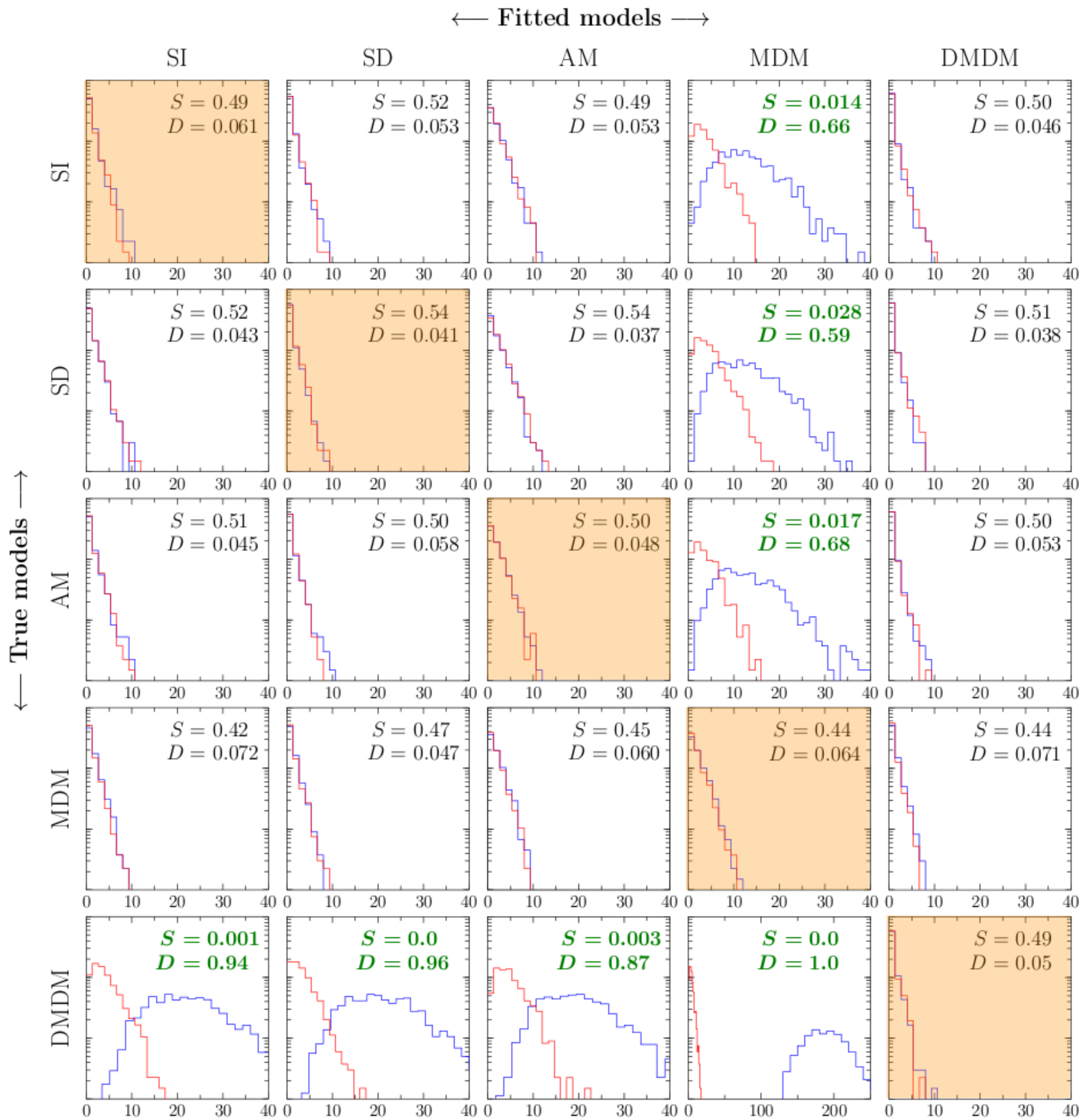

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true model
=
fitted model

S ~ 0.5
D ~ 0.05

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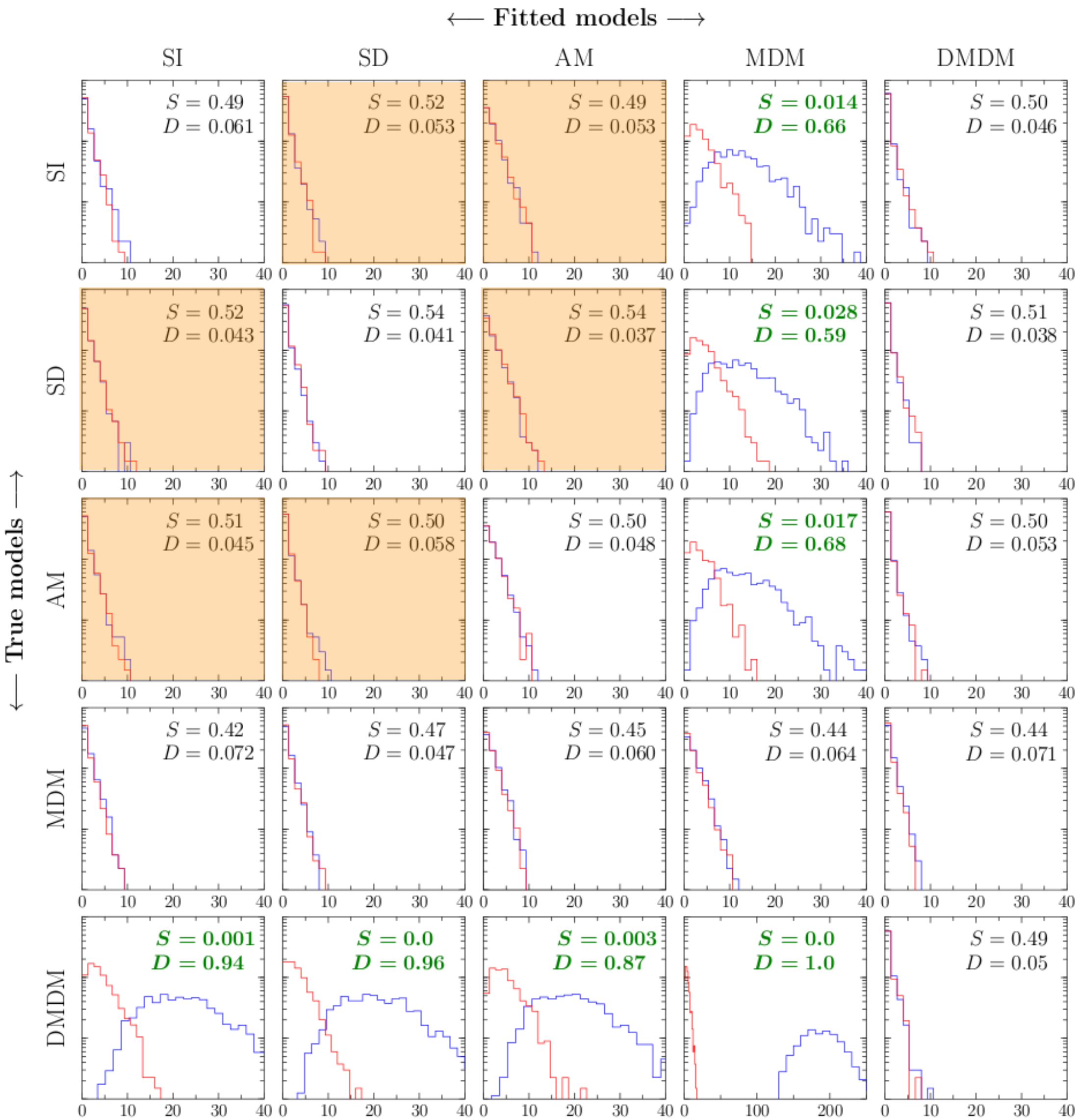

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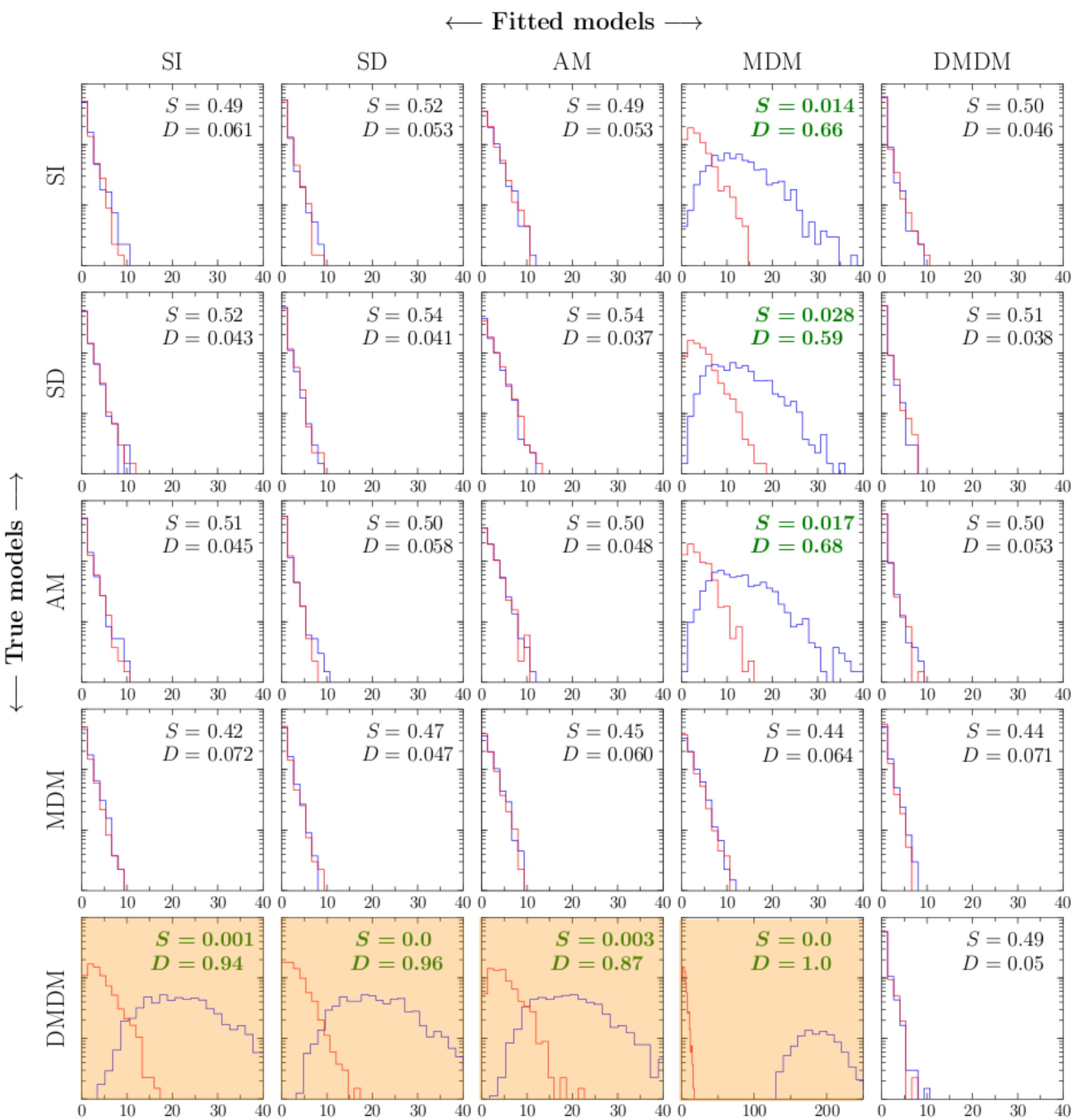
- We find that SI, SD and anapole moment interactions cannot be distinguished with a single xenon experiment.
- For SI and SD this is unsurprising (only the form factors differ).
- For anapole interactions the reason is that (for xenon nuclei) the interactions differ only in their dependence on the velocity integral.

Results for a single xenon experiment



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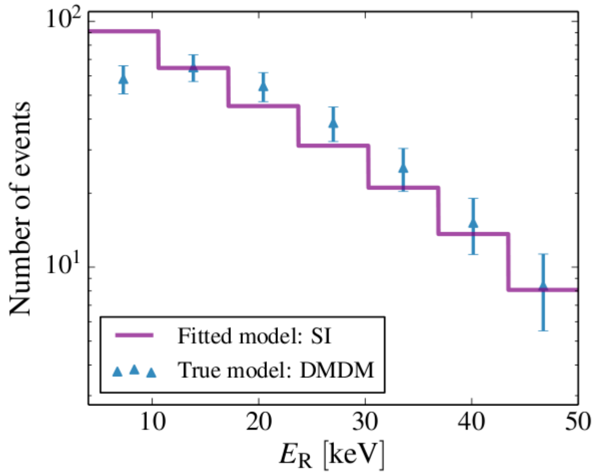
- If DM has a dark magnetic dipole moment, the recoil spectrum has a maximum.



- This cannot be fitted by any other interaction.

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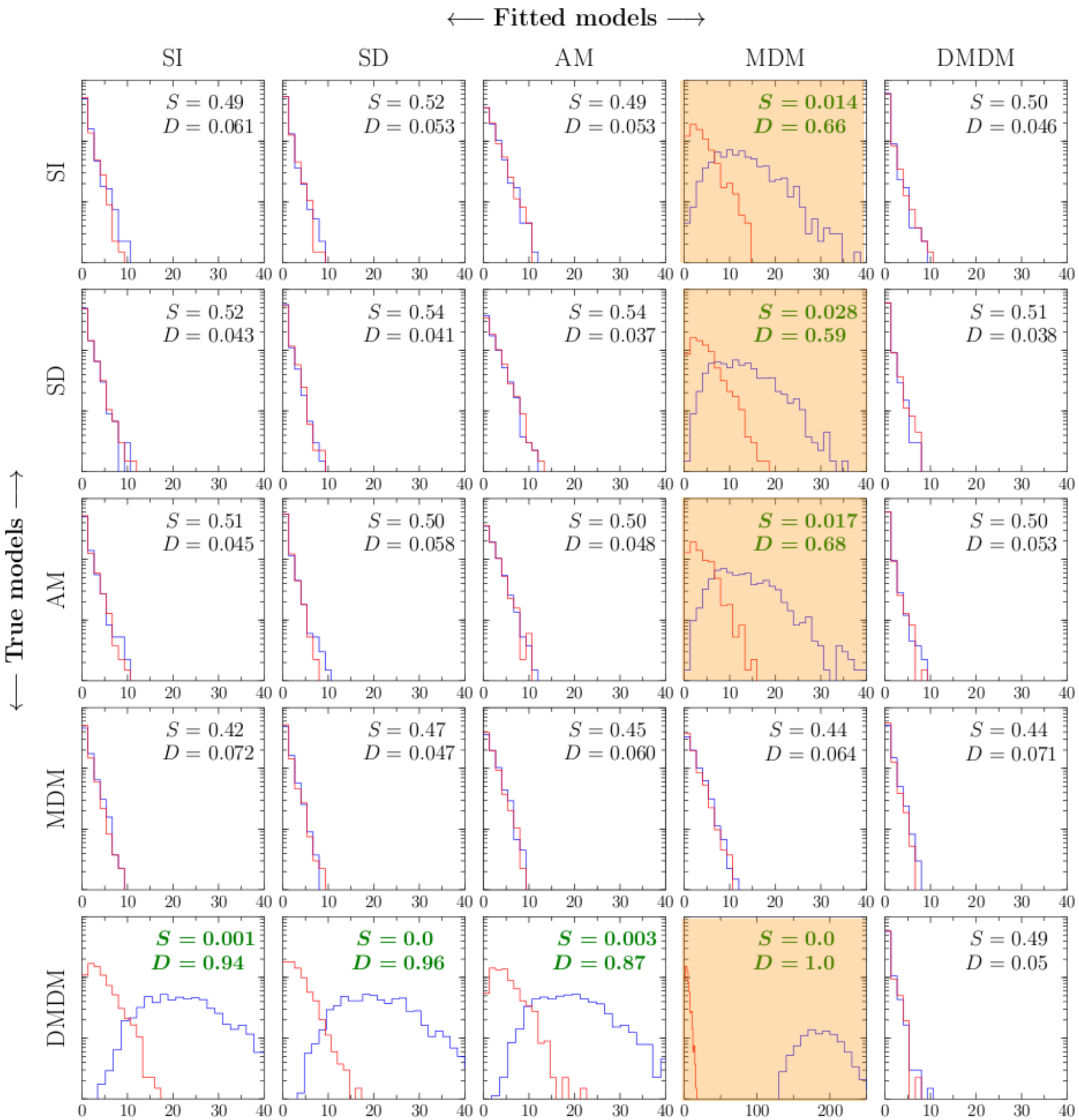

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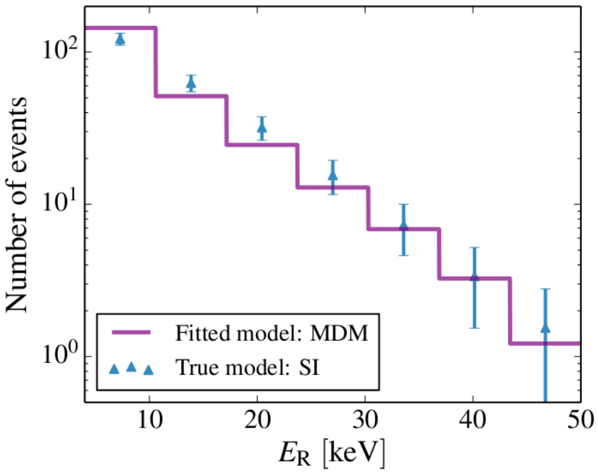
- If DM has a magnetic dipole moment, the recoil spectrum falls very steeply.



- This gives a bad fit to any other interaction.

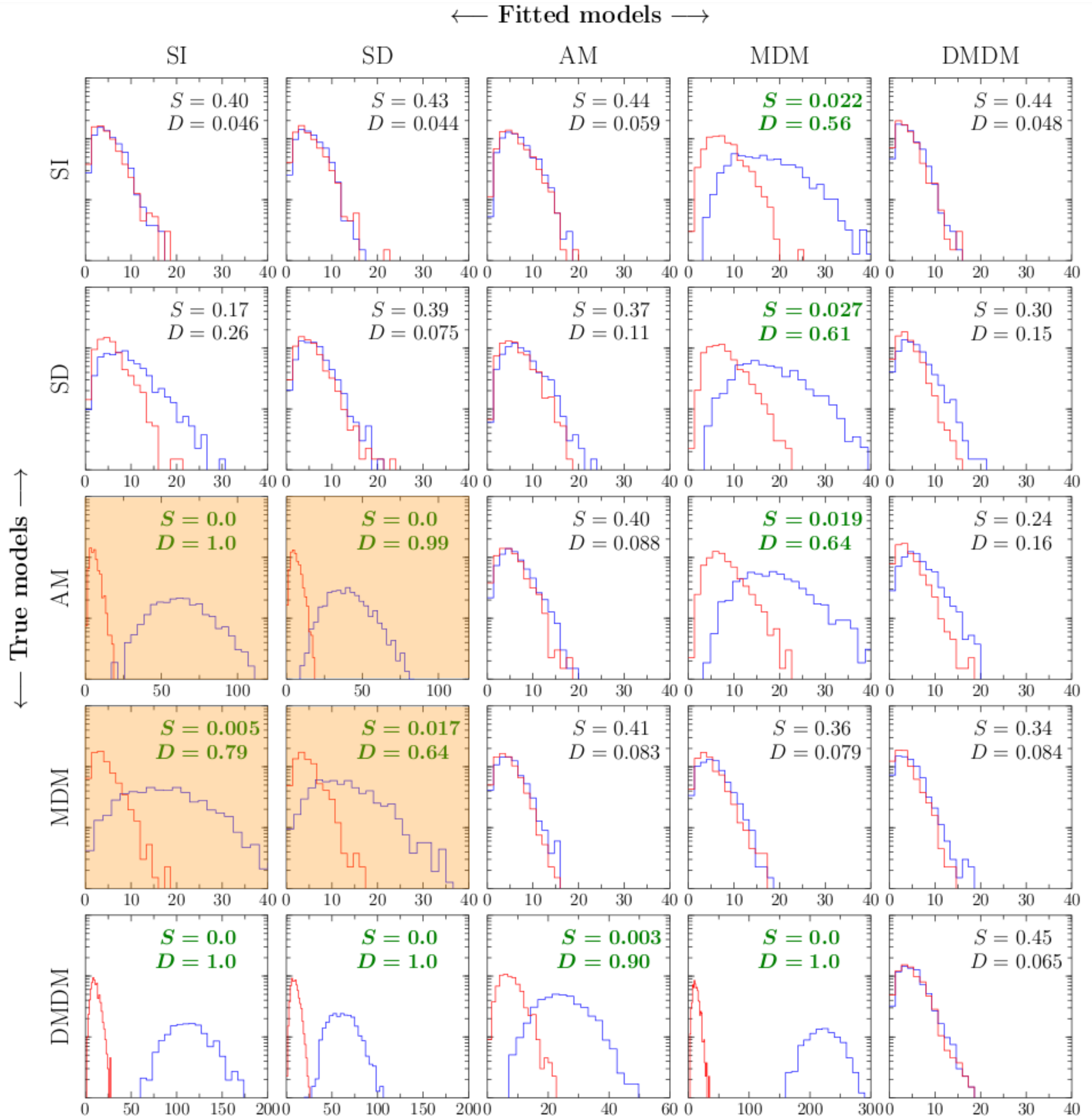
Adding more experiments

- The obvious question now is whether adding more experiments with different targets can help to distinguish certain models.
- One obvious example: Argon is completely insensitive to spin-dependent interactions, so this target allows for perfect discrimination between SI and SD.
- More non-trivial (but important) examples: iodine and germanium

Target	Exposure [kg yr]	Energy range [keV]	Number of bins	Energy resolution [keV]
Xe	2000	4–50	7	$0.6\sqrt{E_R/1\,\mathrm{keV}}$
Ge	200	0.3–50	9	0.06
I	100	10–100	9	$0.15\sqrt{E_R/1\,\mathrm{keV}}$

- We find that for all the models considered, DM interactions look very similar in xenon and germanium, so no additional discrimination power is provided.
- Iodine, on the other hand, can help quite significantly.

Results for xenon+iodine experiments



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- The presence of a second experiment with an iodine target makes it possible to rule out SI or SD interactions if DM has an anapole moment or a magnetic dipole moment.
- The reason is that iodine has a much higher sensitivity to anapole and dipole interactions.

Anapole and dipole interactions

- Anapole:

$$\mathcal{L}_{\text{eff}} = 2\mathcal{A}e \sum_{N=p,n} \left(Q_N \mathcal{O}_8^{(N)} + \tilde{\mu}_N \mathcal{O}_9^{(N)} \right)$$

- Magnetic dipole:

$$\mathcal{L}_{\text{eff}} = \frac{2\mathcal{D}_{\text{magn}}e}{q^2} \sum_{N=p,n} \left[Q_N \left(m_N \mathcal{O}_5^{(N)} - \frac{q^2}{4m_\chi}\mathcal{O}_1^{(N)} \right) + \tilde{\mu}_N \left(m_N \mathcal{O}_6^{(N)} - \frac{q^2}{m_N}\mathcal{O}_4^{(N)} \right) \right]$$

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Magnetic dipole moment of the nucleus

^{129}Xe:	-0.78	(26% abundance)
^{131}Xe:	-0.69	(21% abundance)
^{127}I:	2.81	(100% abundance)

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Distinct momentum and velocity dependence

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For anapole and magnetic dipole interactions, the total rate and the shape of the recoil spectrum are rather different in xenon and iodine, independent of astrophysical uncertainties.

Conclusions

- Astrophysical uncertainties complicate the challenge of interpreting a DM signal in future direct detection experiments.
- A promising way to extract halo-independent information is to parametrise the velocity integral $g(v_{min})$ as a piecewise constant function with many steps.
- For an assumed true model of DM and a model used to fit the mock data, we can define the similarity and distinguishability of the two models.
- In some cases (non-monotonic recoil spectra, long-range interactions), even a single xenon-based experiment may be sufficient to exclude the fitted model in a halo-independent way.
- Additional discrimination power (e.g. for DM with an anapole or dipole moment) is provided by combining the information from two different target materials, such as xenon and iodine.