Theoretical status of the muon $g - 2$

Andreas Nyffeler

PRISMA Cluster of Excellence, Institut für Kernphysik,
Helmholtz-Institut Mainz
Johannes Gutenberg Universität Mainz, Germany
nyffeler@kph.uni-mainz.de

Helmholtz Programme Matter and the Universe (MU)
HIM, Mainz, 12-13 December 2016

Outline

- Basics of the anomalous magnetic moment
- Muon $g-2$: QED, weak interactions, hadronic contributions
- Hadronic vacuum polarization (HVP)
- Hadronic light-by-light scattering (HLbL)
- New Physics contributions to the muon $g-2$
- Conclusions and Outlook

Basics of the anomalous magnetic moment

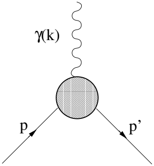
Electrostatic properties of charged particles:
Charge Q, Magnetic moment $\vec{\mu}$, Electric dipole moment $\vec{d}$

For a spin 1/2 particle:

$$\vec{\mu} = g \frac{e}{2m} \vec{s}, \qquad \underbrace{g = 2}_{\text{Dirac}}(1 + a), \qquad a = \frac{1}{2}(g-2) : \text{anomalous magnetic moment}$$

Long interplay between experiment and theory: structure of fundamental forces

In Quantum Field Theory (with C,P invariance):



$$= \quad (-ie)\bar{\mathrm{u}}(p') \left[\gamma^\mu \underbrace{F_1(k^2)}_{\text{Dirac}} + \frac{i\sigma^{\mu\nu} k_\nu}{2m} \underbrace{F_2(k^2)}_{\text{Pauli}} \right] \mathrm{u}(p)$$

$$F_1(0) = 1 \quad \text{and} \quad F_2(0) = a$$

a_e: Test of QED. Most precise determination of $\alpha = e^2/4\pi$.
a_μ: Less precisely measured than a_e, but all sectors of Standard Model (SM), i.e. QED, Weak and QCD (hadronic), contribute significantly.
Sensitive to possible contributions from New Physics. Often (but not always !):

$$a_\ell \sim \left(\frac{m_\ell}{m_{\mathrm{NP}}} \right)^2 \Rightarrow \left(\frac{m_\mu}{m_e} \right)^2 \sim 43000 \text{ more sensitive than } a_e \text{ [exp. precision} \to \text{factor 19]}$$

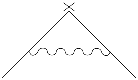
Some theoretical comments

- Anomalous magnetic moment is finite and calculable
  Corresponds to effective interaction Lagrangian of mass dimension 5:

$$\mathcal{L}_{\text{eff}}^{\text{AMM}} = -\frac{e_\ell a_\ell}{4m_\ell} \bar{\psi}(x)\sigma^{\mu\nu}\psi(x)F_{\mu\nu}(x)$$

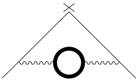
  (mass dimension 6 in SM with $SU(2)_{\mathrm{L}} \times U(1)_{\mathrm{Y}}$ invariant operator)
  $a_\ell = F_2(0)$ can be calculated unambiguously in renormalizable QFT, since there is no counterterm to absorb potential ultraviolet divergence.

- Anomalous magnetic moments are dimensionless
  To lowest order in perturbation theory in quantum electrodynamics (QED):



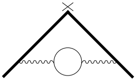
$$= a_e = a_\mu = \frac{\alpha}{2\pi}$$ [Schwinger '48]

- Loops with different masses $\Rightarrow a_e \neq a_\mu$
  - Internal large masses decouple (not always !):



$$= \left[\frac{1}{45}\left(\frac{m_e}{m_\mu}\right)^2 + \mathcal{O}\left(\frac{m_e^4}{m_\mu^4}\ln\frac{m_\mu}{m_e}\right)\right]\left(\frac{\alpha}{\pi}\right)^2$$

  - Internal small masses give rise to large log's of mass ratios:



$$= \left[\frac{1}{3}\ln\frac{m_\mu}{m_e} - \frac{25}{36} + \mathcal{O}\left(\frac{m_e}{m_\mu}\right)\right]\left(\frac{\alpha}{\pi}\right)^2$$

Milestones in measurements of the muon $g-2$

Authors	Lab	Muon Anomaly	
Garwin et al. '60	CERN	0.001 13(14)	
Charpak et al. '61	CERN	0.001 145(22)	
Charpak et al. '62	CERN	0.001 162(5)	
Farley et al. '66	CERN	0.001 165(3)	
Bailey et al. '68	CERN	0.001 166 16(31)	
Bailey et al. '79	CERN	0.001 165 923 0(84)	
Brown et al. '00	BNL	0.001 165 919 1(59)	(μ^+)
Brown et al. '01	BNL	0.001 165 920 2(14)(6)	(μ^+)
Bennett et al. '02	BNL	0.001 165 920 4(7)(5)	(μ^+)
Bennett et al. '04	BNL	0.001 165 921 4(8)(3)	(μ^-)

World average experimental value (dominated by $g-2$ Collaboration at BNL, Bennett et al. '06 + CODATA 2008 value for $\lambda = \mu_\mu/\mu_p$):

$$a_\mu^{\rm exp} = (116\,592\,089 \pm 63) \times 10^{-11} \quad [0.5\text{ppm}]$$

Goal of new planned $g-2$ experiments: $\delta a_\mu = 16 \times 10^{-11}$
Fermilab E989: partly recycled from BNL: moved ring magnet ! (http://muon-g-2.fnal.gov/bigmove/) First beam in 2017, should reach this precision by 2020. J-PARC E34: completely new concept with low-energy muons, not magic γ. Aims in Phase 1 for about $\delta a_\mu = 45 \times 10^{-11}$.

Theory needs to match this precision !

Muon $g-2$: Theory

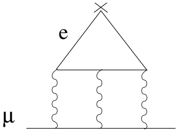
In Standard Model:

$$a_\mu^{\rm SM} = a_\mu^{\rm QED} + a_\mu^{\rm weak} + a_\mu^{\rm had}$$

In contrast to a_e, here the contributions from weak and strong interactions (hadrons) are relevant, since $a_\mu \sim (m_\mu/M)^2$.

QED contributions

- Diagrams with internal electron loops are enhanced.
- At 2-loops: vacuum polarization from electron loops enhanced by QED short-distance logarithm
- At 3-loops: light-by-light scattering from electron loops enhanced by QED infrared logarithm [Aldins et al. '69, '70; Laporta, Remiddi '93]



$$+ \ldots \quad a_\mu^{(3)}\Big|_{\rm lbyl} = \left[\frac{2}{3}\pi^2 \ln\frac{m_\mu}{m_e} + \ldots\right]\left(\frac{\alpha}{\pi}\right)^3 = 20.947\ldots\left(\frac{\alpha}{\pi}\right)^3$$

- Loops with tau's suppressed (decoupling)

QED result up to 5 loops

Include contributions from all leptons (Schwinger '48; ...; Aoyama et al. '12):

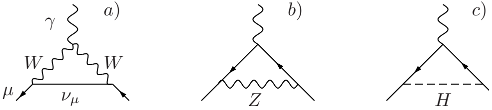
$$
\begin{aligned}
a_\mu^{\rm QED} &= 0.5 \times \left(\frac{\alpha}{\pi}\right) + 0.765\,857\,425 \underbrace{(17)}_{m_\mu/m_{e,\tau}} \times \left(\frac{\alpha}{\pi}\right)^2 \\
&\quad + 24.050\,509\,96 \underbrace{(32)}_{m_\mu/m_{e,\tau}} \times \left(\frac{\alpha}{\pi}\right)^3 + 130.8796 \underbrace{(63)}_{\text{num. int.}} \times \left(\frac{\alpha}{\pi}\right)^4 \\
&\quad + 753.29 \underbrace{(1.04)}_{\text{num. int.}} \times \left(\frac{\alpha}{\pi}\right)^5 \\
&= 116\,584\,718.853 \underbrace{(9)}_{m_\mu/m_{e,\tau}} \underbrace{(19)}_{c_4} \underbrace{(7)}_{c_5} \underbrace{(29)}_{\alpha(a_e)} [36] \times 10^{-11}
\end{aligned}
$$

- Up to 3-loop analytically known (Laporta, Remiddi '93).
- 4-loop: analytical results for electron and tau-loops (asymptotic expansions) by Steinhauser et al. '15 + '16.
- Earlier evaluation of 5-loop contribution yielded $c_5 = 662(20)$ (Kinoshita, Nio '06, numerical evaluation of 2958 diagrams, known or likely to be enhanced). New value is 4.5σ from this leading log estimate and 20 times more precise.
- Aoyama et al. '12: What about the 6-loop term ? Leading contribution from light-by-light scattering with electron loop and insertions of vacuum-polarization loops of electrons into each photon line $\Rightarrow a_\mu^{\rm QED}(\text{6-loops}) \sim 0.1 \times 10^{-11}$

Contributions from weak interaction

Numbers from recent reanalysis by Gnendiger et al. '13.

1-loop contributions [Jackiw, Weinberg '72; ...]:



$$a_\mu^{\text{weak, (1)}}(W) = \frac{\sqrt{2} G_\mu m_\mu^2}{16\pi^2} \frac{10}{3} + \mathcal{O}(m_\mu^2/M_W^2) = 388.70 \times 10^{-11}$$

$$a_\mu^{\text{weak, (1)}}(Z) = \frac{\sqrt{2} G_\mu m_\mu^2}{16\pi^2} \frac{(-1+4s_W^2)^2 - 5}{3} + \mathcal{O}(m_\mu^2/M_Z^2) = -193.89 \times 10^{-11}$$

Contribution from Higgs negligible: $a_\mu^{\text{weak, (1)}}(H) \leq 5 \times 10^{-14}$ for $m_H = 126$ GeV.

$$a_\mu^{\text{weak, (1)}} = (194.80 \pm 0.01) \times 10^{-11}$$

2-loop contributions (1678 diagrams) [Czarnecki et al. '95, '96; ...]:

$$a_\mu^{\text{weak, (2)}} = (-41.2 \pm 1.0) \times 10^{-11}, \quad \text{large since} \sim G_F m_\mu^2 \frac{\alpha}{\pi} \ln \frac{M_Z}{m_\mu}$$

Total weak contribution:

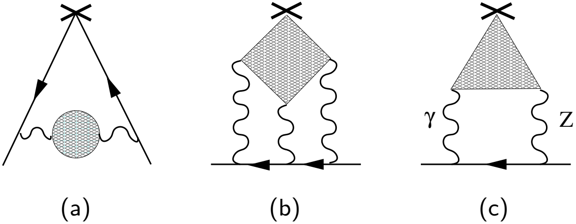
$$a_\mu^{\text{weak}} = (153.6 \pm 1.0) \times 10^{-11}$$

Under control ! With knowledge of $M_H = 125.6 \pm 1.5$ GeV, uncertainty now mostly hadronic $\pm 1.0 \times 10^{-11}$ (Peris et al. '95; Knecht et al. '02; Czarnecki et al. '03, '06). 3-loop effects via RG: $\pm 0.20 \times 10^{-11}$ (Degrassi, Giudice '98; Czarnecki et al. '03).

Hadronic contributions to the muon $g-2$

Largest source of uncertainty in theoretical prediction of a_μ !

Different types of contributions:



Light quark loop not well defined $\rightarrow$ Hadronic "blob"

(a) Hadronic vacuum polarization (HVP) $\mathcal{O}(\alpha^2), \mathcal{O}(\alpha^3), \mathcal{O}(\alpha^4)$

(b) Hadronic light-by-light scattering (HLbL) $\mathcal{O}(\alpha^3), \mathcal{O}(\alpha^4)$

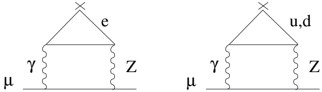
(c) 2-loop electroweak contributions $\mathcal{O}(\alpha G_F m_\mu^2)$

2-Loop EW

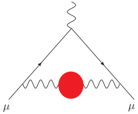
Small hadronic uncertainty from triangle diagrams.

Anomaly cancellation within each generation !

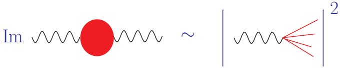
Cannot separate leptons and quarks !



Hadronic vacuum polarization

$a_\mu^{\text{HVP}} =$ 

Optical theorem (from unitarity; conservation of probability) for hadronic contribution $\to$ dispersion relation:



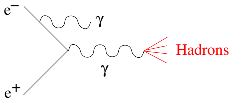
$\sim \quad \sigma(e^+e^- \to \gamma^* \to \text{hadrons})$

$$a_\mu^{\text{HVP}} = \frac{1}{3}\left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty \frac{ds}{s}\, K(s)\, R(s), \qquad R(s) = \frac{\sigma(e^+e^- \to \gamma^* \to \text{hadrons})}{\sigma(e^+e^- \to \gamma^* \to \mu^+\mu^-)}$$

[Bouchiat, Michel ’61; Durand ’62; Brodsky, de Rafael ’68; Gourdin, de Rafael ’69]

$K(s)$ slowly varying, positive function $\Rightarrow$ a_μ^{HVP} positive. Data for hadronic cross section σ at low center-of-mass energies $\sqrt{s}$ important due to factor $1/s$: $\sim 70\%$ from $\pi\pi$ [$\rho(770)$] channel, $\sim 90\%$ from energy region below 1.8 GeV.

Other method instead of energy scan: Radiative return (initial state radiation) at colliders with fixed center-of-mass energy (DAΦNE, B-Factories, BEPC) [Binner et al. ’99; Czyż et al. ’00-’03]

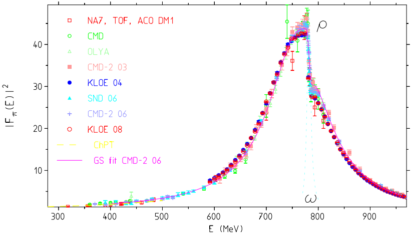


Measured hadronic cross-section

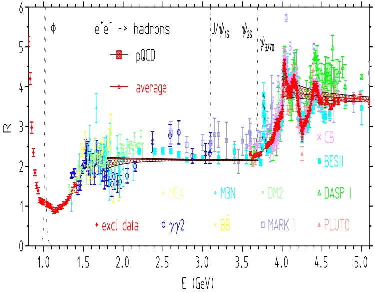
Pion form factor $|F_\pi(E)|^2$
($\pi\pi$-channel)

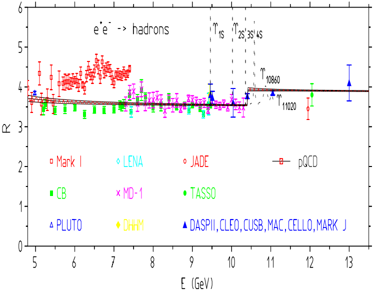
$$R(s) = \frac{1}{4}\left(1-\frac{4m_\pi^2}{s}\right)^{\frac{3}{2}} |F_\pi(s)|^2$$

$$(4m_\pi^2 < s < 9m_\pi^2)$$



R-ratio:

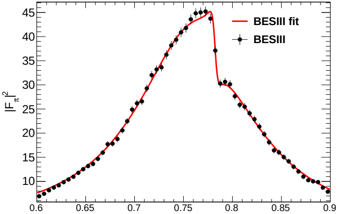




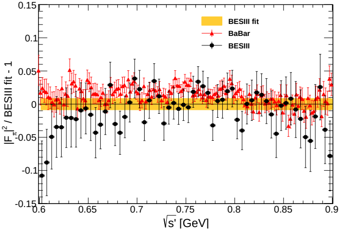
Jegerlehner, AN '09

New results on $e^+e^- \to \pi^+\pi^-$ from BESIII

Ablikim et al. (BESIII Collaboration) '16 (Analysis by group at Mainz (Denig et al.))

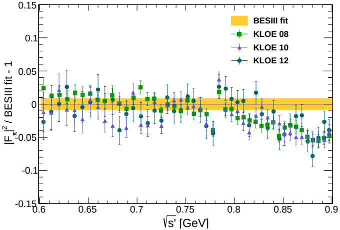


Fit of BESIII data with parametrization of pion form factor by Gounaris-Sakurai. Only statistical errors shown.



BaBar data higher than BESIII below ρ-mass, better agreement above.

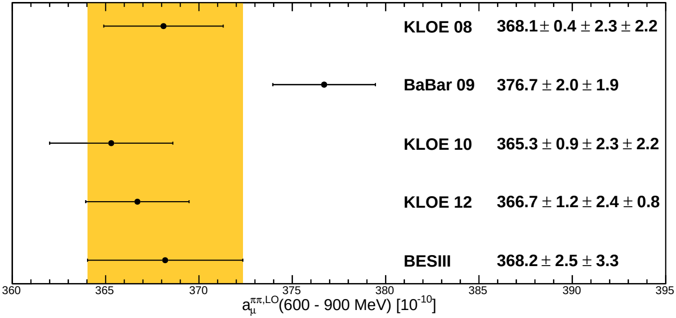
Statistical and systematic errors included in data points. Width of BESIII fit band shows systematic uncertainty only.



Good agreement with KLOE 08 and KLOE 12 up to mass range of $\rho - \omega$ interference, but disagreement with all three data sets at higher energy.

New results on $e^+e^- \to \pi^+\pi^-$ from BESIII (continued)

Comparison of value for $a_\mu^{\mathrm{HVP}}(600 - 900\ \mathrm{MeV})$ from the three experiments using radiative return method (initial state radiation):



Results from BESIII confirm KLOE, disagree with BaBar at level of $1 - 2\ \sigma$ (at least after integration in dispersion integral to get a_μ^{HVP}).

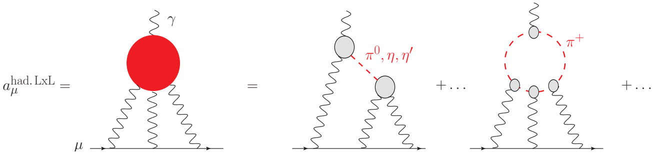
Hadronic vacuum polarization: some recent evaluations

Authors	Contribution to $a_\mu^{\mathrm{HVP}} \times 10^{11}$
Davier et al. '11, '14 (e^+e^-) [+ τ]	6923 ± 42 [7030 ± 44]
Jegerlehner, Szafron '11 (e^+e^-) [+ τ]	6907.5 ± 47.2 [6909.6 ± 46.5]
Hagiwara et al. '11 (e^+e^-)	6949.1 ± 42.7
Benayoun at al. '15 ($e^+e^- + \tau$: BHLS improved)	6818.6 ± 32.0
Jegerlehner '15 (e^+e^-) [+ τ]	6885.7 ± 42.8 [6889.1 ± 35.2]
Davier '16 (e^+e^-)	6926 ± 33

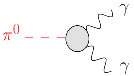
- Precision: $< 1\%$. Non-trivial because of radiative corrections (radiated photons).
- Even if values for a_μ^{HVP} after integration agree quite well, the systematic differences of a few % in the shape of the spectral functions from different experiments (BABAR, BESIII, CMD-2, KLOE, SND) indicate that we do not yet have a complete understanding.
- Use of τ data: additional sources of isospin violation ? Ghozzi, Jegerlehner '04; Benayoun et al. '08, '09; Wolfe, Maltman '09; Jegerlehner, Szafron '11 ($\rho - \gamma$-mixing), also included in BHLS-approach by Benayoun et al. '15 and in Jegerlehner '15.
- Lattice QCD: Various groups are working on it (including here in Mainz), precision at level of about 3-5% (systematics dominated), not yet competitive with phenomenological evaluations.

Hadronic light-by-light scattering

HLbL in muon $g-2$ from strong interactions (QCD):



Coupling of photons to hadrons, e.g. π^0, via form factor:



Relevant scales ($\langle VVVV \rangle$ with offshell photons): $0 - 2$ GeV $\gg m_\mu$ (resonance region)

View before 2014: in contrast to HVP, no direct relation to experimental data
$\rightarrow$ size and even sign of contribution to a_μ unknown !

Approach: use hadronic model at low energies with exchanges and loops of resonances and some (dressed) "quark-loop" at high energies.

Constrain models using experimental data (processes of hadrons with photons: decays, form factors, scattering) and theory (ChPT at low energies; short-distance constraints from pQCD / OPE at high momenta).

Problems: Four-point function depends on several invariant momenta $\Rightarrow$ distinction between low and high energies not as easy as for two-point function in HVP.
Mixed regions: one loop momentum Q_1^2 large, the other Q_2^2 small and vice versa.

HLbL in muon $g-2$

- Frequently used estimates:

$$a_\mu^{\text{HLbL}} = (105 \pm 26) \times 10^{-11} \qquad \text{(Prades, de Rafael, Vainshtein '09)}$$
$$a_\mu^{\text{HLbL}} = (116 \pm 40) \times 10^{-11} \qquad \text{(AN '09; Jegerlehner, AN '09)}$$

  Based almost on same input: calculations by various groups using different models for individual contributions. Error estimates are mostly guesses !

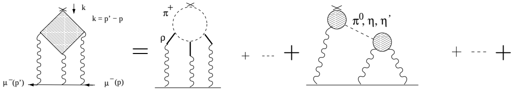
- Need much better understanding of complicated hadronic dynamics to get reliable error estimate of $\pm 20 \times 10^{-11}$ (δa_μ(future exp) $= 16 \times 10^{-11}$).
- Recent new proposal: Colangelo et al. '14, '15; Pauk, Vanderhaeghen '14 (Mainz): use dispersion relations (DR) to connect contribution to HLbL from presumably numerically dominant light pseudoscalars to in principle measurable form factors and cross-sections:

$$\gamma^*\gamma^* \rightarrow \pi^0, \eta, \eta'$$
$$\gamma^*\gamma^* \rightarrow \pi^+\pi^-, \pi^0\pi^0$$

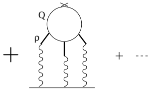
  Could connect HLbL uncertainty to exp. measurement errors, like HVP.
  Note: no data yet with two off-shell photons !

- Future: HLbL from Lattice QCD.
  First steps and results: Blum et al. (RBC-UKQCD) '05, ..., '15, '16.
  Work ongoing by Mainz group: Green et al. '15; Asmussen et al. '16.

HLbL in muon $g-2$: summary of selected results

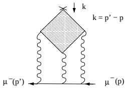


Exchange of other resonances ($f_0, a_1, f_2 \ldots$)

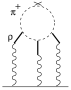


Chiral counting:	p^4	p^6	p^8	p^8
N_C-counting:	1	N_C	N_C	N_C

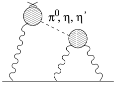
HLbL in muon $g-2$: summary of selected results



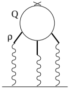
$=$



$+ \cdots +$



$+ \cdots +$ Exchange of other resonances $(f_0, a_1, f_2 \ldots)$ $+$



$+ \cdots$

Chiral counting:		p^4	p^6	p^8	p^8
N_C-counting:		1	N_C	N_C	N_C
Contribution to $a_\mu \times 10^{11}$:					
BPP:	+83 (32)	-19 (13)	+85 (13)	-4 (3) $[f_0, a_1]$	+21 (3)
HKS:	+90 (15)	-5 (8)	+83 (6)	+1.7 (1.7) $[a_1]$	+10 (11)
KN:	+80 (40)		+83 (12)		
MV:	+136 (25)	0 (10)	+114 (10)	+22 (5) $[a_1]$	0
2007:	+110 (40)				
PdRV:	+105 (26)	-19 (19)	+114 (13)	+8 (12) $[f_0, a_1]$	+2.3 [c-quark]
N,JN:	+116 (40)	-19 (13)	+99 (16)	+15 (7) $[f_0, a_1]$	+21 (3)
		ud.: -45	ud.: $+\infty$		ud.: +60

ud. = undressed, i.e. point vertices without form factors

Pseudoscalars: numerically dominant contribution (according to most models !).

Recent reevaluations of axial vector contribution lead to much smaller estimates than in MV '04: $a_\mu^{\mathrm{HLbL;axial}} = (8 \pm 3) \times 10^{-11}$ (Pauk, Vanderhaeghen '14; Jegerlehner '14, '15). Would shift central values of compilations downwards:

$a_\mu^{\mathrm{HLbL}} = (98 \pm 26) \times 10^{-11}$ (PdRV) and $a_\mu^{\mathrm{HLbL}} = (102 \pm 40) \times 10^{-11}$ (N, JN).

Recall (in units of 10^{-11}): $\delta a_\mu(\mathrm{HVP}) \approx 40$; $\delta a_\mu(\mathrm{exp\ [BNL]}) = 63$; $\delta a_\mu(\mathrm{future\ exp}) = 16$

BPP = Bijnens, Pallante, Prades '96, '02; HKS = Hayakawa, Kinoshita, Sanda '96, '98, '02; KN = Knecht, AN '02; MV = Melnikov, Vainshtein '04; 2007 = Bijnens, Prades; Miller, de Rafael, Roberts; PdRV = Prades, de Rafael, Vainshtein '09 (compilation; "Glasgow consensus"); N,JN = AN '09; Jegerlehner, AN '09 (compilation)

Data-driven approach to HLbL using dispersion relations (DR)

Strategy: Split contributions to HLbL into two parts:

I: Data-driven evaluation using DR (hopefully numerically dominant):

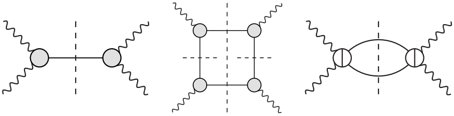
(1) π^0, η, η' poles
(2) $\pi\pi$ intermediate state

II: Model dependent evaluation (hopefully numerically subdominant):

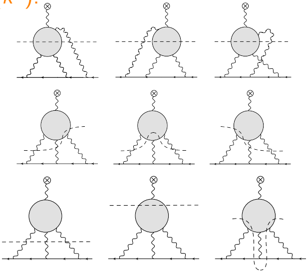
(1) Axial vectors (3π-intermediate state), ...
(2) Quark-loop, matching with pQCD

Error goals: Part I: 10% precision (data driven), Part II: 30% precision.
To achieve overall error of about 20% ($\delta a_\mu^{\text{HLbL}} = 20 \times 10^{-11}$).

Colangelo et al. '14, '15:
Classify intermediate states in 4-point function. Then project onto $g-2$.



Pauk, Vanderhaeghen '14:
DR directly for Pauli form factor $F_2(k^2)$.



$a_\mu^{\mathrm{HLbL;P}}, \mathrm{P} = \pi^0, \eta, \eta'$: impact of precision of form factor measurements

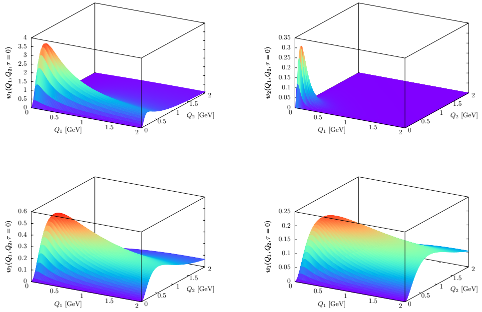
AN '16

In Jegerlehner, AN '09, a 3-dimensional integral representation for the pseudoscalar-pole contribution was derived. Schematically:

$$a_\mu^{\mathrm{HLbL;P}} = \int_0^\infty dQ_1 \int_0^\infty dQ_2 \int_{-1}^{1} d\tau \sum_i w_i(Q_1, Q_2, \tau)\, f_i(Q_1, Q_2, \tau)$$

with universal weight functions w_i (for Euclidean (space-like) momenta: $Q_1 \cdot Q_2 = |Q_1||Q_2|\tau, \tau = \cos\theta$). Dependence on form factors resides in the f_i.

Weight functions w_i:



Top: weight functions $w_{1,2}(Q_1, Q_2, \tau)$ for π^0 with $\theta = 90^\circ (\tau = 0)$.

Bottom: weight functions $w_1(Q_1, Q_2, \tau)$ for η (left) and η' (right).

- Relevant momentum regions below 1 GeV for π^0, below 1.5 GeV for η, η'.
- Analysis of current and future measurement precision of single-virtual $\mathcal{F}_{\mathrm{P}\gamma^*\gamma^*}(-Q^2, 0)$ and double-virtual transition form factor $\mathcal{F}_{\mathrm{P}\gamma^*\gamma^*}(-Q_1^2, -Q_2^2)$, based on Monte Carlo study for BESIII by Denig, Redmer, Wasser (Mainz).
- Data-driven precision for HLbL pseudoscalar-pole contribution that could be achieved in a few years:

$$\delta a_\mu^{\mathrm{HLbL};\pi^0} / a_\mu^{\mathrm{HLbL};\pi^0} = 14\%$$
$$\delta a_\mu^{\mathrm{HLbL};\eta} / a_\mu^{\mathrm{HLbL};\eta} = 23\%$$
$$\delta a_\mu^{\mathrm{HLbL};\eta'} / a_\mu^{\mathrm{HLbL};\eta'} = 15\%$$

Muon $g-2$: current status

Contribution	$a_\mu \times 10^{11}$	Reference
QED (leptons)	116 584 718.853 $\pm$ 0.036	Aoyama et al. '12
Electroweak	153.6 $\pm$ 1.0	Gnendiger et al. '13
HVP: LO	6889.1 $\pm$ 35.2	Jegerlehner '15
NLO	-99.2 $\pm$ 1.0	Jegerlehner '15
NNLO	12.4 $\pm$ 0.1	Kurz et al. '14
HLbL	116 $\pm$ 40	Jegerlehner, AN '09
NLO	3 $\pm$ 2	Colangelo et al. '14
Theory (SM)	116 591 794 $\pm$ 53	
Experiment	116 592 089 $\pm$ 63	Bennett et al. '06
Experiment - Theory	295 $\pm$ 82	3.6 σ

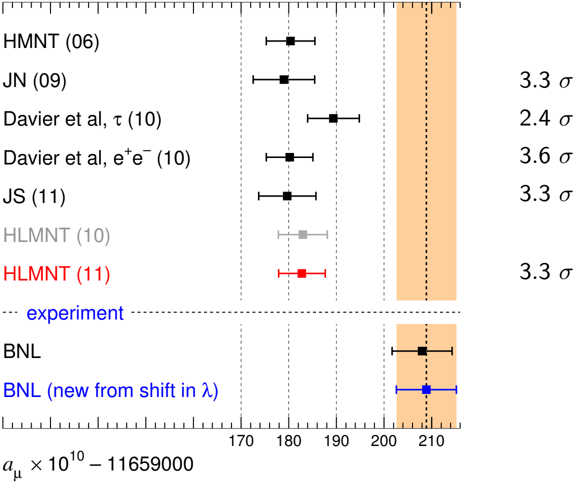
HVP: Hadronic vacuum polarization

HLbL: Hadronic light-by-light scattering

Discrepancy a sign of New Physics ?

Hadronic uncertainties need to be better controlled in order to fully profit from future $g-2$ experiments at Fermilab and J-PARC with $\delta a_\mu = 16 \times 10^{-11}$.
Way forward for HVP seems clear: more precise measurements for $\sigma(e^+e^- \to \text{hadrons})$. Not so obvious how to improve HLbL.

Muon $g-2$: other recent evaluations



Source: Hagiwara et al. '11. Note units of 10^{-10} !

Aoyama et al. '12: $a_\mu^{\rm exp} - a_\mu^{\rm SM} = (249 \pm 87) \times 10^{-11}$ [$2.9\ \sigma$]

Benayoun et al. '15: $a_\mu^{\rm exp} - a_\mu^{\rm SM} = (376.8 \pm 75.3) \times 10^{-11}$ [$5.0\ \sigma$]

Jegerlehner '15: $a_\mu^{\rm exp} - a_\mu^{\rm SM} = (310 \pm 82) \times 10^{-11}$ [$3.8\ \sigma$]

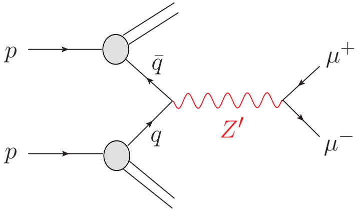
Tests of the Standard Model and search for New Physics

- Standard Model (SM) of particle physics very successful in precise description of a huge amount of experimental data, with a few exceptions ($3-4$ standard deviations).
- Some experimental facts (neutrino masses, baryon asymmetry in the universe, dark matter) and some theoretical arguments, which point to New Physics beyond the Standard Model.
- There are several indications that new particles (forces) should show up in the mass range 100 GeV $-$ 1 TeV.

Tests of the Standard Model and search for New Physics (continued)

Search for New Physics with two complementary approaches:

1. High Energy Physics:
   e.g. Large Hadron Collider (LHC) at CERN
   Direct production of new particles
   e.g. heavy Z' $\Rightarrow$ resonance peak in invariant mass distribution of $\mu^+\mu^-$ at $M_{Z'}$.

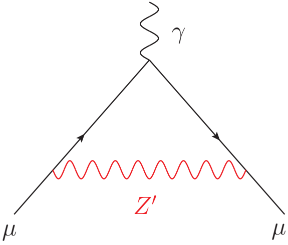


2. Precision physics:
   e.g. anomalous magnetic moments a_e, a_μ
   Indirect effects of virtual particles in quantum corrections
   $\Rightarrow$ Deviations from precise predictions in SM

$$\text{For } M_{Z'} \gg m_\ell : \quad a_\ell \sim \left(\frac{m_\ell}{M_{Z'}}\right)^2$$

Note: there are also non-decoupling contributions of heavy New Physics !
Another example: new light vector meson ("dark photon") with $M_{\gamma'} \sim (10 - 100)$ MeV.

a_e, a_μ allow to exclude some models of New Physics or to constrain their parameter space.



New Physics contributions to the muon $g-2$

Define:

$$\Delta a_\mu = a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = (290 \pm 90) \times 10^{-11} \qquad \text{(Jegerlehner, AN '09)}$$

Absolute size of discrepancy is actually unexpectedly large, compared to weak contribution (although there is some cancellation there):

$$\begin{aligned} a_\mu^{\text{weak}} &= a_\mu^{\text{weak, (1)}}(W) + a_\mu^{\text{weak, (1)}}(Z) + a_\mu^{\text{weak, (2)}} \\ &= (389 - 194 - 41) \times 10^{-11} \\ &= 154 \times 10^{-11} \end{aligned}$$

Assume that New Physics contribution with $M_{\text{NP}} \gg m_\mu$ decouples:

$$a_\mu^{\text{NP}} = \mathcal{C} \frac{m_\mu^2}{M_{\text{NP}}^2}$$

where naturally $\mathcal{C} = \frac{\alpha}{\pi}$, like from a one-loop QED diagram, but with new particles. Typical New Physics scales required to satisfy $a_\mu^{\text{NP}} = \Delta a_\mu$:

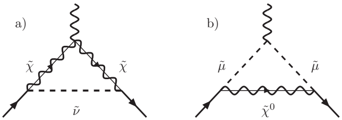
$\mathcal{C}$	1	$\frac{\alpha}{\pi}$	$\left(\frac{\alpha}{\pi}\right)^2$
M_{NP}	$2.0^{+0.4}_{-0.3}$ TeV	100^{+21}_{-13} GeV	5^{+1}_{-1} GeV

Therefore, for New Physics model with particles in $250 - 300$ GeV mass range and electroweak-size couplings $\mathcal{O}(\alpha)$, we need some additional enhancement factor, like large $\tan\beta$ in the MSSM, to explain the discrepancy Δa_μ.

a_μ: Supersymmetry

Supersymmetry for large $\tan\beta, \mu > 0$:

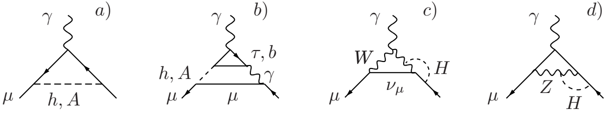
$$a_\mu^{\text{SUSY}} \approx 123 \times 10^{-11} \left(\frac{100\ \text{GeV}}{M_{\text{SUSY}}}\right)^2 \tan\beta$$



(Czarnecki, Marciano, '01)

Explains $\Delta a_\mu = 290 \times 10^{-11}$ if $M_{\text{SUSY}} \approx (93 - 414)$ GeV $(2 < \tan\beta < 40)$.

In some regions of parameter space, large 2-loop contributions (2HDM):



Barr-Zee diagram (b) yields enhanced contribution, which can exceed 1-loop result.
Enhancement factor m_b^2/m_μ^2 compensates suppression by α/π
$((\alpha/\pi) \times (m_b^2/m_\mu^2) \sim 4 > 1)$.

a_μ and Supersymmetry after first LHC run

- LHC so far only sensitive to strongly interacting supersymmetric particles, like squarks and gluinos (ruled out below about 1 TeV).
- Muon $g-2$ and SUSY searches at LHC only lead to tension in constrained MSSM (CMSSM) or NUHM1 / NUHM2 (non-universal contributions to Higgs masses).
- More general SUSY models (e.g. pMSSM10 = phenomenological MSSM with 10 soft SUSY-breaking parameters) with light neutralinos, charginos and sleptons, can still explain muon $g-2$ discrepancy and evade bounds from LHC.

a_e, a_μ: Dark photon

In some dark matter scenarios, there is a relatively light, but massive "dark photon" A'_μ that couples to the SM through mixing with the photon:

$$\mathcal{L}_{\mathrm{mix}} = \frac{\varepsilon}{2} F^{\mu\nu} F'_{\mu\nu}$$

$\Rightarrow$ A'_μ couples to ordinary charged particles with strength $\varepsilon \cdot e$.
$\Rightarrow$ additional contribution of dark photon with mass $m_{\gamma'}$ to the $g-2$ of a lepton (electron, muon) (Pospelov '09):

$$\begin{aligned} a_\ell^{\text{dark photon}} &= \frac{\alpha}{2\pi}\varepsilon^2 \int_0^1 dx \frac{2x(1-x)^2}{\left[(1-x)^2 + \frac{m_{\gamma'}^2}{m_\ell^2} x\right]} \\ &= \frac{\alpha}{2\pi}\varepsilon^2 \times \begin{cases} 1 & \text{for } m_\ell \gg m_{\gamma'} \\ \frac{2m_\ell^2}{3m_{\gamma'}^2} & \text{for } m_\ell \ll m_{\gamma'} \end{cases} \end{aligned}$$

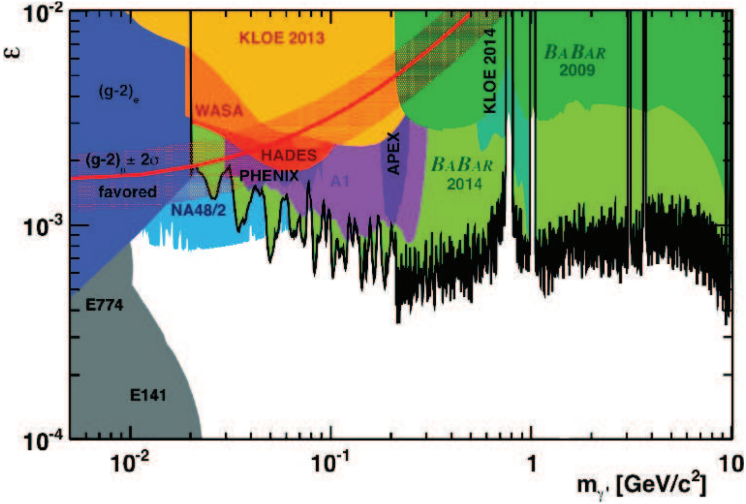
For values $\varepsilon \sim (1-2) \times 10^{-3}$ and $m_{\gamma'} \sim (10-100)$ MeV, the dark photon could explain the discrepancy $\Delta a_\mu \sim 300 \times 10^{-11}$.

Various searches for the dark photon have been performed, are under way or are planned at BABAR, Jefferson Lab, KLOE, MAMI and other experiments.

For a recent overview, see: *Dark Sectors and New, Light, Weakly-Coupled Particles* (Snowmass 2013), Essig et al., arXiv:1311.0029 [hep-ph].

Status of dark photon searches

Essentially all of the parameter space in the $(m_{\gamma'}, \varepsilon)$-plane to explain the muon $g-2$ discrepancy has now been ruled out.



From: F. Curciarello, FCCP15, Capri, September 2015

Different conclusions if dark photon decays (mostly) invisibly !

Conclusions and Outlook

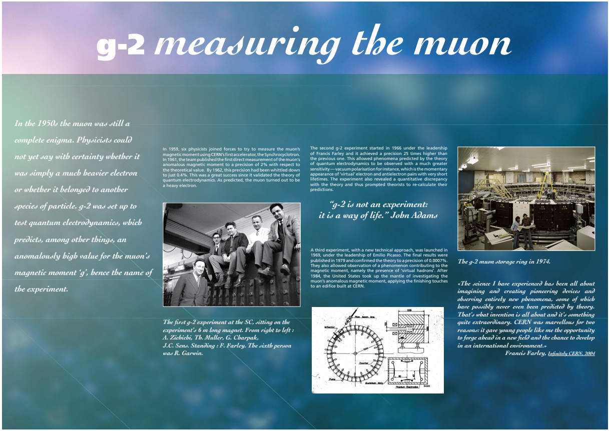
- Over many decades, the (anomalous) magnetic moments of the electron and the muon have played a crucial role in atomic and elementary particle physics.
- Gained important insights into the structure of the fundamental interactions and matter in the universe (quantum field theory).
- a_μ: Test of Standard Model, potential window to New Physics.
- Current situation:

$$a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = (295 \pm 82) \times 10^{-11} \qquad [3.6\ \sigma]$$

  Hadronic effects ? Sign of New Physics ?

- Two new planned $g-2$ experiments at Fermilab and J-PARC with goal of $\delta a_\mu^{\text{exp}} = 16 \times 10^{-11}$ (factor 4 improvement)
- Theory needs to match this precision !
- Concerted effort needed of experiments (measuring processes with hadrons and photons), phenomenology (modelling and data-driven using dispersion relations) and lattice QCD to improve HVP and HLbL estimates with reliable uncertainties.

And finally:



Source: CERN (50 Years of Proton Synchrotron)

"$g-2$ is not an experiment: it is a way of life."

John Adams (Head of the Proton Synchrotron at CERN (1954-61) and Director General of CERN (1960-1961))

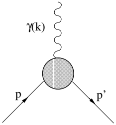
This statement also applies to many theorists working on the $g-2$!

Backup slides

Anomalous magnetic moment in quantum field theory

Quantized spin 1/2 particle interacting with external, classical electromagnetic field

4 form factors in vertex function
(momentum transfer $k = p' - p$, not assuming parity or charge conjugation invariance)



$$
\begin{aligned}
&\equiv \quad i\langle p', s'|j^{\mu}(0)|p, s\rangle \\
&= \quad (-ie)\bar{\mathrm{u}}(p', s')\Big[\gamma^{\mu}\underbrace{F_1(k^2)}_{\text{Dirac}} + \frac{i\sigma^{\mu\nu}k_{\nu}}{2m}\underbrace{F_2(k^2)}_{\text{Pauli}} \\
&\qquad + \gamma^5\frac{\sigma^{\mu\nu}k_{\nu}}{2m}F_3(k^2) + \gamma^5\left(k^2\gamma^{\mu} - \not{k}\; k^{\mu}\right)F_4(k^2)\Big]\mathrm{u}(p, s)
\end{aligned}
$$

$\not{k} = \gamma^{\mu}k_{\mu}$. Real form factors for spacelike $k^2 \leq 0$. Non-relativistic, static limit:

$$
\begin{aligned}
F_1(0) &= 1 \qquad \text{(renormalization of charge } e\text{)} \\
\mu &= \frac{e}{2m}(F_1(0) + F_2(0)) \qquad \text{(magnetic moment)} \\
a &= F_2(0) \qquad \text{(anomalous magnetic moment)} \\
d &= -\frac{e}{2m}F_3(0) \qquad \text{(electric dipole moment, violates P and CP)} \\
F_4(0) &= \text{anapole moment (violates P)}
\end{aligned}
$$

HLbL scattering: selected results for $a_\mu^{\mathrm{HLbL}} \times 10^{11}$

Contribution	BPP	HKS, HK	KN	MV	BP, MdRR	PdRV	N, JN
π^0, η, η'	85 ± 13	82.7 ± 6.4	83 ± 12	114 ± 10	–	114 ± 13	99 ± 16
axial vectors	2.5 ± 1.0	1.7 ± 1.7	–	22 ± 5	–	15 ± 10	22 ± 5
scalars	-6.8 ± 2.0	–	–	–	–	-7 ± 7	-7 ± 2
π, K loops	-19 ± 13	-4.5 ± 8.1	–	–	–	-19 ± 19	-19 ± 13
π, K loops +subl. N_C	–	–	–	0 ± 10	–	–	–
quark loops	21 ± 3	9.7 ± 11.1	–	–	–	2.3 (c-quark)	21 ± 3
Total	83 ± 32	89.6 ± 15.4	80 ± 40	136 ± 25	110 ± 40	105 ± 26	116 ± 40

BPP = Bijnens, Pallante, Prades '95, '96, '02; HKS = Hayakawa, Kinoshita, Sanda '95, '96; HK = Hayakawa, Kinoshita '98, '02; KN = Knecht, AN '02; MV = Melnikov, Vainshtein '04; BP = Bijnens, Prades '07; MdRR = Miller, de Rafael, Roberts '07; PdRV = Prades, de Rafael, Vainshtein '09; N = AN '09, JN = Jegerlehner, AN '09

- Pseudoscalar-exchanges dominate numerically. Other contributions not negligible. Cancellation between π, K-loops and quark loops !
- Note that recent reevaluations of axial vector contribution lead to much smaller estimates than in MV: $a_\mu^{\mathrm{HLbL;axial}} = (8 \pm 3) \times 10^{-11}$ (Pauk, Vanderhaeghen '14; Jegerlehner '14, '15). This would shift central values of compilations downwards: $a_\mu^{\mathrm{HLbL}} = (98 \pm 26) \times 10^{-11}$ (PdRV) and $a_\mu^{\mathrm{HLbL}} = (102 \pm 40) \times 10^{-11}$ (N, JN).
- PdRV: Analyzed results obtained by different groups with various models and suggested new estimates for some contributions (shifted central values, enlarged errors). Do not consider dressed light quark loops as separate contribution. Added all errors in quadrature !
- N, JN: New evaluation of pseudoscalar exchange contribution imposing new short-distance constraint on off-shell form factors. Took over most values from BPP, except axial vectors from MV. Added all errors linearly.

HLbL: recent developments

- Most calculations for neutral pion and all light pseudoscalars agree at level of 15%, but some are quite different:

$$a_\mu^{\mathrm{HLbL};\pi^0} = (50-80)\times 10^{-11}$$
$$a_\mu^{\mathrm{HLbL;PS}} = (59-114)\times 10^{-11}$$

- New estimates for axial vectors (Pauk, Vanderhaeghen '14; Jegerlehner '14, '15):

$$a_\mu^{\mathrm{HLbL;axial}} = (6-8)\times 10^{-11}$$

  Substantially smaller than in MV '04 !

- First estimate for tensor mesons (Pauk, Vanderhaeghen '14):

$$a_\mu^{\mathrm{HLbL;tensor}} = 1\times 10^{-11}$$

- Open problem: Dressed pion-loop
  Potentially important effect from pion polarizability and a_1 resonance
  (Engel, Patel, Ramsey-Musolf '12; Engel '13; Engel, Ramsey-Musolf '13):

$$a_\mu^{\mathrm{HLbL};\pi-\mathrm{loop}} = -(11-71)\times 10^{-11}$$

  Maybe large negative contribution, in contrast to BPP '96, HKS '96.
  Not confirmed by recent reanalysis by Bijnens, Relefors '15, '16. Essentially get again old central value from BPP, but smaller error estimate:

$$a_\mu^{\mathrm{HLbL};\pi-\mathrm{loop}} = (-20\pm 5)\times 10^{-11}$$

- Open problem: Dressed quark-loop
  Dyson-Schwinger equation approach (Fischer, Goecke, Williams '11, '13):

$$a_\mu^{\mathrm{HLbL;quark-loop}} = 107\times 10^{-11} \qquad \text{(still incomplete !)}$$

  Large contribution, no damping seen, in contrast to BPP '96, HKS '96.

Electron $g-2$: Theory

Main contribution in Standard Model (SM) from mass-independent Feynman diagrams in QED with electrons in internal lines (perturbative series in α):

$$
\begin{aligned}
a_e^{\mathrm{SM}} \quad = \quad & \sum_{n=1}^{5} c_n \left(\frac{\alpha}{\pi}\right)^n \\
& +2.7478(2) \times 10^{-12} \quad [\text{Loops in QED with } \mu, \tau] \\
& +0.0297(5) \times 10^{-12} \quad [\text{weak interactions}] \\
& +1.706(15) \times 10^{-12} \quad [\text{strong interactions / hadrons}]
\end{aligned}
$$

The numbers are from Aoyama et al. '15.

QED: mass-independent contributions to a_e

- α: 1-loop, 1 Feynman diagram; Schwinger '48:

  $c_1 = \frac{1}{2}$

- α^2: 2-loops, 7 Feynman diagrams; Petermann '57, Sommerfield '57:

  $c_2 = \frac{197}{144} + \frac{\pi^2}{12} - \frac{\pi^2}{2}\ln 2 + \frac{3}{4}\zeta(3) = -0.32847896557919378\ldots$

- α^3: 3-loops, 72 Feynman diagrams; ..., Laporta, Remiddi '96:

$$
\begin{aligned}
c_3 &= \frac{28259}{5184} + \frac{17101}{810}\pi^2 - \frac{298}{9}\pi^2\ln 2 + \frac{139}{18}\zeta(3) - \frac{239}{2160}\pi^4 \\
&\quad + \frac{83}{72}\pi^2\zeta(3) - \frac{215}{24}\zeta(5) + \frac{100}{3}\left\{\mathrm{Li}_4\left(\frac{1}{2}\right) + \frac{1}{24}\ln^4 2 - \frac{1}{24}\pi^2\ln^2 2\right\} \\
&= 1.181241456587\ldots
\end{aligned}
$$

- α^4: 4-loops, 891 Feynman diagrams; Kinoshita et al. '99, ..., Aoyama et al. '08; '12, '15:

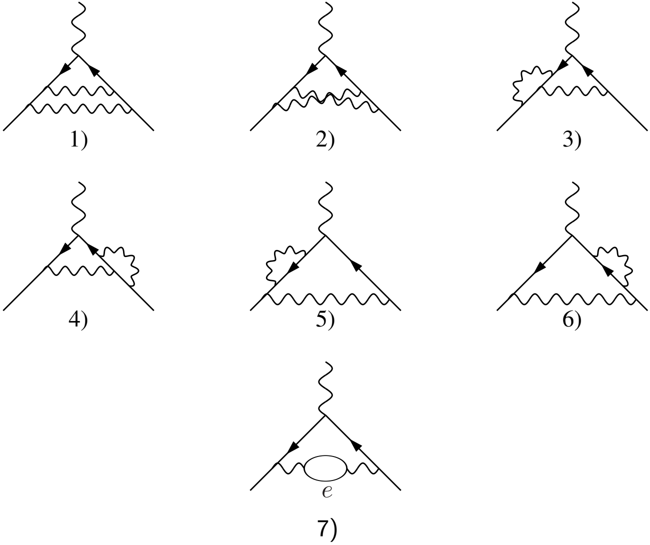
  $c_4 = -1.91298(84)$ (numerical evaluation)

- α^5: 5-loops, 12672 Feynman diagrams; Aoyama et al. '05, ..., '12, '15:

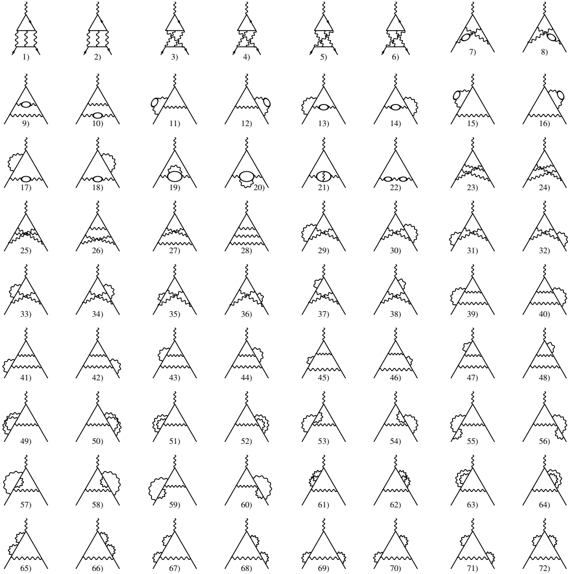
  $c_5 = 7.795(336)$ (numerical evaluation)

  Replaces earlier rough estimate $c_5 = 0.0 \pm 4.6$.
  Result removes biggest theoretical uncertainty in a_e !

Mass-independent 2-loop Feynman diagrams in a_e



Mass-independent 3-loop Feynman diagrams in a_e



Determination of fine-structure constant α from $g-2$ of electron

- Recent measurement of α via recoil-velocity of Rubidium atoms in atom interferometer (Bouchendira et al. '11 and recent CODATA input):

$$\alpha^{-1}(\mathrm{Rb}) = 137.035\,999\,049(90) \quad [0.66\mathrm{ppb}]$$

This leads to (Aoyama et al. '15):

$$a_e^{\mathrm{SM}}(\mathrm{Rb}) = 1\,159\,652\,181.643\,\underbrace{(25)}_{c_4}\,\underbrace{(23)}_{c_5}\,\underbrace{(16)}_{\mathrm{had}}\,\underbrace{(763)}_{\alpha(\mathrm{Rb})}\,[764] \times 10^{-12} \quad [0.67\mathrm{ppb}]$$

$$\Rightarrow a_e^{\mathrm{exp}} - a_e^{\mathrm{SM}}(\mathrm{Rb}) = -0.91(0.82) \times 10^{-12} \quad [\text{Error from } \alpha(\mathrm{Rb}) \text{ dominates !}]$$

$\rightarrow$ Test of QED !

- Use a_e^{exp} to determine α from series expansion in QED (contributions from weak and strong interactions under control !). Assume: Standard Model "correct", no New Physics (Aoyama et al. '15):

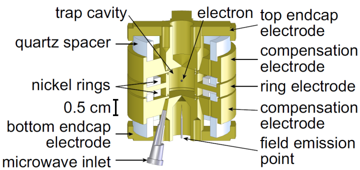
$$\alpha^{-1}(a_e) = 137.035\,999\,1570\,\underbrace{(29)}_{c_4}\,\underbrace{(27)}_{c_5}\,\underbrace{(18)}_{\mathrm{had+EW}}\,\underbrace{(331)}_{a_e^{\mathrm{exp}}}\,[334] \quad [0.25\mathrm{ppb}]$$

The uncertainty from theory has been improved considerably by Aoyama et al. '12, '15, the experimental uncertainty in a_e^{exp} is now the limiting factor.

- Today the most precise determination of the fine-structure constant α, a fundamental parameter of the Standard Model.

Electron $g-2$: Experiment

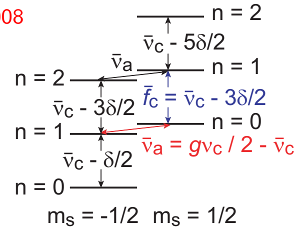
Latest experiment: Hanneke, Fogwell, Gabrielse, 2008



Cylindrical Penning trap for single electron

(1-electron quantum cyclotron)

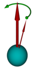
Source: Hanneke et al.



Cyclotron and spin precession levels of electron in Penning trap

Source: Hanneke et al.

$$\frac{g_e}{2} = \frac{\nu_s}{\nu_c} \simeq 1 + \frac{\bar{\nu}_a - \bar{\nu}_z^2/(2\bar{f}_c)}{\bar{f}_c + 3\delta/2 + \bar{\nu}_z^2/(2\bar{f}_c)} + \frac{\Delta g_{cav}}{2}$$



ν_s = spin precession frequency; $\nu_c, \bar{\nu}_c$ = cyclotron frequency: free electron, electron in Penning trap; $\delta/\nu_c = h\nu_c/(m_e c^2) \approx 10^{-9}$ = relativistic correction

4 quantities are measured precisely in experiment:

$\bar{f}_c = \bar{\nu}_c - \frac{3}{2}\delta \approx 149$ GHz; $\qquad \bar{\nu}_a = \frac{g}{2}\nu_c - \bar{\nu}_c \approx 173$ MHz;

$\bar{\nu}_z \approx 200$ MHz = oscillation frequency in axial direction;

Δg_{cav} = corrections due to oscillation modes in cavity

$\Rightarrow a_e^{\rm exp} = 0.00115965218073(28)$ [0.24 ppb ≈ 1 part in 4 billions]

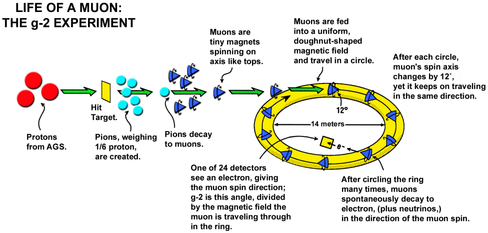
[Kusch & Foley, 1947/48: 4% precision]

Precision in $g_e/2$ even 0.28 ppt ≈ 1 part in 4 trillions !

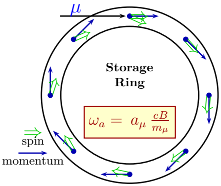
The Brookhaven Muon $g-2$ Experiment

The first measurements of the anomalous magnetic moment of the muon were performed in 1960 at CERN, $a_\mu^{\rm exp} = 0.00113(14)$ (Garwin et al.) [12% precision] and improved until 1979: $a_\mu^{\rm exp} = 0.0011659240(85)$ [7 ppm] (Bailey et al.)

In 1997, a new experiment started at the Brookhaven National Laboratory (BNL):



Source: BNL Muon $g-2$ homepage



actual precession × 2

Angular frequencies for cyclotron precession ω_c and spin precession ω_s:

$$\omega_c = \frac{eB}{m_\mu\,\gamma}\,, \quad \omega_s = \frac{eB}{m_\mu\,\gamma} + a_\mu\,\frac{eB}{m_\mu}\,, \quad \omega_a = a_\mu\,\frac{eB}{m_\mu}$$

$\gamma = 1/\sqrt{1-(v/c)^2}$. With an electric field to focus the muon beam one gets:

$$\vec{\omega}_a = \frac{e}{m_\mu}\left(a_\mu \vec{B} - \left[a_\mu - \frac{1}{\gamma^2-1} \right] \vec{v} \times \vec{E} \right)$$

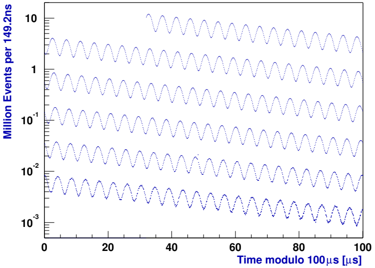
Term with $\vec{E}$ drops out, if $\gamma = \sqrt{1+1/a_\mu} = 29.3$: "magic γ" $\rightarrow p_\mu = 3.094$ GeV$/c$

The Brookhaven Muon $g-2$ Experiment: storage ring

Source: BNL Muon $g-2$ homepage

The Brookhaven Muon $g-2$ Experiment: determination of a_μ

Histogram with 3.6 billion decays of μ^-:



Bennett et al. 2006

$$N(t) = N_0(E)\,\exp\left(\frac{-t}{\gamma\tau_\mu}\right) \times [1 + A(E)\sin(\omega_a t + \phi(E))]$$

Exponential decay with mean lifetime: $\tau_{\mu,\text{lab}} = \gamma\tau_\mu = 64.378\mu s$ (in lab system).

Oscillations due to angular frequency $\omega_a = a_\mu eB/m_\mu$.

$$a_\mu = \frac{R}{\lambda - R} \text{ where } R = \frac{\omega_a}{\omega_p} \text{ and } \lambda = \frac{\mu_\mu}{\mu_p}$$

Brookhaven experiment measures ω_a and ω_p (spin precession frequency for proton). λ from hyperfine splitting of muonium (μ^+e^-) (external input).